

**About the article**  
**PARTICLE CONDUCTIVITY IN A TWO-DIMENSIONAL RANDOM POTENTIAL**  
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**Abstract**

These are the recollections about the circumstances under which this article was written and a discussion about the further development of the subject.

**I. An April First Story**

The story begun in late March 1979. The issue of Physical Review Letters with the paper [1] by Abrahams et al appeared in Chernogolovka, and I was able to read it. The paper made a very strong impression. At its four pages, the author painted a broad and universal picture. It is not surprising that their narration contained a number of gaps, which inspired a desire to feel these gaps.

Strongly excited, I discussed the issue - separately - with L.P. Gorkov and A.I. Larkin. This finally ended up with our joint paper, the only paper in which L.P. and A.I. were coauthors. After the simplest part - the calculation of the quantum correction to conductivity - had been performed, the question arose what is the next. And Tolya Larkin suggested that we must calculate the contribution to conductivity at low frequency  $\omega$ , which would be proportional to  $\log^2 \omega \tau$ . If the Renormalization group conjectured by the authors of Ref [1] are correct, then the coefficient at the front of  $\log^2 \omega \tau$  must vanish. The calculation turned out to be lengthy and tedious and took good two months. Finally, after numerous mistakes, I found a missing term, and the whole collection of different contributions collapsed cancelling each other. I showed this to Larkin and convinced him, and Tolya said to Gor'kov. Time of Summer vacations approached, and we decided to write a text of the paper, for which I carefully checked all the coefficients.

When I returned from my white-water rafting trip to Sayan Mountains, I discovered that the manuscript of our paper - written a months ago - was brought to Editorial office of JETP Letters at the last week. As Tolya explained to me, Gor'kov was busy. When he got time, he took several pages of clean paper and reproduced all the calculations in order to check the formulae, written in our draft. All the coefficients coincided, and the manuscript went to the typists and further to the Journal. The article begun with the sacral words that the aim of the authors was to understand the validity of the claims made by the authors of Ref [1].

**II. Fast Development**

Development begun for us with the Soviet-American Symposium, hold on 1-21 September 1979 at Sevan Lake in Armenia. This was a response to the similar Symposium hold at Aspen, Colorado in the Summer 1977. American delegation consisted of J.R. Schrieffer, J. Sak, A. Lüther, C. Pethick, T.C. Lubensky, P.A. Lee, D.S. Fisher, S. Kirkpatrick, M.J. Stephen, K. Maki, G. Mazenko, J. Hertz, W.F. Brinkman. As for Soviet group, it was much larger and included both the veterans and very young theorists. Among the youngest, one yet PhD students Boris Altshuler from Leningrad was there. Unfortunately, Arkady Aronov could not come. All participants live in an empty Hotel, belonging to the Council of Ministers of Armenia, which was staying just at the Lake shore. Nobody was around. Armenian early autumn - dry and warm - bare hills around and the lake at 2000 meters above sea level, an opportunity of a very informal contacts - all created unique atmosphere, I cannot forget even now, 35 years later.

From the conversation - on my very bad English - with Americans, I understood that the results of calculation of the quantum correction to conductivity, reported in our paper, is known to them, although

the relevant article [2] was not yet published<sup>1</sup>. Still, there were no agreement among the American theorists whether the Renormalization group, conjectured by the authors of Ref [1], is valid. The most outspoken was Patrick Lee, who reported at Sevan the results of his numerical studies [3]. Patrick claimed that the results of these calculations demonstrate that the G4-conjecture is wrong<sup>2</sup>. In light of these doubts, the calculation of the  $\log^2$  terms in correction to conductivity and the claim that these terms cancel each other, in agreement with the G4-conjecture, sounded a revelation. Being away from the libraries, we did not know about the paper [4] by Wegner<sup>3</sup>, in which he conjectured the new Field Theory for Localization - the Non-Linear  $\sigma$ -Model. It was known for some time, that the models of this kind are renormalizable and the Renormalization group equation coincides with that conjectured by G4.

Wegner's paper reduced, therefore, the meaning of our efforts to check the validity of G4-conjecture to a shire banality. Still, the ability to calculate the quantum correction brought numerous benefits. First of all, it was discovered at Sevan that the quantum correction to conductivity is extremely sensitive to magnetic field. As the result, although the quantum correction is small in comparison with the total value of conductivity, its dependence on magnetic field determines the dependence of the whole conductivity, leading to negative magnetoresistance. This result was later published in the paper [5] co-authored by Boris Altshuler, Patrick Lee, Larkin and me.

On next month, during his visit to Japan, Larkin, together with Hikami and Nagaoka, studied effect of spin-orbit interaction. These authors showed [6] that the spin-orbit interaction leads to inversion of the sign of the quantum correction to conductivity and to transformation of the negative magneto-resistance into positive one. It turned out that the study of magneto-resistance at low temperatures provide the best way of observing the quantum corrections. It also provided a new method of spectroscopy of different rates of relaxation: of the spin and of the phase of electron wave function. Another outcome of our Sevan discovery was a short paper [7] by Larkin and me, in which we determined the shift of critical concentration of impurities for the metal-insulator transition in weak magnetic field.

Necessity to present the results of the theory to the broad community of experimentalists demanded a different language, which had been developed not immediately. I gave several talks [8] on this subject, in which new qualitative language had been exposed. Qualitative picture turned out to be very useful for theoretical work. I would especially mentioned here our papers [9] with Arkady Aronov and Boris Altshuler about effect of high-frequency electromagnetic field on conductivity and about rate of dephasing of electrons in disordered conductors.

In conclusion, the lessons brought by this paper is that, if you have a question you don't know the answer to, try to work out the answer. An immediate result could be not so spectacular. Still, this allows to you to see a broader picture and ask new questions.

## References

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<sup>1</sup>I guess, would Gor'kov, Larkin and I know about this, our paper would never been written.

<sup>2</sup>Patrick begun his talk at Sevan by showing several Chinese hieroglyphes and explaining that this script means "Down the Gang of Four". This is where the slogan G4 for the authors of the paper [1] came from.

<sup>3</sup>It is important to remind to modern readers, that the personal computers were unknown in 1979 as well as electronic mail, Internet, and all that

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