# The paper "Line and point singularities in superfluid He$^3$" (G.E.Volovik and V.P.Mineev, JETP Lett. 24, 562 (1976))

The discovery of superfluid phases of liquid $^3$He in 1972 put forward the problem of mathematical description of vortices in these superfluids. Vortices in a fluid are characterized by circulation of velocity $\mathbf{v}$ given by integral $\oint_\gamma \mathbf{v}d\mathbf{l}$ over closed contour $\gamma$. There was known that unlike to ordinary fluids where the circulation of velocity can take an arbitrary value the circulation of superfluid velocity in superfluid $^4$He is quantized being equal to $N\frac{h}{m}$. Here, $h$ is the Planck constant and $m$ is mass of $^4$He atom. Hence, the vortices differ each other by the integer number of circulation quanta N. This in particular means that vortex lines are either closed or terminated at the walls or on the free surface of the helium.

The situation in superfluid $^3$He has proved to be different. There was shown that the superfluid $^3$He-A admits the existence of the vortex lines with free ends[1]. The superfluid velocity field around such vortices coincides with the field of the vector potential of the Dirac monopole[2]. This astonishing theoretical discovery pointed out that one need to search some general mathematical approach to description of singular and nonsingular order parameter distributions in superfluid phases of $^3$He. This was done in the paper by Volovik and Mineev[3].

The idea is simple: the mathematical description of order parameter distributions is given in terms of mappings between real space filled by an ordered media for example by the superfluid $^3$He and the space of order parameter variations called *the space of degeneracy* which leave the energy invariant but not the state of the superfluid. The stable singularities and rules of their coalescence were classified in correspondence with elements of homotopy group of the particular space of degeneracy and the rules of their multiplications. Unlike to similar approach developed at the same time by french scientists G.Toulouse and M. Kleman[4] in the paper[3] there was stressed that the topological stability determined by the energy of relevant interactions. The latter are different at different space scales. As result the types of topologically stable defects is also scale dependent.

Most of the exotic singular and nonsingular orders in superfluid phases of $^3$He described theoretically in the 1976 have been experimentally discovered in the 1980s and 1990s by means of Nuclear Magnetic Resonance on liquid helium under rotation[5,6]. Among the more recent experimental achievements based on the predictions done in the paper[3] it is necessary to mention the discoveries of half-quantum vortices: in mesoscopic samples of spin-triplet superconductor $Sr_2RuO_4$[7], in exciton-polariton condensate[8], in antiferromagnetic spinor Bose-Einstein condensate[9], in the polar phase of superfluid $^3$He[10].

Passed about four decades since the development of the topological approach to the classification of defects in ordered media. The use of topology for the treatment of unusually complex ordering in superfluid phases of $^3$He was innovative and offered new areas of applications. As it was with other mathematical tools, topological methods have been proved very effective for the description of many phenomena in different branches of physics. Chern classes, skyrmions and instantons are encountered in theory of quantum Hall effect and in quantum field theory. Monopole-like objects have been observed in liquid crystals and in spin ice media and were discovered recently in the Bose-Einstein condensate of cold gas of $^{87}$Rb atoms[11]. The braid groups are applied in theory of quantum computers. Another new and vast area of topological applications is opened with discovery of so called topological insulators and theoretical studies of topological superfluids and superconductors[12].

---


[1] G. E. Volovik and V. P. Mineev, Pis'ma Zh.Exp. Teor. Fiz. **23**, 647 (1976)[JETP Lett. **23**, 593 (1976)].
[2] P.A.M.Dirac, Proc. R.Soc. A **133**, 60 (1931).
[3] G. E. Volovik and V. P. Mineev, Pis'ma Zh.Exp. Teor. Fiz. **24**, 605 (1976)[JETP Lett. **24**, 562 (1976)].
[4] G. Toulouse and M. Kleman, J. de Phys. Lettr. **37**, L-149 (1076).
[5] M. M. Salomaa, G. E.Volovik, Rev. Mod. Phys. **59**, 533 (1987).
[6] O. V. Lounasmaa, E. Thuneberg, Proc. Natl. Acad. Sci. **96**, 7760 (1999).
[7] J.Jang, D. G. Ferguson, V. Vakaryuk et al., Science **331**, 186 (2011).
[8] K. G. Lagourdakis, T.Ostatnicky, A.V. Kavokin et al., Science **326** 974 (2009).
[9] Sang Wong Seo, Seji Kang, Woo Jin Kwon, and Yong-il Shin, Phys. Rev.Lett. **115**, 015301 (2015).
[10] S.Autti, V. V. Dmitriev,V.B. Eltsov et al, arXiv:1508.02197[cond-mat.]
[11] M. W. Ray, E. Ruokokoski, S.Kandel et al, Nature **505**, 657 (2014).
[12] T. Mizushima, Y. Tsutsumi, T. Kawakami et al, arXiv:1508.00787 [cond-mat.]