The Fermi condensate near the saddle point and in the vortex core

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The case in which the Fermi surface is near the saddle point of the electron spectrum is favorable for formation of the fermionic condensate—a flat plateau in the quasiparticle energy at the Fermi level. This may explain the results of the angle-resolved photoemission experiments on YBaCuO and BiSrCaCuO in the normal state, which revealed an "extended saddle-point singularity"[1] or "flat band"[2] in the vicinity of the Fermi surface. The Fermi condensate can also appear in the core of quantized vortices.

Recently it was found (in the Hartree–Fock approximation) that at a large enough interaction the Landau Fermi liquid is unstable toward the Fermi condensate.[3-5] In this new state the particles with the Fermi energy form a three-dimensional (3D) flat band, instead of a 2D Fermi surface of conventional Fermi liquid. While the residual interaction can in principle lift this degeneracy, some topologically stable features can nevertheless be preserved.[6] On the other hand, recent photoemission experiments on high-T_c materials in the normal state revealed the existence of a flat band at the Fermi level.[2] This singular behavior was shown to occur in the vicinity of the saddle point of the electronic spectrum[1] and was interpreted as an extended saddle point singularity. In Sec. 1 we show that the case in which the Fermi surface is near the saddle point is the most favorable one for the formation of the fermionic condensate. It is possible that it manifests itself as a flat band in the photoemission experiments. In Sec. 2, we discuss an example of the 1D flat band at the Fermi level, which occurs in the core of some quantized vortices. The existence of this 1D Fermi condensate is supported by the symmetry and topology of the vortex.

1. Fermi condensate in the vicinity of the saddle point

For simplicity, following Ref. 4, we consider the extreme case of the contact interaction between the particles

$$E=\sum_{\vec{k}}\left(\xi_{\vec{k}}n_{\vec{k}}+\frac{1}{2}Un_{\vec{k}}^2\right), \tag{1.1}$$

while the noncontact part is absorbed into the quasiparticle spectrum $\xi_{\vec{k}}$. We assume that the contact interaction U is positive and small. Let us compare two different cases. (i) In the conventional isotropic case the bare quasiparticle energy is $\xi_{\vec{k}}=v_F(k-k_F)$. (ii) Near the saddle point we have $\xi_{\vec{k}}=(k_xk_y/m)-\mu$, where the chemical potential μ is counted from the saddle point. Here we assumed that the spectrum in the CuO_2 plane is a 2D spectrum; i.e., $\xi_{\vec{k}}$ does not depend on the momentum k_z along the c axis, and we ignored the anisotropy of the mass in the x,y plane.

Minimization of the energy in Eq. (1.1) gives the Fermi condensate: The Fermi surface $\xi_{\vec{k}}=0$, which appears at $U=0$, becomes smeared at $T=0$, producing a flat plateau between the two edge surfaces. On the plateau the quasiparticle energy is exactly zero:

$$\varepsilon_{\vec{k}}=\frac{\delta E}{\delta n_{\vec{k}}}=\xi_{\vec{k}}+Un_{\vec{k}}=0, \tag{1.2}$$

and the particle distribution, which follows from Eq. (1.2), is

$$n_{\vec{k}}=1/2-\frac{\xi_{\vec{k}}}{U}. \tag{1.3}$$

(Here we changed the chemical potential by an amount $U/2$ to have the same total number of particles as in the case $U=0$.) The plateau is limited by two surfaces, $\xi_{\vec{k}}=U/2$ and $\xi_{\vec{k}}=-U/2$, at which $n_{\vec{k}}$ reaches the limiting values 0 and 1. In the conventional isotropic case the width of the flat band is small, $\delta k=k_2-k_1=U/v_F$. This width can now be compared with the width σ of the quasiparticle interaction, which was assumed to be zero in Eq. (1.1). According to Ref. 4, the flat band exists only if σ is less than the critical value, $\sigma^*\sim U/v_F$. If $\sigma>\sigma^*$, the Fermi liquid behavior is restored.

Let us now consider the case of the saddle point, where in the $\sigma=0$ limit the Fermi condensate is concentrated in the region

$$\mu-\frac{1}{2}U<\frac{1}{m}k_xk_y<\mu+\frac{1}{2}U. \tag{1.4}$$

Let us also consider the effect of finite σ. For $|\mu|\gg U/2$, the maximal width of the Fermi condensate is $\delta k\sim U(m/|\mu|)^{1/2}$. When μ approaches the saddle point, this width increases and finally approaches the maximal value, $\delta k\sim(mU)^{1/2}$, when $|\mu|\sim U$. The increase in the width of the Fermi condensate in the vicinity of the saddle point makes it less vulnerable to the effect of finite σ. The critical value of σ is now much larger than in the isotropic case: $\sigma^*\sim(mU)^{1/2}\gg U/v_F$.

Let us now consider how the topology of the Lifshitz transition, which occurs when the chemical potential crosses the saddle point, changes when the Fermi condensate is taken into account. Without the Fermi condensate, i.e., at $\sigma>\sigma^*$, the transition takes place at one point $\mu=0$, where the reconstruction of the Fermi surface takes place (Fig. 1). If $\sigma<\sigma^*$, the spectrum can have four consecutive reconstructions. First, at $\mu_1\simeq mU^2/\sigma^2$ two Fermi condensates are formed in the vicinity of two Fermi surfaces (Fig. 2a). Then, at $\mu_2=U/2$, these two condensates merge (Fig. 2b). At $\mu_3=-U/2$, the condensates again become separated (Fig. 2d) and finally disappear at $\mu_4\simeq-mU^2/\sigma^2$.

2. One-dimensional Fermi condensate in the vortex core

Here we consider the 1D fermions which are localized in the core of the vortex in superfluids and superconductor and which were first discussed in Ref. 7. The energy $E(Q,k_z)$ of these fermions depends on the momentum k_z along the symmetry axis and on the quantum number Q, the eigenstate of the generator of the "axial" symmetry of the vortex.[8,9] Here Q is similar to the angular momentum. It can be either an integer or a half-integer, which depends on the type of the vortex and on the pairing state. The

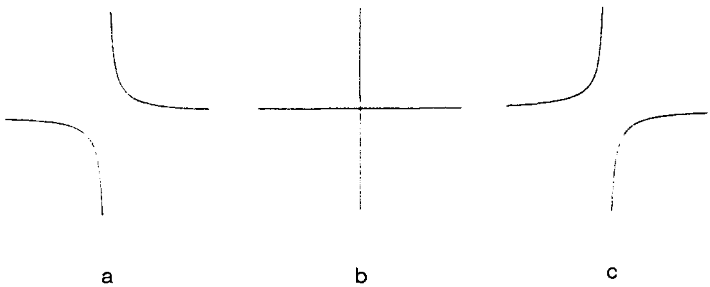

FIG. 1. Reconstruction of the Fermi surface in the conventional Lifshitz saddle-point transition: (a) before the transition; (b) the Fermi surface at the moment of crossing the saddle point; (c) after the transition.

spectrum contains anomalous branches which as functions of discrete Q cross zero energy. In a large class of vortices the energy of such a branch is odd in Q:

$$E(Q,k_z)=Q\epsilon(k_z), \tag{2.1}$$

and it changes sign together with Q. In an ordinary (s-wave) superconductor Q is a half-integer for an ordinary vortex with a single circulation quantum ($m=1$) and the lowest excitation energy corresponds to $Q=1/2$ and is $(1/2)\epsilon(k_z=0)\sim\Delta^2/E_F\ll\Delta$ (Ref. 7).

In general, Eq. (2.1) does not hold, but the main features are preserved. The existence of branches which cross zero as a function of Q is prescribed by the topological arguments[10] in a similar way as the existence of chiral fermions within the strings in

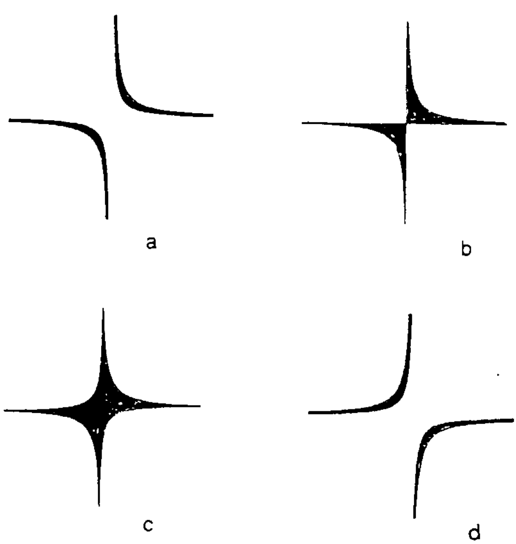

FIG. 2. Intermediate states with the Fermi condensate. (a) Two separated Fermi condensates (the blackened area) exist in the region $\mu_2<\mu<\mu_1$ of the chemical potential; (b) condensates merge at $\mu=\mu_2$ and (c) one condensate exists in the region $\mu_3<\mu<\mu_2$, which (d) splits again in the region $\mu_4<\mu<\mu_3$. At $\mu<\mu_4$ and $\mu>\mu_1$ there are pure Fermi surfaces in Figs. 1a and 1b, correspondingly.

relativistic theories:[11] The number of anomalous branches is defined by the winding number m of the vortex, $N=2m$ ($N=m$ for the ^{3}He–A$_1$, where the pair-correlated state contains only one spin component).

In some vortices Q is an integer. We see this, for example, in $m=2$ vortices in an ordinary (s-wave) superconductor, and in $m=1$ vortices in some non-s-wave systems.[9,12] In this case Q can be zero, and if we assume that Eq. (2.1) holds, we obtain an interesting result: all fermions with $Q=0$ have zero energy, $E(0,k_z)=0$; i.e., the absolutely flat 1D band with Fermi energy exists in the vortex. Such 1D Fermi condensate was found in the $m=1$ vortex of some special type in a ^{3}He–A superfluid.[12] Here we show that the existence of a flat band is not an artifact of the models used for calculation of the fermionic spectrum but is prescribed by the symmetry and topology of the vortex. The flat bands can exist in different p-wave and d-wave pair-correlated systems. Let us discuss the conditions under which the 1D Fermi condensate exists.

(i) The quantum number Q must be an integer for the fermionic quasiparticles. Let us consider the case in which this condition is satisfied. Here Q is the eigenvalue of the generator Q of the unbroken Abelian symmetry group of the axisymmetric vortices. This generator is different for s-wave superconductors and for the A, B and the planar phases of ^{3}He. It also depends on the circulation number m of the vortex:[8]

$$\mathrm{Q}_s=\mathrm{L}_z-m\mathrm{I}, \quad \mathrm{Q}_A=\mathrm{L}_z-(m+l_z)\mathrm{I}, \quad \mathrm{Q}_B=\mathrm{Q}_{\mathrm{planar}}=\mathrm{L}_z+\mathrm{S}_z-m\mathrm{I}. \tag{2.2}$$

Here L_z is the generator of the orbital rotations, which includes the internal (isotopic) rotation of the Cooper pair around the center of mass and the external rotation related to the motion of the center of mass, S_z is the generator of spin rotations; I is the generator of "gauge rotations," which takes the value 1/2 for a Bogoliubov–Nambu particle, $-1/2$ for a hole, and 1 for a Cooper pair; and l_z is the z component of the unit orbital vector $\hat{l}$ in bulk ^{3}He–A.

The vacuum state of the system with a given vortex has the quantum number $Q=0$, while the excitations (collective modes of the vortex and fermionic quasiparticles) are described by half-integer or integer values of Q. Since fermionic quasiparticles have $S_z=\pm 1/2$ and $I=\pm 1/2$, their Q take the values $k-(1/2)m$ in s-wave superconductors, $k-(1/2)(m+l_z)$ in ^{3}He–A, and $k+(1/2)(1-m)$ in ^{3}He–B and the planar state, where k is an integer. Therefore, in s-wave superconductors Q is an integer for fermions on vortices with even m and in ^{3}He–B and the planar state it is an integer for vortices with odd m. For ^{3}He–A Q is an integer for vortices with even $m+l_z$, which is precisely the case in the $m=1$ vortex considered in Ref. 12, since the $\hat{l}$ vector is oriented along the vortex axis. The same condition, $m+l_z$ is even, is satisfied for the $m=1$ vortex in the d-wave superconductor with the gap function $\propto k_z(k_x+ik_y)$, which is believed to be the case in heavy fermionic UPt_3 (see review[13]).

(ii) Some discrete symmetry should be satisfied. The symmetry group of the vacuum with a vortex line contains discrete symmetries, in addition to the continuous axial symmetry Q. These are the space inversion symmetry P and the combined TU_2 symmetry, which corresponds to a flipping of the vortex axis with a simultaneous time inversion: Circulation does not change under this combined operation. One more important symmetry is related to the Bogoliubov fermions: the symmetry under operation C of the

transformation of the Bogoliubov particle into the Bogoliubov hole. Transformation of the quasiparticle spectrum under these three operations is (see Ref. 9)

$$\mathrm{C}\, E(Q,k_z)=-E(-Q,-k_z), \quad \mathrm{P}\, E(Q,k_z)=E(Q,-k_z), \quad \mathrm{TU}_2\, E(Q,k_z) =E(Q,k_z).$$

The symmetry C is satisfied for any vortex state, while the P and TU_2 symmetries are often spontaneously broken in the vortex core.[8] We are interested only in vortices in which either the symmetry P or PTU_2 is conserved. This results in the equation $E(Q,k_z)=E(Q,-k_z)$. Applying the symmetry C, we obtain

$$E(Q,k_z)=-E(-Q,k_z). \tag{2.4}$$

This means that for each branch $E(Q,k_z)$ we can find the branch with opposite Q and E: $E_p(Q,k_z)=-E_q(-Q,k_z)$.

(iii) Chirality of fermions on vortices. In the ^{3}He$-$A$_1$ with one spin population, the $m=1$ vortex has only one low-energy branch ($N=1$). Therefore, Eq. (2.4) inevitably gives the flat band with $Q=0$: $E_\uparrow(0,k_z)=-E_\uparrow(0,k_z)=0$. The existence of only one branch which crosses zero as a function of Q is the result of the fact that the fermions on the low-energy branch are chiral: they have a positive E for a positive Q and a negative E for a negative Q. The chirality is the direct consequence of the topology and takes place only if $m\neq 0$.

Let us now verify that the flat band occurs for $m=1$ ^{3}He$-$A vortices which were discussed in Ref. 12. Conditions (i) and (ii) are satisfied because $m+l_z$ is either 2 or 0 for their vortex and the P symmetry is conserved. As for the condition (iii), there are two chiral branches ($N=2$). The equation $E_p(Q,k_z)=-E_q(-Q,k_z)$ therefore does not automatically produce the flat band. However, these chiral branches are degenerate in spin, $E_\uparrow(Q,k_z)=E_\downarrow(Q,k_z)$, if the small spin–orbit coupling is ignored. It therefore follows from Eq. (2.4) that $E_\uparrow(0,k_z)=E_\downarrow(0,k_z)=0$ for all k_z, which accounts for the fact that the flat zero mode calculated in Ref. 12 in a simple model survives any perturbation if it does not violate the vortex symmetry. The same 1D fermionic condensate should exist in the UPt_3 vortices if the order parameter in this heavy-fermion superconductor is correctly identified.

In summary, we discussed two physical systems in which the Fermi condensate is most likely to occur: the 3D condensate can arise in the metals if the Fermi surface is close to the saddle point of the electron spectrum and the 1D condensate can form in the cores of vortices in superfluids and superconductors if some moderately restrictive symmetries are satisfied.

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