

Supplemental material to the article

“Full replica symmetry breaking in p -spin-glass-like systems”

I. Free energy expansion. To describe the RSB-solution near the bifurcation point temperature T_0 , where RS “transforms” into RSB, we expand the expression for the free energy up to the fourth order of $\delta q^{\alpha\beta}$. The deviation $\Delta F(p)$ of the free energy from its RS-part is:

$$\begin{aligned}
 \frac{\Delta F(p)}{NkT} = \lim_{n \rightarrow 0} \frac{1}{n} & \left\{ \frac{t^2 p(p-1)}{4} q_{\text{RS}}^{(p-2)} [\lambda(p)_{(\text{RS}) \text{ repl}}] \sum'_{\alpha, \beta} (\delta q^{\alpha\beta})^2 - \frac{t^4}{2} L(p) \sum'_{\alpha, \beta, \delta} \delta q^{\alpha\beta} \delta q^{\alpha\delta} - t^6 \left[B_2(p) \sum'_{\alpha, \beta, \gamma, \delta} \delta q^{\alpha\beta} \delta q^{\alpha\gamma} \delta q^{\beta\delta} + \right. \right. \\
 & + B'_2(p) \sum'_{\alpha, \beta, \gamma, \delta} \delta q^{\alpha\beta} \delta q^{\alpha\gamma} \delta q^{\alpha\delta} + B_3(p) \sum'_{\alpha, \beta, \gamma} \delta q^{\alpha\beta} \delta q^{\beta\gamma} \delta q^{\gamma\alpha} + B'_3(p) \sum'_{\alpha, \beta, \gamma} (\delta q^{\alpha\beta})^2 \delta q^{\alpha\gamma} + B_4(p) \sum'_{\alpha, \beta} (\delta q^{\alpha\beta})^3 \left. \right] + \\
 & + t^8 \left[D_2(p) \sum'_{\alpha, \beta} (\delta q^{\alpha\beta})^4 + D_{31}(p) \sum'_{\alpha, \beta, \gamma} (\delta q^{\alpha\beta})^3 \delta q^{\alpha\gamma} + D_{32}(p) \sum'_{\alpha, \beta, \delta} (\delta q^{\alpha\beta})^2 (\delta q^{\alpha\delta})^2 + D_{33}(p) \sum'_{\alpha, \beta, \gamma} (\delta q^{\alpha\beta})^2 \delta q^{\alpha\gamma} \delta q^{\gamma\beta} + \right. \\
 & + D_{42}(p) \sum'_{\alpha, \beta, \gamma, \delta} (\delta q^{\alpha\beta})^2 \delta q^{\alpha\gamma} \delta q^{\alpha\delta} + D_{43}(p) \sum'_{\alpha, \beta, \gamma, \delta} (\delta q^{\alpha\beta})^2 \delta q^{\alpha\gamma} \delta q^{\beta\delta} + D_{45}(p) \sum'_{\alpha, \beta, \gamma, \delta} (\delta q^{\alpha\beta})^2 \delta q^{\alpha\gamma} \delta q^{\gamma\delta} + \\
 & + D_{46}(p) \sum'_{\alpha, \beta, \gamma, \delta} \delta q^{\alpha\beta} \delta q^{\alpha\gamma} \delta q^{\alpha\delta} \delta q^{\beta\gamma} + D_{47}(p) \sum'_{\alpha, \beta, \gamma, \delta} \delta q^{\alpha\beta} \delta q^{\beta\gamma} \delta q^{\gamma\delta} \delta q^{\delta\alpha} + D_{53}(p) \sum'_{\alpha, \beta, \gamma, \delta, \mu} \delta q^{\alpha\beta} \delta q^{\alpha\gamma} \delta q^{\alpha\delta} q^{\alpha\mu} + \\
 & \left. + D_{54}(p) \sum'_{\alpha, \beta, \gamma, \delta, \mu} \delta q^{\alpha\beta} \delta q^{\alpha\gamma} \delta q^{\alpha\delta} q^{\beta\mu} + D_{55}(p) \sum'_{\alpha, \beta, \gamma, \delta, \mu} \delta q^{\alpha\beta} \delta q^{\alpha\gamma} \delta q^{\gamma\delta} q^{\delta\mu} \right\}, \tag{1}
 \end{aligned}$$

where the coefficients are given below:

$$\begin{aligned}
 L(p) &= \left[\frac{p(p-1)}{2} q_{\text{RS}}^{(p-2)} \right]^2 \langle \hat{U}_1^2 \hat{U}_2 \hat{U}_3 \rangle - \langle \hat{U}_1 \hat{U}_2 \hat{U}_3 \hat{U}_4 \rangle = \\
 &= \left[\frac{p(p-1)}{2} q_{\text{RS}}^{(p-2)} \right]^2 \int dz^G \left[\frac{\text{Tr} \hat{U} e^{\hat{\theta}_{\text{RS}}}}{\text{Tr} e^{\hat{\theta}_{\text{RS}}}} \right]^2 \left\{ \frac{\text{Tr} (\hat{U}^2 e^{\hat{\theta}_{\text{RS}}})}{\text{Tr} e^{\hat{\theta}_{\text{RS}}}} - \left[\frac{\text{Tr} \hat{U} e^{\hat{\theta}_{\text{RS}}}}{\text{Tr} e^{\hat{\theta}_{\text{RS}}}} \right]^2 \right\}; \tag{2}
 \end{aligned}$$

$$\begin{aligned}
 t^6 B_4(p) &= t^6 \left[\frac{p(p-1)}{2} q_{\text{RS}}^{(p-2)} \right]^3 \left\{ \frac{1}{3} \langle \hat{U}_1 \hat{U}_2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \hat{U}_6 \rangle - \langle \hat{U}_1^2 \hat{U}_2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \rangle + \frac{1}{3} \langle \hat{U}_1^3 \hat{U}_2 \hat{U}_3 \hat{U}_4 \rangle + \right. \\
 & \left. + \frac{3}{4} \langle \hat{U}_1^2 \hat{U}_2^2 \hat{U}_3 \hat{U}_4 \rangle - \frac{1}{2} \langle \hat{U}_1^3 \hat{U}_2^2 \hat{U}_3 \rangle + \frac{1}{12} \langle \hat{U}_1^3 \hat{U}_2^3 \rangle \right\} - \frac{t^2 p(p-1)(p-2)}{12} q_{\text{RS}}^{(p-3)} \left[1 - t^2 \frac{3}{2} W \right]; \tag{3}
 \end{aligned}$$

$$W(p) = \left[\frac{p(p-1)}{2} q_{\text{RS}}^{(p-2)} \right] \left\{ \langle \hat{U}_1^2 \hat{U}_2^2 \rangle - 2 \langle \hat{U}_1^2 \hat{U}_2 \hat{U}_3 \rangle + \langle \hat{U}_1 \hat{U}_2 \hat{U}_3 \hat{U}_4 \rangle \right\}; \tag{4}$$

$$B_3(p) = \left[\frac{p(p-1)}{2} q_{\text{RS}}^{(p-2)} \right]^3 \left\{ \frac{1}{6} \langle \hat{U}_1^2 \hat{U}_2^2 \hat{U}_3 \rangle - \frac{1}{2} \langle \hat{U}_1^2 \hat{U}_2^2 \hat{U}_3 \hat{U}_4 \rangle - \frac{1}{6} \langle \hat{U}_1 \hat{U}_2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \hat{U}_6 \rangle + \frac{1}{2} \langle \hat{U}_1^2 \hat{U}_2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \rangle \right\}; \tag{5}$$

$$\begin{aligned}
 D_2(p) &= - \left[\frac{p(p-1)}{2} q_{\text{RS}}^{(p-2)} \right]^4 \frac{1}{48} \int dz^G \left\{ \frac{\text{Tr} (\hat{U}^4 e^{\hat{\theta}_{\text{RS}}})}{\text{Tr} e^{\hat{\theta}_{\text{RS}}}} - 3 \left[\frac{\text{Tr} \hat{U}^2 e^{\hat{\theta}_{\text{RS}}}}{\text{Tr} e^{\hat{\theta}_{\text{RS}}}} \right]^2 - \right. \\
 & - 6 \left[\frac{\text{Tr} \hat{U} e^{\hat{\theta}_{\text{RS}}}}{\text{Tr} e^{\hat{\theta}_{\text{RS}}}} \right]^4 + 12 \left[\frac{\text{Tr} \hat{U} e^{\hat{\theta}_{\text{RS}}}}{\text{Tr} e^{\hat{\theta}_{\text{RS}}}} \right]^2 \left[\frac{\text{Tr} \hat{U}^2 e^{\hat{\theta}_{\text{RS}}}}{\text{Tr} e^{\hat{\theta}_{\text{RS}}}} \right] - 4 \left[\frac{\text{Tr} \hat{U} e^{\hat{\theta}_{\text{RS}}}}{\text{Tr} e^{\hat{\theta}_{\text{RS}}}} \right] \left[\frac{\text{Tr} \hat{U}^3 e^{\hat{\theta}_{\text{RS}}}}{\text{Tr} e^{\hat{\theta}_{\text{RS}}}} \right] \left. \right\}^2 + \frac{p(p-1)(p-2)(p-3)}{32t^6} q_{\text{RS}}^{p-4} -
 \end{aligned}$$

$$\begin{aligned}
& - \left[\frac{p(p-1)}{4t^2} q_{RS}^{p-3} \right]^2 \frac{(p-2)(7p-18)}{12} \left\{ \langle \hat{U}_1^2 \hat{U}_2^2 \rangle - 2 \langle \hat{U}_1^2 \hat{U}_2 \hat{U}_3 \rangle + \langle \hat{U}_1 \hat{U}_2 \hat{U}_3 \hat{U}_4 \rangle \right\} - \\
& - \frac{3}{16t^2} [p(p-1)q_{RS}^{p-2}]^2 [p(p-1)(p-2)q_{RS}^{p-3}] \left\{ \frac{1}{3} \langle \hat{U}_1 \hat{U}_2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \hat{U}_6 \rangle - \langle \hat{U}_1^2 \hat{U}_2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \rangle + \frac{1}{3} \langle \hat{U}_1^3 \hat{U}_2 \hat{U}_3 \hat{U}_4 \rangle + \right. \\
& \quad \left. + \frac{3}{4} \langle \hat{U}_1^2 \hat{U}_2^2 \hat{U}_3 \hat{U}_4 \rangle - \frac{1}{2} \langle \hat{U}_1^3 \hat{U}_2^2 \hat{U}_3 \rangle + \frac{1}{12} \langle \hat{U}_1^3 \hat{U}_2^3 \rangle \right\}; \tag{6}
\end{aligned}$$

$$\begin{aligned}
D_{33}(p) = & - \left[\frac{p(p-1)}{2} q_{RS}^{(p-2)} \right]^4 \left[\frac{1}{4} \langle \hat{U}_1^3 \hat{U}_2^3 \hat{U}_3^2 \rangle + \frac{1}{4} \langle \hat{U}_1^3 \hat{U}_2^3 \hat{U}_3 \hat{U}_4 \rangle + \frac{3}{2} \langle \hat{U}_1^3 \hat{U}_2^2 \hat{U}_3^2 \hat{U}_4 \rangle - \frac{5}{2} \langle \hat{U}_1^3 \hat{U}_2^2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \rangle - \frac{9}{4} \langle \hat{U}_1^2 \hat{U}_2^2 \hat{U}_3^2 \hat{U}_4 \hat{U}_5 \rangle + \right. \\
& \quad \left. + \langle \hat{U}_1^3 \hat{U}_2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \hat{U}_6 \rangle + \frac{21}{4} \langle \hat{U}_1^2 \hat{U}_2^2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \hat{U}_6 \rangle - 4 \langle \hat{U}_1^2 \hat{U}_2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \hat{U}_6 \hat{U}_7 \rangle + \langle \hat{U}_1 \hat{U}_2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \hat{U}_6 \hat{U}_7 \hat{U}_8 \rangle \right] - \\
& - \frac{3}{16t^2} [p(p-1)q_{RS}^{p-2}]^2 [p(p-1)(p-2)q_{RS}^{p-3}] \left\{ \frac{1}{6} \langle \hat{U}_1^2 \hat{U}_2^2 \hat{U}_3^2 \rangle - \frac{1}{2} \langle \hat{U}_1^2 \hat{U}_2^2 \hat{U}_3 \hat{U}_4 \rangle - \frac{1}{6} \langle \hat{U}_1 \hat{U}_2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \hat{U}_6 \rangle + + \right. \\
& \quad \left. + \frac{1}{2} \langle \hat{U}_1^2 \hat{U}_2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \rangle \right\}; \tag{7}
\end{aligned}$$

$$D_{47}(p) = - \left[\frac{p(p-1)}{2} q_{RS}^{(p-2)} \right]^4 \frac{1}{8} \int dz^G \left\{ \frac{\text{Tr} \left(\hat{U}^2 e^{\hat{\theta}_{RS}} \right)}{\text{Tre}^{\hat{\theta}_{RS}}} - \left[\frac{\text{Tr} \hat{U} e^{\hat{\theta}_{RS}}}{\text{Tre}^{\hat{\theta}_{RS}}} \right]^2 \right\}^4. \tag{8}$$

We have also,

$$\begin{aligned}
D_{31}(p) = & \left[\frac{p(p-1)}{2} q_{RS}^{(p-2)} \right]^4 \left[-\frac{1}{6} \langle \hat{U}_1^4 \hat{U}_2^3 \hat{U}_3 \rangle + \frac{1}{2} \langle \hat{U}_1^4 \hat{U}_2^2 \hat{U}_3 \hat{U}_4 \rangle + \frac{2}{3} \langle \hat{U}_1^3 \hat{U}_2^3 \hat{U}_3 \hat{U}_4 \rangle + \frac{1}{2} \langle \hat{U}_1^3 \hat{U}_2^2 \hat{U}_3^2 \hat{U}_4 \rangle - 4 \langle \hat{U}_1^3 \hat{U}_2^2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \rangle - \right. \\
& \quad \left. - \frac{3}{2} \langle \hat{U}_1^2 \hat{U}_2^2 \hat{U}_3^2 \hat{U}_4 \hat{U}_5 \rangle - \frac{1}{3} \langle \hat{U}_1^4 \hat{U}_2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \rangle + 7 \langle \hat{U}_1^2 \hat{U}_2^2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \hat{U}_6 \rangle + \frac{7}{3} \langle \hat{U}_1^3 \hat{U}_2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \hat{U}_6 \rangle - \right. \\
& \quad \left. - 7 \langle \hat{U}_1^2 \hat{U}_2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \hat{U}_6 \hat{U}_7 \rangle + 2 \langle \hat{U}_1 \hat{U}_2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \hat{U}_6 \hat{U}_7 \hat{U}_8 \rangle \right] - \frac{2(p-2)(p-3)}{3t^4} \left[\frac{p(p-1)}{4} q_{RS}^{p-3} \right]^2 \left\{ \langle \hat{U}_1^2 \hat{U}_2 \hat{U}_3 \rangle - \langle \hat{U}_1 \hat{U}_2 \hat{U}_3 \hat{U}_4 \rangle \right\} - \\
& - \frac{3}{16t^2} [p(p-1)q_{RS}^{p-2}]^2 [p(p-1)(p-2)q_{RS}^{p-3}] \left\{ -\frac{2}{3} \langle \hat{U}_1 \hat{U}_2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \hat{U}_6 \rangle + \frac{5}{3} \langle \hat{U}_1^2 \hat{U}_2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \rangle - \frac{1}{3} \langle \hat{U}_1^3 \hat{U}_2 \hat{U}_3 \hat{U}_4 \rangle - \right. \\
& \quad \left. - \langle \hat{U}_1^2 \hat{U}_2^2 \hat{U}_3 \hat{U}_4 \rangle + \frac{1}{3} \langle \hat{U}_1^3 \hat{U}_2^2 \hat{U}_3 \rangle \right\}; \tag{9}
\end{aligned}$$

$$\begin{aligned}
D_{32}(p) = & \left[\frac{p(p-1)}{2} q_{RS}^{(p-2)} \right]^4 \left[-\frac{1}{8} \langle \hat{U}_1^4 \hat{U}_2^2 \hat{U}_3^2 \rangle + \frac{1}{4} \langle \hat{U}_1^4 \hat{U}_2^2 \hat{U}_3 \hat{U}_4 \rangle + \frac{3}{8} \langle \hat{U}_1^2 \hat{U}_2^2 \hat{U}_3^2 \hat{U}_4^2 \rangle + \frac{1}{2} \langle \hat{U}_1^3 \hat{U}_2^2 \hat{U}_3^2 \hat{U}_4 \rangle - \right. \\
& \quad \left. - \langle \hat{U}_1^3 \hat{U}_2^2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \rangle - \frac{9}{4} \langle \hat{U}_1^2 \hat{U}_2^2 \hat{U}_3^2 \hat{U}_4 \hat{U}_5 \rangle - \frac{1}{8} \langle \hat{U}_1^4 \hat{U}_2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \rangle + \frac{33}{8} \langle \hat{U}_1^2 \hat{U}_2^2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \hat{U}_6 \rangle + \frac{1}{2} \langle \hat{U}_1^3 \hat{U}_2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \hat{U}_6 \rangle - \right. \\
& \quad \left. - 3 \langle \hat{U}_1^2 \hat{U}_2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \hat{U}_6 \hat{U}_7 \rangle + \frac{3}{4} \langle \hat{U}_1 \hat{U}_2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \hat{U}_6 \hat{U}_7 \hat{U}_8 \rangle \right] - \frac{1}{2t^4} \left[\frac{p(p-1)(p-2)}{4} q_{RS}^{p-3} \right]^2 \left\{ \langle \hat{U}_1^2 \hat{U}_2 \hat{U}_3 \rangle - \langle \hat{U}_1 \hat{U}_2 \hat{U}_3 \hat{U}_4 \rangle \right\} - \\
& - \frac{3}{16t^2} [p(p-1)q_{RS}^{p-2}]^2 [p(p-1)(p-2)q_{RS}^{p-3}] \left\{ -\frac{1}{3} \langle \hat{U}_1 \hat{U}_2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \hat{U}_6 \rangle + \frac{5}{6} \langle \hat{U}_1^2 \hat{U}_2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \rangle - \frac{1}{6} \langle \hat{U}_1^3 \hat{U}_2 \hat{U}_3 \hat{U}_4 \rangle - \right. \\
& \quad \left. - \frac{1}{2} \langle \hat{U}_1^2 \hat{U}_2^2 \hat{U}_3 \hat{U}_4 \rangle + \frac{1}{6} \langle \hat{U}_1^3 \hat{U}_2^2 \hat{U}_3 \rangle \right\}; \tag{10}
\end{aligned}$$

$$D_{42}(p) = \left[\frac{p(p-1)}{2} q_{RS}^{(p-2)} \right]^4 \left[-\frac{1}{4} \langle \hat{U}_1^4 \hat{U}_2^2 \hat{U}_3 \hat{U}_4 \rangle + \frac{1}{4} \langle \hat{U}_1^4 \hat{U}_2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \rangle + \langle \hat{U}_1^3 \hat{U}_2^2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \rangle + \frac{3}{4} \langle \hat{U}_1^2 \hat{U}_2^2 \hat{U}_3^2 \hat{U}_4 \hat{U}_5 \rangle - \right.$$

$$\begin{aligned}
& -\frac{5}{4}\langle\hat{U}_1^2\hat{U}_2^2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\rangle - \langle\hat{U}_1^3\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\rangle + \frac{9}{2}\langle\hat{U}_1^2\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\hat{U}_7\rangle - \frac{3}{2}\langle\hat{U}_1\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\hat{U}_7\hat{U}_8\rangle \Big] - \\
& -\frac{3}{16t^2} [p(p-1)q_{\text{RS}}^{p-2}]^2 [p(p-1)(p-2)q_{\text{RS}}^{p-3}] \left\{ \frac{1}{3}\langle\hat{U}_1\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\rangle + \frac{1}{6}\langle\hat{U}_1^3\hat{U}_2\hat{U}_3\hat{U}_4\rangle - \frac{1}{2}\langle\hat{U}_1^2\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\rangle \right\}; \quad (11)
\end{aligned}$$

$$\begin{aligned}
D_{43}(p) &= \left[\frac{p(p-1)}{2} q_{\text{RS}}^{(p-2)} \right]^4 \left[-\frac{1}{4}\langle\hat{U}_1^3\hat{U}_2^3\hat{U}_3\hat{U}_4\rangle + \frac{3}{2}\langle\hat{U}_1^3\hat{U}_2^2\hat{U}_3\hat{U}_4\hat{U}_5\rangle - \frac{9}{4}\langle\hat{U}_1^2\hat{U}_2^2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\rangle - \right. \\
& \quad \left. - \langle\hat{U}_1^3\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\rangle + 3\langle\hat{U}_1^2\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\hat{U}_7\rangle - \langle\hat{U}_1\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\hat{U}_7\hat{U}_8\rangle \right] - \\
& -\frac{3}{16t^2} [p(p-1)q_{\text{RS}}^{p-2}]^2 [p(p-1)(p-2)q_{\text{RS}}^{p-3}] \left\{ \frac{1}{6}\langle\hat{U}_1\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\rangle - \frac{1}{3}\langle\hat{U}_1^2\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\rangle + \frac{1}{6}\langle\hat{U}_1^2\hat{U}_2^2\hat{U}_3\hat{U}_4\rangle \right\}; \quad (12)
\end{aligned}$$

$$\begin{aligned}
D_{45}(p) &= \left[\frac{p(p-1)}{2} q_{\text{RS}}^{(p-2)} \right]^4 \left[-\frac{1}{2}\langle\hat{U}_1^3\hat{U}_2^2\hat{U}_3^2\hat{U}_4\rangle + \langle\hat{U}_1^3\hat{U}_2^2\hat{U}_3\hat{U}_4\hat{U}_5\rangle + \frac{3}{2}\langle\hat{U}_1^2\hat{U}_2^2\hat{U}_3^2\hat{U}_4\hat{U}_5\rangle - 4\langle\hat{U}_1^2\hat{U}_2^2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\rangle - \right. \\
& \quad \left. - \frac{1}{2}\langle\hat{U}_1^3\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\rangle + \frac{7}{2}\langle\hat{U}_1^2\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\hat{U}_7\rangle - \langle\hat{U}_1\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\hat{U}_7\hat{U}_8\rangle \right] - \\
& -\frac{3}{16t^2} [p(p-1)q_{\text{RS}}^{p-2}]^2 [p(p-1)(p-2)q_{\text{RS}}^{p-3}] \left\{ \frac{1}{3}\langle\hat{U}_1\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\rangle - \frac{2}{3}\langle\hat{U}_1^2\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\rangle + \frac{1}{3}\langle\hat{U}_1^2\hat{U}_2^2\hat{U}_3\hat{U}_4\rangle \right\}; \quad (13)
\end{aligned}$$

$$\begin{aligned}
D_{46}(p) &= \left[\frac{p(p-1)}{2} q_{\text{RS}}^{(p-2)} \right]^4 \left[-\frac{1}{2}\langle\hat{U}_1^3\hat{U}_2^2\hat{U}_3^2\hat{U}_4\rangle + \langle\hat{U}_1^3\hat{U}_2^2\hat{U}_3\hat{U}_4\hat{U}_5\rangle + \frac{3}{2}\langle\hat{U}_1^2\hat{U}_2^2\hat{U}_3^2\hat{U}_4\hat{U}_5\rangle - 4\langle\hat{U}_1^2\hat{U}_2^2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\rangle - \right. \\
& \quad \left. - \frac{1}{2}\langle\hat{U}_1^3\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\rangle + \frac{7}{2}\langle\hat{U}_1^2\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\hat{U}_7\rangle - \langle\hat{U}_1\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\hat{U}_7\hat{U}_8\rangle \right]; \quad (14)
\end{aligned}$$

$$\begin{aligned}
D_{53}(p) &= \left[\frac{p(p-1)}{2} q_{\text{RS}}^{(p-2)} \right]^4 \left[-\frac{1}{24}\langle\hat{U}_1^4\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\rangle + \frac{1}{6}\langle\hat{U}_1^3\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\rangle + \frac{1}{8}\langle\hat{U}_1^2\hat{U}_2^2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\rangle - \right. \\
& \quad \left. - \frac{1}{2}\langle\hat{U}_1^2\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\hat{U}_7\rangle + \frac{1}{4}\langle\hat{U}_1\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\hat{U}_7\hat{U}_8\rangle \right]; \quad (15)
\end{aligned}$$

$$\begin{aligned}
D_{54}(p) &= \left[\frac{p(p-1)}{2} q_{\text{RS}}^{(p-2)} \right]^4 \left[-\frac{1}{2}\langle\hat{U}_1^3\hat{U}_2^2\hat{U}_3\hat{U}_4\hat{U}_5\rangle + \frac{1}{2}\langle\hat{U}_1^3\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\rangle + \frac{3}{2}\langle\hat{U}_1^2\hat{U}_2^2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\rangle - \right. \\
& \quad \left. - \frac{5}{2}\langle\hat{U}_1^2\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\hat{U}_7\rangle + \langle\hat{U}_1\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\hat{U}_7\hat{U}_8\rangle \right]; \quad (16)
\end{aligned}$$

$$\begin{aligned}
D_{55}(p) &= \left[\frac{p(p-1)}{2} q_{\text{RS}}^{(p-2)} \right]^4 \times \\
& \times \left[-\frac{1}{2}\langle\hat{U}_1^2\hat{U}_2^2\hat{U}_3^2\hat{U}_4\hat{U}_5\rangle + \frac{3}{2}\langle\hat{U}_1^2\hat{U}_2^2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\rangle - \frac{3}{2}\langle\hat{U}_1^2\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\hat{U}_7\rangle + \frac{1}{2}\langle\hat{U}_1\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\hat{U}_7\hat{U}_8\rangle \right]. \quad (17)
\end{aligned}$$