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DYON CONDENSATION AND AHARONOV – BOHM EFFECT

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We derive the string representation of the Abelian Higgs theory in which dyons are condensed. It occurs that in such representation the topological interaction exists in the expectation value of the Wilson loop. Due to this interaction the dynamics of the string spanned on the Wilson loop is non-trivial.

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The method of abelian projections [1] is one of the popular approaches to the confinement problem [2] in non-abelian gauge theories. Numerous computer simulations of the lattice gluodynamics in the abelian projection (see, e.g. Refs. [3]) show that the vacuum of gluodynamics behaves as a dual superconductor [4]. The key role in the dual superconductor model of the Quantum chromodynamics (QCD) vacuum is played by abelian monopoles [1]. In the abelian projection quarks are electrically charged particles, and if monopoles are condensed the dual Abrikosov string carrying the electric flux is formed between quark and antiquark. Due to a non-zero string tension the quarks are confined by the linear potential.

The abelian monopole currents in gluodynamics are correlated [5] with (anti-)instantons. For the (anti-)self-dual fields the abelian monopoles become abelian dyons [6]. Moreover, in the vacuum of lattice gluodynamics the local correlator of the topological charge density and the product of the electric and magnetic currents is positive [7]. This means that the abelian monopoles have the electric charge. The sign of this electric charge coincides with the sign of the product of the magnetic charge and the topological charge density. Thus the infrared properties of the QCD in the abelian projection can be described by the Abelian Higgs model (AHM) in which dyons are condensed. The electric charge of the dyons fluctuates[2].

Note that there exists the model of the QCD vacuum [2] in which the *nonabelian* dyons are responsible for the confinement. The nonabelian dyons (as instantons) give rise to the abelian dyons in the abelian projection.

Below we study the properties of the Abrikosov – Nielsen – Olesen (ANO) strings in the abelian model in which dyons are condensed. We consider the abelian dyons which have a constant electric charge. This model can be a zero approximation for the realistic effective model of the QCD vacuum in which the electric charge of the condensed dyons fluctuates.

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[2] Note that according to the Schwinger quantization rule the electric charge e of the dyon is not fixed while magnetic charge g is quantized: $e_0 g \in 2\pi\mathcal{N}$, e_0 is an elementary electric charge of an external electric particle, see eq.(6).

We start with the following expression for the partition function in the Euclidian space–time[3]:

$$Z_{dyon} = \int \mathcal{D}A_\mu \mathcal{D}B_\mu \mathcal{D}\Phi \exp\Big\{ - \int d^4x\, \mathcal{L}_{dyon}(A,B,\Phi)\Big\}\,, \tag{1}$$

where the dyon Lagrangian is:

$$\mathcal{L}_{dyon}(A,B,\Phi) = \mathcal{L}_{gauge}(A,B) + 1/2|(\partial_\mu - ieA_\mu - igB_\mu)\Phi|^2 + \lambda(|\Phi|^2 - \eta^2)^2\,. \tag{2}$$

The field B_μ is the magnetic gauge potential, which is dual to the electric gauge potential A_μ, and Φ is the dyon field with the electric charge e and magnetic charge g. It was shown in [9], that it is possible to write the lagrangian in which both fields A_μ and B_μ are regular:

$$\mathcal{L}_{gauge}(A,B) = \frac{1}{2}[n\cdot(\partial\wedge A)]^2 + \frac{1}{2}[n\cdot(\partial\wedge B)]^2 +$$
$$+\frac{i}{2}[n\cdot(\partial\wedge A)]^\nu[n\cdot^*(\partial\wedge B)]_\nu - \frac{i}{2}[n\cdot(\partial\wedge B)]^\nu[n\cdot^*(\partial\wedge A)]_\nu\,,$$

where $[a\cdot(b\wedge c)]^\nu \equiv a_\mu(b^\mu c^\nu - b^\nu c^\mu)$, $[a\cdot^*(b\wedge c)]^\nu \equiv a_\mu \epsilon^{\mu\nu\alpha\beta}(b_\alpha c_\beta)$ and n_μ is an arbitrary unit four-vector, $n^2=1$.

The partition function (1) can be represented as the partition function of the AHM. The lagrangian $\mathcal{L}_{gauge}$ is invariant under the linear transformation of the fields A and B [9]:

$$\begin{pmatrix} A \\ B \end{pmatrix} \to \begin{pmatrix} A' \\ B' \end{pmatrix} = \begin{pmatrix} \cos v & -\sin v \\ \sin v & \cos v \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix}, \tag{3}$$

where v is an arbitrary constant. Applying this transformation with the parameter

$$v = -\arctan\frac{g}{e}\,, \tag{4}$$

to eqs.(1), (2) and integrating over the field A' we get the partition function of the AHM [9]:

$$Z_{dyon} \propto Z_{AHM} = \int \mathcal{D}B'_\mu \mathcal{D}\Phi \exp\Big\{-\int d^4x \mathcal{L}_{AHM}(B',\Phi)\Big\}\,,$$
$$\mathcal{L}_{AHM}(B',\Phi) = \frac{1}{4}(\partial_{[\mu}B'_{\nu]})^2 + \frac{1}{2}|(\partial_\mu - i\tilde{g}B'_\mu)\Phi|^2 + \lambda(|\Phi|^2-\eta^2)^2\,, \tag{5}$$

the Higgs field Φ has the magnetic charge[4] $\tilde{g} = \sqrt{e^2+g^2}$.

Consider the quantum average of the Wilson loop in the dyon theory (1):

$$< W^C_e >_{dyon} = \frac{1}{Z_{dyon}} \int \mathcal{D}A_\mu \mathcal{D}B_\mu \mathcal{D}\Phi \exp\Big\{-\int d^4x\, \mathcal{L}_{dyon}(A,B,\Phi)\Big\} W^C_e(A)\,, \tag{6}$$
$$W^C_e(A) = \exp\Big\{ie_0 \int d^4x\, j_\mu A^\mu\Big\}\,, \qquad j_\mu(x) = \oint_C d\tilde{x}_\mu\, \delta^{(4)}(x-\tilde{x}(\tau))\,,$$

[3] The theory with $e=0$ (monopoles are condensed) has been investigated as an effective abelian theory of QCD in Refs. [8].

[4] We call B'_μ as the dual gauge field (thus Φ carries magnetic charge) since we consider (5) as the abelian effective model of the QCD vacuum. Really after the transformation (3) this is the matter of convention.

which creates the particle with the electric charge e_0 on the world trajectory[5] C.

Applying the transformation (3), (4) to the quantum average (6) and integrating over the field A'_μ we get:

$$< W_e^C >_{dyon} = < K^C_{(q_e,q_m)} >_{AHM}, \tag{7}$$

where the expectation value in the r.h.s. of this equation is calculated in the AHM with the lagrangian (5). The operator K is the product of the t'Hooft loop [10] H^C and the Wilson loop W^C:

$$K^C_{(q_e,q_m)}(B') = H^C_{q_e}(B') W^C_{q_m}(B'), \qquad q_e = e_0 g/\tilde{g}, \qquad q_m = e_0 e/\tilde{g}. \tag{8}$$

The operator $H^C_{q_e}$ is defined as follows:

$$H^C_{q_e}(B') = \exp\Big\{-\frac{1}{4}\int d^4x \left[\left(\partial_{[\mu}B'_{\nu]} - q_e \cdot \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}G^C_{\alpha\beta}\right)^2 - \left(\partial_{[\mu}B'_{\nu]}\right)^2\right]\Big\}, \tag{9}$$

where the tensor $G^C_{\mu\nu} = (n\cdot\partial)^{-1} j_{[\mu}n_{\nu]}$ satisfies the relation $\partial_\nu G^C_{\mu\nu} = j_\mu$. The tensor $F^d_{\mu\nu} = q_e G^C_{\mu\nu}$ plays a role of the dual field strength tensor: $\partial_\nu F^d_{\mu\nu} = q_e j_\mu$. In the string representation of the AHM [11] the operator $H^C_{q_e}$ creates the string spanned on the loop C, this string carries the flux q_e.

The product K^C of the operators H^C and W^C creates the dyon loop with electric charge q_e and magnetic charge q_m on the world trajectory C in the AHM (5).

Now we discuss the string representation for the AHM (5) [11, 12]. In the center of the ANO strings the field $\Phi = |\Phi|e^{i\theta}$ vanishes, $\mathrm{Im}\Phi = \mathrm{Re}\Phi = 0$, and the phase θ is singular on the two dimensional surfaces, which are world-sheets of the ANO strings. The measure of the integration over the fields Φ can be rewritten as follows: $\mathcal{D}\Phi = const\cdot\mathcal{D}|\Phi|^2\,\mathcal{D}\theta$. $\int\mathcal{D}\theta$ contains the integration over functions which are singular on two-dimensional manifolds, and we subdivide θ into the regular θ^r and the singular θ^s parts: $\theta = \theta^r + \theta^s$, here θ^s is defined by:

$$\begin{gathered}\partial_{[\mu,}\partial_{\nu]}\theta^s(x,\tilde{x}) = 2\pi\epsilon_{\mu\nu\alpha\beta}\Sigma_{\alpha\beta}(x,\tilde{x}),\\ \Sigma_{\alpha\beta}(x,\tilde{x}) = \int_\Sigma d^2\sigma\,\epsilon^{ab}\partial_a\tilde{x}_\alpha\partial_b\tilde{x}_\beta\,\delta^{(4)}[x-\tilde{x}(\sigma)], \quad \partial_a = \frac{\partial}{\partial\sigma^a},\end{gathered} \tag{10}$$

the vector function $\tilde{x}_\mu$ is the position of the string, Σ is the collection of all closed surfaces, $\sigma = (\sigma_1,\sigma_2)$ is the parametrization of the string surface; the measure $\mathcal{D}\theta$ can be decomposed as follows: $\mathcal{D}\theta = \mathcal{D}\theta^r\,\mathcal{D}\theta^s$.

For simplicity we consider below the London limit of the AHM ($\lambda\to\infty$). In this limit the radial part of the field Φ is fixed everywhere except for the centers of the ANO strings. All expression below can be generalized to the case of an arbitrary λ; this leads to an additional functional integral over the radial part $|\Phi|$.

Performing the transformations as in Refs.[12, 11] we get the following string theory for the quantum average (6) of the Wilson loop:

$$< W_e^C >_{dyon} = \frac{1}{Z_{str}}\int[\mathcal{D}\tilde{x}]\cdot J(\tilde{x})\cdot\exp\Big\{-\int d^4x\int d^4y\times$$

[5] This average corresponds to the quark Wilson loop if we consider (1) as an effective theory of QCD.

$$\times\left[\frac{q_m^2}{2}j_\mu(x)D_m(x-y)j_\mu(y)+\pi i\zeta\cdot j_\mu(x)D_m(x-y)\partial_\nu\epsilon_{\mu\nu\alpha\beta}\left(\Sigma_{\alpha\beta}(y)+\mathcal{N}G^C_{\mu\nu}(y)\right)+\right.$$

$$\left.+\pi^2\eta^2\left(\Sigma_{\mu\nu}(x)+\mathcal{N}G^C_{\mu\nu}(x)\right)D_m(x-y)\left(\Sigma_{\mu\nu}(y)+\mathcal{N}G^C_{\mu\nu}(y)\right)\right]+2\pi i\zeta\mathbb{L}(\Sigma,C)\Bigg\}, \quad (11)$$

where

$$\mathcal{N}=\frac{e_0 g}{2\pi}, \qquad \zeta=\frac{e_0 e}{\tilde{g}^2}=\frac{e_0 e}{e^2+g^2}, \quad (12)$$

$D_m(x)$ is the scalar Yukawa propagator, $(\Delta+m^2)D_m(x)=\delta^{(4)}(x)$, and $m^2=2\tilde{g}^2\eta^2$ is the mass of the dual gauge boson (B').

The measure $[\mathcal{D}\tilde{x}_\mu]$ assumes both integration over all possible positions and summation over all topologies of the string's world-sheets Σ; $J(\tilde{x})$ is the Jacobian of the transformation from the field θ^s to the string position $\tilde{x}_\mu$. The Jacobian $J(\tilde{x})$ was estimated in [11] for string with spherical or disc topology.

First three terms in the exponent in eq. (11) describe the short-range interaction and the self-interaction of the ANO strings and dyon-anti-dyon pair through the exchange of the massive gauge boson. The constant $\mathcal{N}$ which appears in these terms has a physical meaning. It is equal to the number of the elementary fluxes in the string which connects the dyon-anti-dyon pair introduced by the operator K, (8). By definition, $\mathcal{N}=q_e/\Psi_0$, where q_e is equal to the total electric flux from the dyon and $\Psi_0=2\pi/\tilde{g}$ is the flux carried by the elementary string in the AHM (5). Since this number of the elementary fluxes $\mathcal{N}$ must be integer, we get the charge quantization rule: $e_0 g\in 2\pi\mathcal{N}$, $\mathcal{N}\in\mathbb{Z}$ [9].

The last term in eq.(11),

$$\mathbb{L}(\Sigma,C)=\frac{1}{4\pi^2}\int d^4x\int d^4y\,\epsilon_{\mu\nu\alpha\beta}\,\Sigma_{\mu\nu}(x)\,j_\alpha(y)\,\frac{(x-y)_\beta}{|x-y|^4}$$

is the linking number of the string world sheet Σ and the trajectory C of the dyon. This formula represents the long-range interaction which describes the *dual* four-dimensional analogue [13] of the dual Aharonov – Bohm effect: strings correspond to electric solenoids which scatter magnetic charges of abelian dyons. This linking number term is important for the infrared properties of the theory since it may induce an additional long range potential between quark and anti-quark [14]. It also leads to non-trivial commutation relations between different operators in the theory [11].

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