

## Supplemental Material to the article

# “Stability of the coexistence phase of chiral superconductivity and noncollinear spin ordering with nontrivial topology with regard to strong electron correlations”

**A. Spin and electron Green functions in the Hubbard-I approximation.** Let introduce the spin Green functions in the Matsubara representation:

$$D^{++}(x-x') = -\langle T_\tau X_f^{\uparrow\downarrow}(\tau) X_{f'}^{\downarrow\uparrow}(\tau') S(\beta) \rangle_{0,c}, \quad (1)$$

$$D^{+-}(x-x') = -\langle T_\tau X_f^{\uparrow\downarrow}(\tau) X_{f'}^{\uparrow\downarrow}(\tau') S(\beta) \rangle_{0,c}, \quad (2)$$

$$D^{-+}(x-x') = -\langle T_\tau X_f^{\downarrow\uparrow}(\tau) X_{f'}^{\downarrow\uparrow}(\tau') S(\beta) \rangle_{0,c}, \quad (3)$$

$$D^{--}(x-x') = -\langle T_\tau X_f^{\downarrow\uparrow}(\tau) X_{f'}^{\uparrow\downarrow}(\tau') S(\beta) \rangle_{0,c}, \quad (4)$$

where  $S(\beta = 1/T)$  is the temperature scattering matrix.

In the simplest loopless approximation the matrix function composed of the spin Green functions for the transformed Hamiltonian (10\*) (see the main text) can be presented in the form:

$$\begin{pmatrix} D^{++} & D^{+-} \\ D^{-+} & D^{--} \end{pmatrix} = \begin{pmatrix} G_0^{++} & G_0^{+-} \\ G_0^{-+} & G_0^{--} \end{pmatrix} \begin{pmatrix} 2M & 0 \\ 0 & -2M \end{pmatrix}, \quad (5)$$

where  $M$  is the amplitude of the magnetic order parameter. Then, the Dyson equation for the function  $\hat{G}_0$  in the loopless approximation is

$$\hat{G}_0 \equiv \begin{pmatrix} G_0^{++} & G_0^{+-} \\ G_0^{-+} & G_0^{--} \end{pmatrix} = \begin{pmatrix} G^{(0)} & 0 \\ 0 & -(G^{(0)})^* \end{pmatrix} + \begin{pmatrix} G^{(0)} & 0 \\ 0 & -(G^{(0)})^* \end{pmatrix} \begin{pmatrix} 2M & 0 \\ 0 & -2M \end{pmatrix} \frac{1}{2} \begin{pmatrix} A_q^+ & A_q^- \\ A_q^- & A_q^+ \end{pmatrix} \hat{G}_0, \quad (6)$$

where  $G^{(0)} = (i\omega_m - 2h_Q)^{-1}$ . Solving this equation we obtain

$$\hat{G}_0^{-1} = \begin{pmatrix} i\omega_m - 2h_Q - MA_q^+ & -MA_q^- \\ MA_q^- & i\omega_m + 2h_Q + MA_q^+ \end{pmatrix}, \quad (7)$$

where  $h_Q = -MJ_Q$ ,  $A_q^\pm = J_q \pm (J_{q-Q} + J_{q+Q})/2$  as in the main text.

The spin-wave spectrum is defined from the equation  $\det(\hat{G}_0^{-1}) = 0$  substituting  $i\omega_m \rightarrow \omega + i\delta$  and has a form:

$$\omega_{0q} = 2M\gamma_q = 2M\sqrt{(J_q - J_Q) \left[ \frac{J_{q-Q} + J_{q+Q}}{2} - J_Q \right]}. \quad (8)$$

The electron Green functions are defined by the expressions:

$$D_{\sigma 2, \sigma 2}(x-x') = -\langle T_\tau X_f^{\sigma 2}(\tau) X_{f'}^{2\sigma}(\tau') S(\beta) \rangle_{0,c}, \quad (9)$$

$$D_{\sigma 2, \bar{\sigma} 2}(x-x') = -\langle T_\tau X_f^{\sigma 2}(\tau) X_{f'}^{2\bar{\sigma}}(\tau') S(\beta) \rangle_{0,c}. \quad (10)$$

Applying the unitary transformation the expressions for the irreducible parts of the electron Green functions in the magnetic phase takes a form:

$$G_{\sigma 2, \sigma 2}(p, i\omega_n) = \frac{i\omega_n - \xi_p^+ + \eta_\sigma M (t_p^+ - J_Q)}{(i\omega_n - \varepsilon_{1p})(i\omega_n - \varepsilon_{2p})}, \quad G_{\sigma 2, \bar{\sigma} 2}(p, i\omega_n) = \frac{F_{\sigma 2} t_p^-}{(i\omega_n - \varepsilon_{1p})(i\omega_n - \varepsilon_{2p})}, \quad (11)$$

$$G_{2\sigma, 2\sigma}(p, i\omega_n) = -G_{\sigma 2, \sigma 2}(-p, -i\omega_n), \quad G_{2\sigma, 2\bar{\sigma}}(p, i\omega_n) = -G_{\sigma 2, \bar{\sigma} 2}(-p, -i\omega_n), \quad (12)$$

where  $D_{\sigma_2, \sigma_2}(p, i\omega_n) = G_{\sigma_2, \sigma_2}(p, i\omega_n)F_{\sigma_2}$ ,  $D_{\sigma_2, \bar{\sigma}_2}(p, i\omega_n) = G_{\sigma_2, \bar{\sigma}_2}(p, i\omega_n)F_{\bar{\sigma}_2}$  and so on. Here we have used the notation as in the main text:  $t_p^\pm = (t_{p-Q/2} \pm t_{p+Q/2})/2$  and introduced  $\xi_p^+ = \xi_0 + nt_p^+/2$ ,  $\xi_0 = \varepsilon + U - \mu + J_0(1 - n/2) + V_0n$ ,  $F_{\sigma_2} = \langle X_f^{\sigma\sigma} + X_f^{22} \rangle = n/2 + \eta_\sigma M$ . The Fermi spectrum in the magnetic phase is

$$\varepsilon_{1,2p} = \xi_p^\pm \mp \sqrt{(nt_p^-/2)^2 + M^2(t_{p-Q/2} - J_Q)(t_{p+Q/2} - J_Q)}. \quad (13)$$

The analytic expression obtained from the diagrammatic series for the average  $N_\uparrow = \langle X_f^{\uparrow\uparrow} \rangle$  (Fig. 1 in the main text) has a form:

$$N_\uparrow = \langle X_f^{\uparrow\uparrow} \rangle_0 + M \frac{T}{N} \sum_{qm} G^0(i\omega_m) [A_q^+ G_{++}(q, i\omega_m) - A_q^- G_{+-}(q, i\omega_m)] + \\ - \frac{T}{N} \sum_{pn} G_{\uparrow 2}^0(i\omega_n) [t_p^+ G_{\uparrow 2, \uparrow 2}(p, i\omega_n) F_{\uparrow 2} + t_p^- G_{\uparrow 2, \downarrow 2}(p, i\omega_n) F_{\downarrow 2}], \quad (14)$$

where  $G_{\uparrow 2}^{(0)}(i\omega_n) = (i\omega_n - \xi_\downarrow)^{-1}$  and  $\xi_\sigma = \xi_0 - \eta_\sigma h_Q$ . Calculating the sum over Matsubara frequencies we obtain the Eq. (11\*) (see the main text).

**B. Calculation of the topological invariant  $\tilde{N}_3$  and the excitation spectrum for the cylinder topology.** The matrix Green function for fermions in the coexistence phase of superconductivity and noncollinear magnetism has a form:

$$\hat{G}^{-1}(i\omega, p) = \\ = \begin{bmatrix} i\omega - \xi_p & -\Delta_p^* & -R_{p-Q} & 0 \\ -\Delta_p & i\omega + \xi_p & 0 & R_{p-Q} \\ -R_p & 0 & i\omega - \xi_{p-Q} & \Delta_{-p+Q}^* \\ 0 & R_p & \Delta_{-p+Q} & i\omega + \xi_{p-Q} \end{bmatrix}.$$

Substituting the obtained expression into the definition of the topological invariant  $\tilde{N}_3$  (13\*) (see the main text) and calculating the trace of the product of the matrices we obtain integral in the following form:

$$\tilde{N}_3 = \frac{i}{4\pi^2} \int_{-\infty}^{\infty} d\omega \int_{-\pi}^{\pi} dp_1 dp_2 \frac{a(p)\omega^4 - id(p)\omega^3 - b(p)\omega^2 + ig(p)\omega + c(p)}{(\omega^2 + E_{1p}^2)^2 (\omega^2 + E_{2p}^2)^2}. \quad (15)$$

In this form the integral over energy can be calculated analytically:

$$\tilde{N}_3 = -\frac{i}{8\pi} \int_{-\pi}^{\pi} dp_1 dp_2 \sum_{j=1,2} \left\{ \frac{3a(p)E_{jp}^4 + b(p)E_{jp}^2 - c(p)}{4E_{jp}^3 \nu_p^4} - (-1)^j \cdot \frac{a(p)E_{jp}^4 + b(p)E_{jp}^2 + c(p)}{2E_{jp} \nu_p^6} \right\}. \quad (16)$$

It should be noted that the terms proportional to the odd power of the energy in (15) are omitted as they do not contribute to the result. The coefficients  $d(p)$  and  $g(p)$  appear when the corrections to the exchange field due to the kinematic interaction are taken into account and they are proportional to  $\partial_j R_p$ ,  $\partial_j R_{p-Q}$ ,  $j = 1, 2$ .

The coefficient  $a(p)$  do not depend on the effective exchange field:

$$a(p) = [\xi_p \cdot \partial_1 \Delta_p^* \cdot \partial_2 \Delta_p + \xi_{p-Q} \cdot \partial_1 \Delta_{-p+Q}^* \cdot \partial_2 \Delta_{-p+Q} + \Delta_p \cdot (\partial_1 \xi_p \cdot \partial_2 \Delta_p^* - \partial_2 \xi_p \cdot \partial_1 \Delta_p^*) + \\ + \Delta_{-p+Q} \cdot (\partial_1 \xi_{p-Q} \cdot \partial_2 \Delta_{-p+Q}^* - \partial_2 \xi_{p-Q} \cdot \partial_1 \Delta_{-p+Q}^*)] - [\text{h.c.}], \quad (17)$$

where h.c. is hermitian conjugate.

The remaining coefficients can be represented in the following form:

$$b(p) = A(p) \cdot R_p R_{p-Q} + B(p) + C(p) \cdot R_p + D(p) \cdot R_{p-Q}, \quad (18)$$

$$c(p) = \alpha(p) \cdot R_p^2 R_{p-Q}^2 + \beta(p) \cdot R_p R_{p-Q} + \gamma(p) - C(p) \cdot R_p^2 R_{p-Q} - D(p) \cdot R_p R_{p-Q}^2 + \eta(p) \cdot R_p + \zeta(p) R_{p-Q}, \quad (19)$$

where

$$\begin{aligned}
A(p) &= a(p) + [(\xi_p + \xi_{p-Q}) (\partial_1 \Delta_p^* \cdot \partial_2 \Delta_{-p+Q} - \partial_2 \Delta_p^* \cdot \partial_1 \Delta_{-p+Q}) + 3\xi_p \cdot \partial_1 \Delta_{-p+Q}^* \cdot \partial_2 \Delta_{-p+Q} + \\
&+ 3\xi_{p-Q} \cdot \partial_1 \Delta_p^* \cdot \partial_2 \Delta_p + 3\Delta_{-p+Q} (\partial_1 \xi_p \cdot \partial_2 \Delta_p^* - \partial_2 \xi_p \cdot \partial_1 \Delta_p^*) + \\
&+ 3\Delta_p (\partial_1 \xi_{p-Q} \cdot \partial_2 \Delta_{-p+Q}^* - \partial_2 \xi_{p-Q} \cdot \partial_1 \Delta_{-p+Q}^*) + \\
&+ (\Delta_p + \Delta_{-p+Q}) (\partial_1 \xi_p \cdot \partial_2 \Delta_{-p+Q}^* + \partial_1 \xi_{p-Q} \cdot \partial_2 \Delta_p^* - \partial_2 \xi_p \cdot \partial_1 \Delta_{-p+Q}^* - \partial_2 \xi_{p-Q} \cdot \partial_1 \Delta_p^*)] - \{h.c.\}, \quad (20)
\end{aligned}$$

$$\begin{aligned}
B(p) &= \{2(\xi_p^2 + |\Delta_p^2|) [\Delta_{-p+Q} (\partial_2 \xi_{p-Q} \cdot \partial_1 \Delta_{-p+Q}^* - \partial_1 \xi_{p-Q} \cdot \partial_2 \Delta_{-p+Q}^*) + \xi_{p-Q} \cdot \partial_1 \Delta_{-p+Q} \cdot \partial_2 \Delta_{-p+Q}^*] + \\
&+ 2(\xi_{p-Q}^2 + |\Delta_{-p+Q}^2|) [\Delta_p (\partial_2 \xi_p \cdot \partial_1 \Delta_p^* - \partial_1 \xi_p \cdot \partial_2 \Delta_p^*) + \xi_p \cdot \partial_1 \Delta_p \cdot \partial_2 \Delta_p^*]\} - \{h.c.\}, \quad (21)
\end{aligned}$$

$$\begin{aligned}
C(p) &= \{(\Delta_p \xi_{p-Q} - \Delta_{-p+Q} \xi_p) [\partial_1 R_{p-Q} (\partial_2 \Delta_p^* - \partial_2 \Delta_{-p+Q}^*) - \partial_2 R_{p-Q} (\partial_1 \Delta_p^* - \partial_1 \Delta_{-p+Q}^*)] - \\
&- \Delta_p \Delta_{-p+Q}^* [\partial_1 R_{p-Q} (\partial_2 \xi_p - \partial_2 \xi_{p-Q}) - \partial_2 R_{p-Q} (\partial_1 \xi_p - \partial_1 \xi_{p-Q})]\} - \{h.c.\}, \quad (22)
\end{aligned}$$

$$D(p) = C(p)|_{\partial_j R_{p-Q} \rightarrow \partial_j R_p}, \quad (23)$$

$$\begin{aligned}
\alpha(p) &= \{(\xi_p + \xi_{p-Q}) (\partial_1 \Delta_p^* \cdot \partial_2 \Delta_{-p+Q} - \partial_2 \Delta_p^* \cdot \partial_1 \Delta_{-p+Q}) - \xi_p \cdot \partial_1 \Delta_{-p+Q}^* \cdot \partial_2 \Delta_{-p+Q} - \xi_{p-Q} \cdot \partial_1 \Delta_p^* \cdot \partial_2 \Delta_p + \\
&+ (\Delta_p + \Delta_{-p+Q}) [\partial_1 \xi_p \cdot \partial_2 \Delta_{-p+Q}^* + \partial_1 \xi_{p-Q} \cdot \partial_2 \Delta_p^* - \partial_2 \xi_p \cdot \partial_1 \Delta_{-p+Q}^* - \partial_2 \xi_{p-Q} \cdot \partial_1 \Delta_p^*] \\
&+ \Delta_p (\partial_2 \xi_{p-Q} \cdot \partial_1 \Delta_{-p+Q}^* - \partial_1 \xi_{p-Q} \cdot \partial_2 \Delta_{-p+Q}^*) + \Delta_{-p+Q} (\partial_2 \xi_p \cdot \partial_1 \Delta_p^* - \partial_1 \xi_p \cdot \partial_2 \Delta_p^*) - \{h.c.\}, \quad (24)
\end{aligned}$$

$$\begin{aligned}
\beta(p) &= \{(\xi_p - \xi_{p-Q}) [(\xi_p^2 + |\Delta_p|^2) \cdot \partial_1 \Delta_{-p+Q} \cdot \partial_2 \Delta_{-p+Q}^* - (\xi_{p-Q}^2 + |\Delta_{-p+Q}|^2) \cdot \partial_1 \Delta_p \cdot \partial_2 \Delta_p^*] + \\
&+ (\xi_p \xi_{p-Q} + \Delta_p \Delta_{-p+Q}^*) (\xi_p + \xi_{p-Q}) (\partial_2 \Delta_p^* \partial_1 \Delta_{-p+Q} - \partial_1 \Delta_p^* \cdot \partial_2 \Delta_{-p+Q}) + \\
&+ (\xi_p^2 + |\Delta_p|^2) (\Delta_p - \Delta_{-p+Q}) (\partial_2 \xi_{p-Q} \cdot \partial_1 \Delta_{-p+Q}^* - \partial_1 \xi_{p-Q} \cdot \partial_2 \Delta_{-p+Q}^*) + \\
&+ (\xi_{p-Q}^2 + |\Delta_{-p+Q}|^2) (\Delta_p - \Delta_{-p+Q}) (\partial_1 \xi_p \partial_2 \Delta_p^* - \partial_2 \xi_p \partial_1 \Delta_p^*) + \\
&+ [(\xi_p^2 - |\Delta_p|^2 - \Delta_{-p+Q} \Delta_p^*) \Delta_{-p+Q} - (\xi_{p-Q}^2 + 2\xi_p \xi_{p-Q}) \Delta_p] (\partial_1 \xi_p \partial_2 \Delta_{-p+Q}^* - \partial_2 \xi_p \partial_1 \Delta_{-p+Q}^*) \\
&+ [(\xi_{p-Q}^2 - |\Delta_{-p+Q}|^2 - \Delta_p \Delta_{-p+Q}^*) \Delta_p - (\xi_p^2 + 2\xi_p \xi_{p-Q}) \Delta_{-p+Q}] (\partial_1 \xi_{p-Q} \partial_2 \Delta_p^* - \partial_2 \xi_{p-Q} \partial_1 \Delta_p^*) \\
&+ (\xi_p \Delta_{-p+Q} - \xi_{p-Q} \Delta_p) (\Delta_p + \Delta_{-p+Q}) (\partial_1 \Delta_p^* \cdot \partial_2 \Delta_{-p+Q}^* - \partial_2 \Delta_p^* \cdot \partial_1 \Delta_{-p+Q}^*) \\
&+ 2(\xi_p + \xi_{p-Q}) \Delta_p \Delta_{-p+Q}^* (\partial_1 \xi_p \cdot \partial_2 \xi_{p-Q} - \partial_2 \xi_p \cdot \partial_1 \xi_{p-Q})\} - \{h.c.\}, \quad (25)
\end{aligned}$$

$$\begin{aligned}
\gamma(p) &= \{(\xi_p^2 + |\Delta_p|^2)^2 \cdot [\xi_{p-Q} \cdot \partial_1 \Delta_{-p+Q}^* \cdot \partial_2 \Delta_{-p+Q} + \Delta_{-p+Q} (\partial_1 \xi_{p-Q} \cdot \partial_2 \Delta_{-p+Q}^* - \partial_2 \xi_{p-Q} \cdot \partial_1 \Delta_{-p+Q}^*)] + \\
&+ (\xi_{p-Q}^2 + |\Delta_{-p+Q}|^2)^2 \cdot [\xi_p \cdot \partial_1 \Delta_p^* \cdot \partial_2 \Delta_p + \Delta_p (\partial_1 \xi_p \cdot \partial_2 \Delta_p^* - \partial_2 \xi_p \cdot \partial_1 \Delta_p^*)]\} - \{h.c.\}, \quad (26)
\end{aligned}$$

$$\begin{aligned}
\eta(p) &= \{(\xi_p^2 + |\Delta_p|^2) (\xi_p \Delta_{-p+Q} - \xi_{p-Q} \Delta_p) (\partial_2 R_{p-Q} \cdot \partial_1 \Delta_{-p+Q}^* - \partial_1 R_{p-Q} \cdot \partial_2 \Delta_{-p+Q}^*) + \\
&+ (\xi_{p-Q}^2 + |\Delta_{-p+Q}|^2) (\xi_{p-Q} \Delta_p - \xi_p \Delta_{-p+Q}) (\partial_2 R_{p-Q} \cdot \partial_1 \Delta_p^* - \partial_1 R_{p-Q} \cdot \partial_2 \Delta_p^*) + \\
&+ (\xi_p^2 + |\Delta_p|^2) \Delta_p \Delta_{-p+Q}^* (\partial_1 \xi_{p-Q} \cdot \partial_2 R_{p-Q} - \partial_2 \xi_{p-Q} \cdot \partial_1 R_{p-Q}) + \\
&+ (\xi_{p-Q}^2 + |\Delta_{-p+Q}|^2) \Delta_p^* \Delta_{-p+Q} (\partial_1 \xi_p \cdot \partial_2 R_{p-Q} - \partial_2 \xi_p \cdot \partial_1 R_{p-Q})\} - \{h.c.\}, \quad (27)
\end{aligned}$$

$$\zeta(p) = \eta(p)|_{\partial_j R_{p-Q} \rightarrow \partial_j R_p}. \quad (28)$$

For the case of periodic boundary conditions along the direction  $\mathbf{a}_2$  of the triangular lattice we obtain the system of equations for the Green functions:

$$\begin{pmatrix} i\omega_n - \hat{\xi}_{k_2} & \hat{h}_{k_2 - Q_2}(Q_1) & \hat{0} & \hat{D}_{k_2} \\ \hat{h}_{k_2}(-Q_1) & i\omega_n - \hat{\xi}_{k_2 - Q_2} & -\hat{D}_{k_2 - Q_2} & \hat{0} \\ \hat{0} & -\hat{D}_{k_2 - Q_2}^\dagger & i\omega_n + \hat{\xi}_{k_2 - Q_2} & -\hat{h}_{k_2}(-Q_1) \\ \hat{D}_{k_2}^\dagger & \hat{0} & -\hat{h}_{k_2 - Q_2}(Q_1) & i\omega_n + \hat{\xi}_{k_2} \end{pmatrix} \cdot \begin{bmatrix} G_{\downarrow 2, \downarrow 2}(k_2, k_2; l'; i\omega_n) \\ G_{\uparrow 2, \downarrow 2}(k_2 - Q_2, k_2; l'; i\omega_n) \\ G_{\downarrow 2, \downarrow 2}(k_2 - Q_2, k_2; l'; i\omega_n) \\ G_{\uparrow 2, \downarrow 2}(k_2, k_2; l'; i\omega_n) \end{bmatrix} = \begin{bmatrix} \hat{\delta} \\ \hat{0} \\ \hat{0} \\ \hat{0} \end{bmatrix}, \quad (29)$$

where  $N_1$  by  $N_1$  ( $N_1$  is the number of sites along  $\mathbf{a}_1$  direction) matrices  $\hat{\xi}_{k_2}$ ,  $\hat{D}_{k_2}$  and  $\hat{h}_{k_2}(Q_1)$  have the form

$$\hat{\xi}_{k_2} = \begin{pmatrix} \xi_0 + F_2 t_{k_2} & F_2 T_{k_2} & 0 & 0 \\ F_2 T_{-k_2} & \ddots & \ddots & 0 \\ 0 & \ddots & \ddots & T_{k_2} \\ 0 & 0 & F_2 T_{-k_2} & \xi_0 + F_2 t_{k_2} \end{pmatrix}, \quad \hat{D}_{k_2} = - \begin{pmatrix} \Delta_{k_2}^* & \psi_{-k_2}^* & \Delta_{22}^* e^{ik_2} & 0 & 0 \\ \psi_{k_2}^* & \ddots & \ddots & \ddots & 0 \\ \Delta_{22}^* e^{-ik_2} & \ddots & \ddots & \ddots & \Delta_{22}^* e^{ik_2} \\ 0 & \ddots & \ddots & \ddots & \psi_{-k_2}^* \\ 0 & 0 & \Delta_{22}^* e^{-ik_2} & \psi_{k_2}^* & \Delta_{k_2}^* \end{pmatrix}.$$

$$\hat{h}_{k_2}(Q_1) = -M \begin{pmatrix} (t_{k_2} - J_Q) e^{iQ_1} & T_{k_2} e^{iQ_1} & 0 & 0 \\ T_{-k_2} e^{i2Q_1} & \ddots & \ddots & 0 \\ 0 & \ddots & \ddots & T_{k_2} e^{i(N_1 - 1)Q_1} \\ 0 & 0 & T_{-k_2} e^{iN_1 Q_1} & (t_{k_2} - J_Q) e^{iN_1 Q_1} \end{pmatrix}.$$

Here  $F_2 = n/2$  is the Hubbard renormalization,  $t_{k_2} = 2t_1 \cos(k_2)$ ,  $\Delta_{k_2} = 2\Delta_{21} \cos(k_2)$ ,  $T_{k_2} = t_1 (1 + \exp(ik_2))$ , and

$$\psi_{k_2} = \Delta_{21} \exp(i2\pi/3) (1 + \exp(i2\pi/3 + ik_2)) + \Delta_{22} \exp(i2\pi/3) (\exp(i2k_2) + \exp(i2\pi/3 - ik_2)).$$

Solving the Eq. (29) we obtain the excitation spectrum containing the Majorana mode and the structure of such mode as described in the main text.