EXCHANGE INTERACTION EFFECTS IN INTER-LANDAU LEVEL AUGER SCATTERING IN A TWO-DIMENSIONAL ELECTRON GAS

E.Tsitsishvili*, Y.Levinson

*Department of Condensed Matter Physics, The Weizmann Institute of Science,
76100 Rehovot, Israel*

**Institute of Cybernetics
380086 Tbilisi,Georgia*

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We consider the influence of spin effects on the inter-Landau level electron-electron scattering rate in a two-dimensional electron gas. Due to the exchange spin splitting, the Landau levels are not equidistant. This leads to the suppresion of Auger processes and a nonlinear dependence of the lifetime on the concentration of the excited electrons even at very low excitation levels.

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In the past years a considerable amount of work has been done on the determination of the electron lifetime in the excited Landau levels (LL) in a two-dimensional electron gas (2DEG) [1 − 5]. It was found that electron-electron (*ee*) scattering is the dominant relaxation mechanism, when the emission of LO phonons is suppressed off the magnetophonon resonance conditions [6, 7]. In this case the electron lifetime is determined by Auger processes, in which two excited electrons in the same LL are scattered, deexciting one to a lower LL, and exciting the second to a higher LL. The decrease of the measured lifetimes with an increase in the *excited* electrons' concentration, n_{exc}, has proved a convincing argument for this conclusion.

One can think that the probability for an Auger process to occur, τ_{ee}^{-1}, increases linearly with n_{exc}. However, the experiments do not confirm this conclusion[4]. We will show that this "naive" picture is not complete, and that the nonlinear dependence of τ_{ee}^{-1} on n_{exc} is due to spin effects.

In a 2DEG the LLs are equally spaced if one neglects spin effects (and nonparabolicity). The exchange interaction violates the equidistant LL spacing. The exchange energy in the $N\sigma$ LL ($\sigma = \uparrow, \downarrow$) is usually presented as $-\Sigma_{N\sigma}$, where $\Sigma_{N\sigma} = E_0\, \nu_{N\sigma}$, with $E_0 > 0$, and $\nu_{N\sigma}$ is the corresponding filling factor[8]. The observed values of E_0 are of the order of a several meV, e.g., in GaAs $E_0 = 3$ meV ÷ 6 meV at 10 T [9 − 12]. The Zeeman spin splitting is important for the empty LLs only, since it is much smaller (~ 0.2 meV at 10 T) than the exchange energy.

We will consider a situation that is similar to the experiment[4], in which the 2DEG is spin polarized. At equilibrium the electrons occupy the lowest LL, $0\uparrow$, with a filling factor $\nu_{0\uparrow} = \nu < 1$. Due to cyclotron absorption some of the electrons are excited to the higher LLs, $N\uparrow$. Since the Auger processes preserve the initial spin orientation due to total spin conservation during the scattering event, the LLs $N\downarrow$ are empty. They coincide with the "bare" levels and are equally spaced, see the left part of Fig.1. The LLs N↑ are occupied with filling factors $\nu_{0\uparrow} > \nu_{1\uparrow} > \nu_{2\uparrow} > ...$, and $\nu = \nu_{0\uparrow} + \nu_{1\uparrow} + \nu_{2\uparrow} + ...$. Due to the exchange

interaction the LLs $N\uparrow$ are shifted down depending on the level occupation by the energy $\Sigma_{N\uparrow} = E_0\nu_{N\uparrow}$, see the right side of Fig.1. If the excitation is not very strong, and also due to nonparabolicity, one can consider a three-level model with $\nu_{0\uparrow} + \nu_{1\uparrow} + \nu_{2\uparrow} = \nu$, as is shown in Fig.1. Since $\nu_{0\uparrow} > \nu_{1\uparrow} > \nu_{2\uparrow}$, the energy shifts $\Sigma_{0\uparrow} > \Sigma_{1\uparrow} > \Sigma_{2\uparrow}$, and hence, the LLs $0\uparrow$, $1\uparrow$, and $2\uparrow$ are nonequidistant.
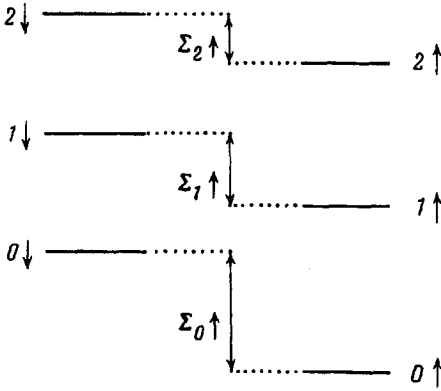

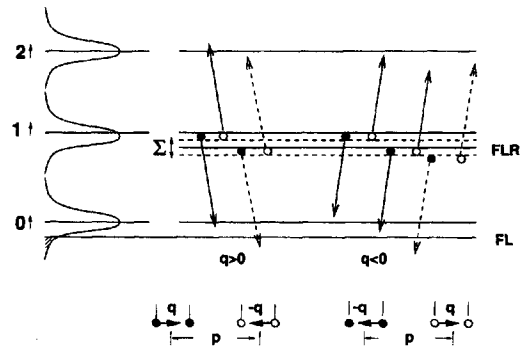
Fig.1. The Landau level ladder in the spin polarized case

Fig.2. The Auger process: $1\uparrow + 1\uparrow \rightarrow 0\uparrow + 2\uparrow$. The suppresed processes are shown with dashed arrows

The lifetime of the photoelectrons in level $1\uparrow$, is defined by the Auger process $1\uparrow + +1\uparrow \rightarrow 0\uparrow + 2\uparrow$. It is evident that since the LLs are not equidistant this process is forbidden by energy conservation and can happen only due to the LL broadening. Hence it is clear that the Auger transitions are well suppressed if the disbalance in the LLs spacing $\Sigma = (\Sigma_{0\uparrow} - \Sigma_{1\uparrow}) - (\Sigma_{1\uparrow} - \Sigma_{2\uparrow}) = E_0(\nu - 3\nu_{1\uparrow})$ is larger than the LL width Δ. In this case only the tails of the LLs density of states (DOS) are effective. One can suppose that the greatest possibility for energy conservation occurs when the scattering partners are situated in the middle between the centers of the lowest and highest LLs, $0\uparrow$ and $2\uparrow$. At very low excitation, one can assume that in first approximation Σ is independent of the excitation intensity. Then, the probability for the Auger process $1\uparrow +1\uparrow \rightarrow 0\uparrow + +2\uparrow$, is $\tau_{ee}^{-1} \sim \nu_{1\uparrow}$. When $\nu_{1\uparrow}$ is further increased the exchange energy $\Sigma_{0\uparrow}$ decreases, while $\Sigma_{1\uparrow}$ and $\Sigma_{2\uparrow}$ increase, and hence the disbalance Σ decreases. This obviously causes an increase in the scattering rate, resulting in a superlinear dependence of τ_{ee}^{-1} on $\nu_{1\uparrow}$. One can expect that the nonlinear enhancement of the scattering rate will be essential when the disbalance change, $\delta\Sigma = 3E_0\nu_{1\uparrow}$, approaches the LL width Δ, and hence, the crossover filling factor is $\nu_{1\uparrow}^* \simeq \Delta/3E_0$. Since the LL width, $\Delta 1$ meV[13], is appreciably smaller than the exchange energy E_0, one finds $\nu_{1\uparrow}^* \ll 1$. Thus, the nonlinear dependence of the scattering rate τ_{ee}^{-1} on $\nu_{1\uparrow}$ can be pronounced already at low concentrations of the excited electrons.

As an illustration we calculate the scattering rate τ_{ee}^{-1} of the Auger process $1\uparrow + +1\uparrow \rightarrow 0\uparrow + 2\uparrow$ using the approaches given in Refs.[14, 15]. We consider a 2DEG in a strong magnetic field and a random statistically homogeneous potential with a correlator: $\langle U(y)U(0) \rangle = \Delta^2 \exp\left(-y^2/\Lambda^2\right)$, where the correlation length, Λ, is much larger than the magnetic length $l_B = (eB/\hbar c)^{1/2}$. The correlation length is of the order of the spacer $\Lambda \simeq d$, while typical values of the magnetic length for fields B between 5 and 15 T are

~ 100 Å. Hence the random potential can be considered as a smooth one in samples with a spacer $d \geq 200$ Å.

We assume that the LLs follow the random potential in space, and the DOS, $\varrho(\varepsilon) = (\sqrt{2\pi}\Delta)^{-1} \exp(-\varepsilon^2/2\Delta^2)$, where the energy ε is referenced to the LL center, renormalized by the exchange energy. In the calculation of the scattering rate only relative coordinates of the interacting electrons are important. We choose the gauge $A = (-By, 0, 0)$. Let $y_1 = l_B^2 k_1$, and $y_2 = l_B^2 k_2$ be the guiding centers before scattering, and $y_1' = l_B^2 k_1'$, $y_2' = l_B^2 k_2'$ - the guiding centers after scattering. k_1, k_2, k_1', k_2' are the corresponding momenta. The shifts of the electrons in the scattering event are $(y_1' - y_1) = q$, and $(y_2' - y_2) = -q$, and the "average" distance between the scattering partners is $[(y_2' + y_2)/2 - (y_1' + y_1)/2] = p$. These quantities define the scattering probability. The averaged scattering rate of a test electron in the 1↑ LL with an energy ε referenced to its center is

$$\left\langle \frac{1}{\tau_{ee}} \right\rangle_\varepsilon = \int \int_{-\infty}^{+\infty} \frac{dpdq}{2\pi\hbar l_B^2} |M(p,q) - \bar{M}(p,q)|^2 \left(S_\varepsilon(p,q) + S_\varepsilon(q,p) \right), \quad (1)$$

where $M(p,q)$ and $\bar{M}(p,q)$ are the scattering matrix elements for the direct and the exchange electrons' collisions, respectively. The functions $S_\varepsilon(p,q)$ and $S_\varepsilon(q,p)$ are due to the statistical factors and the energy conservation. $S_\varepsilon(p,q)$ is related to the "deexciting" Auger transition, in which the test electron is deexcited to the lower level 0↑, and its partner is excited to the upper level 2↑, while $S_\varepsilon(q,p)$ corresponds to the "exciting" transition, in which the test electron is excited to the upper level 2↑, and its partner is deexcited to the lower level 0↑.

It is easy to check that $\bar{M}(p,q) = M(q,p)$, and

$$M(p,q) = \frac{1}{l_B^2} \int_{-\infty}^{+\infty} d\eta \, V(q,\eta) \, F(q, p-\eta), \quad (2)$$

where

$$V(q,\eta) = \int_{-\infty}^{+\infty} d\xi \, V(\sqrt{\xi^2 + \eta^2}) \, \exp\left(\frac{iq\xi}{l_B^2}\right) \quad (3)$$

is the Fourier transform of the *ee*-interaction potential $V(r)$. The *ee*-interaction is chosen as $V(\sqrt{x^2 + y^2}) = V_0 \exp\{-(x^2+y^2)/l_{sc}^2\}$, where l_{sc} is the screening length, $V_0 \simeq e^2/\kappa l_B$, and κ is the dielectric constant. The function $F(q,p)$ is defined as follows:

$$F(q,p) = \frac{1}{4} \exp\left(-\frac{p^2}{2l_B^2} - \frac{q^2}{2l_B^2}\right) \int_{-\infty}^{+\infty} dz e^{-2z^2} H_1(z+s) H_1(z-s) H_2(z-r), \quad (4)$$

where $s = (p+q)/2l_B$, $r = (p-q)/2l_B$, and $H_n(y)$ is the Hermite polynomial. Since the initial and final states must overlap, the electron shift in the scattering event is of the order of or smaller than the magnetic length l_B, and hence ql_B. Thus, the Auger transitions are quasivertical in space. The "average" distance between interacting electrons is limited by the screening length l_{sc} of the *ee*-interaction, pl_{sc}. In the situation we consider l_{sc} is defined by the electrons in the 0↑ LL, and since this level is only partially occupied, the screening is strong and $l_{sc} \simeq l_B$ [16]. In this case, the matrix elements $M(p,q)$ and $\bar{M}(p,q)$ that enter Eq.(1) are exponentially small if $p, q \gg l_B$[15], and therefore the main

contribution to the integrals in Eq.(1) arises from $p, q l_B$. Note also, that due to the spatial homogeneity, $M(p,q) = M(-p,-q)$ and $S_\varepsilon(p,q) = S_\varepsilon(-p,-q)$, and one can restrict the integration over p in Eq.(1) to $p > 0$.

We will consider the low-excitation limit, such that $\nu_{1\uparrow} \ll \nu_{0\uparrow}$, and $\nu_{2\uparrow} = 0$. In this case the electron concentration in the $0\uparrow$ LL changes only slightly with pumping, and one can assume that the initial Fermi distribution in this level is not perturbed. The energy distribution of the photoelectrons in level $1\uparrow$ depends on the relation between the inter- and intra-LL relaxation times. In order to simplify the calculations we will consider the case in which the inter-LL relaxation is faster, and thus the excited electrons are not at equilibrium. We suppose that they are distributed uniformly in space and are excited in a rather wide spectral interval, thus their occupation numbers are assumed to be constant and equal to $\nu_{1\uparrow}$.

With this in mind the function $S_\varepsilon(p,q)$ in Eq.(1) is

$$S_\varepsilon(p,q) = \nu_{1\uparrow} \Big\langle \delta\Big(\varepsilon + \Sigma + U(p+q) - U(q) - U(p)\Big)\Big[1 - f\big(U(q)\big)\Big]\Big\rangle, \qquad (5)$$

where $\langle .... \rangle$ stands for the statistical average[14], and $f(\varepsilon)$ is the Fermi distribution in LL $0\uparrow$. Performing the average over the random potential realizations[15] in Eq.(5) at the limit of zero temperature, $T = 0$, one obtains

$$S_\varepsilon(p,q) = \frac{\nu_{1\uparrow}}{8\sqrt{\pi}\Delta} \frac{\Lambda^2}{|pq|} \exp\Big\{-\frac{1}{4\Delta^2}\Big[\varepsilon - \frac{\Lambda^2\Sigma}{2pq}\Big]^2\Big\}\Big\{1 - \Phi\Big[-\frac{\Lambda}{2\,\Delta\,|q|}\Big(\varepsilon - \varepsilon_F - \frac{\Sigma q}{2p}\Big)\Big]\Big\}, \qquad (6)$$

where $\Phi(x) = (2/\sqrt{\pi}) \int_0^x e^{-t^2}\,dt$ is the probability integral [17], and ε_F is the Fermi energy referenced to the center of the $0\uparrow$ LL.

The factor $\{1 - \Phi\}$ in Eq.(6) is due to the occupation of the $0\uparrow$ LL. Let us introduce the Fermi-level replica (FLR), which is given by $\varepsilon_F + \hbar\omega_B - \Sigma_{1\uparrow}$ and is thus pinned to level $1\uparrow$, see Fig. 2. The energy difference $\varepsilon - \varepsilon_F$ in Φ is the test electron energy referenced to the FLR. Consider first the Auger transitions with $q > 0$, i.e., when the scattering partners are closer in space after scattering. The factor $\{1 - \Phi\}$ shows that these transitions are strong if the test electron is above the FLR by an energy $\simeq \Sigma/2$, i.e., at $\varepsilon - \varepsilon_F \Sigma/2$, and weak if $\varepsilon - \varepsilon_F \Sigma/2$, see the left part of the Fig. 2. Similarly, the Auger transitions with $q < 0$ (i.e., when the scattering partners are closer in space before scattering) are strong at $\varepsilon - \varepsilon_F - \Sigma/2$, and weak if $\varepsilon - \varepsilon_F - \Sigma/2$, see the right part of the Fig. 2. In both cases the crossover scale is $\Delta(l_B/\Lambda) \equiv \Delta_B \ll \Delta$, much smaller than the LL width. Thus, due to the occupation of the lowest LL, $0\uparrow$, all Auger processes for large negative energies of the test electron are quenched.

The origin of the exponential factor is as follows. During the scattering event the total momentum and energy are conserved, i.e., $[\varepsilon + U(p+q) - 2\Sigma_{1\uparrow}] - [U(p) + U(q) - \Sigma_{0\uparrow}] = 0$. Since the Auger transitions are quasivertical ($q \ll \Lambda$), the random potential $U(y)$ can be expanded in powers of q and p. Assuming the test electron is at $y = 0$, i.e., $\varepsilon = U(0)$, one obtains $U''(0)pq = -\Sigma$, where $\Sigma = \Sigma_{0\uparrow} - 2\Sigma_{1\uparrow}$ is the disbalance in the LL spacing. The exponential factor in Eq.(6) is proportional to the conditional probability $\mathrm{Probc}\{U(0) = \varepsilon | U''(0) = -\Sigma/pq\}$ that if the random potential $U(y)$ at $y = 0$ is $U(0) = \varepsilon$, then its second derivative at the same point is $U''(0) = -\Sigma/pq$. Typical magnitudes of the second derivative of the random potential at energies $|\varepsilon|\Delta$ are $|U''| \simeq \Delta/\Lambda^2$, which is much smaller than Σ/l_B^2 for typical Σ. Hence energy conservation can not be satisfied for

$|\varepsilon|\Delta$, and can be obeyed only in the tails of the DOS where the probability to find large $|U''|$ is not small. This can be seen from the exponential factor entering Eq.(6), which has its maximum values at $\varepsilon = \pm\varepsilon_s \simeq (\Lambda/l_B)^2\Sigma \gg \Delta$, i.e., in the tails of the DOS.

Note, that the results obtained in the case of a smooth random potential differ from the general predictions given above. Namely, the scattering rate in this case is very sensitive to nonequal distances between the LLs, and is suppressed not only when the disbalance in the LL spacing is comparable to the LL width, but also at much smaller $\Sigma\Delta(l_B/\Lambda)^2 \equiv \Delta_s$ with $\Delta_s \ll \Delta$. In addition when the occupation of the excited LL 1↑ increases, the scattering rate responds to a much smaller change of the disbalance than predicted, $\delta\Sigma \simeq \Delta$. Indeed, as follows from the exponential factor in Eq.(6), the scattering rate τ_{ee}^{-1} is sensitive to the decrease in Σ, when $\delta\Sigma \simeq 8\Delta_s^2/\Sigma \ll \Delta$. The crossover filling factor is also much smaller, $\nu_{1\uparrow}^* \simeq \Delta_s^2/\Sigma E_0 \ll \Delta/E_0$. For example, at $\Delta_s \simeq 0.1$ meV, $E_0 \simeq 2$ meV, and $\Sigma \simeq 0.5$ meV, $\nu_{1\uparrow}^* \simeq 0.01$.
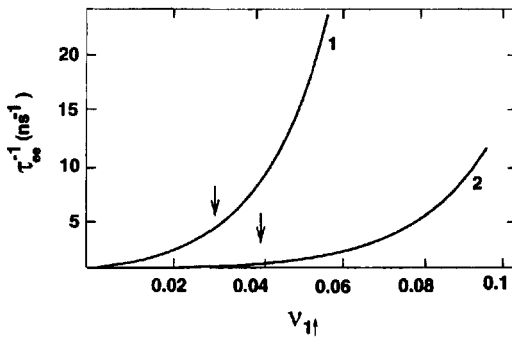

Fig.3. The Auger scattering rate, τ_{ee}^{-1}, for the process: 1↑ + 1↑→ 0↑ + 2↑ at $\varepsilon = \varepsilon_F$, $\nu = 0.31$ (curve 1), and $\nu = 0.5$ (curve 2), versus the filling factor $\nu_{1\uparrow}$. The arrows indicate the crossover filling factors $\nu_{1\uparrow}^*$ (see also text)

The dependence of the scattering rate on the concentration of the excited electrons at the test electron energy $\varepsilon = \varepsilon_F$ is shown in Fig. 3. The magnetic field is $B = 6$ T, and the curves correspond to two different electron concentrations: $n = 4.5 \cdot 10^{10}$ cm^{-2}, i.e., $\nu = 0.31$ (curve 1), and $n = 7.3 \cdot 10^{10}$ cm^{-2}, i.e., $\nu = 0.5$ (curve 2). The other parameters are as follows: $\kappa = 12$, $\Delta = 1$ meV, $\Lambda = 300$ Å, and $E_0 = 2$ meV[12]. ¿From Fig. 3 it can be seen that in both cases the scattering rate τ_{ee}^{-1} changes linearly with the filling factor $\nu_{1\uparrow}$ only at very small $\nu_{1\uparrow}$. The arrows in Fig. 3 indicate the crossover values of $\nu_{1\uparrow}^*$, when the deviation from the linear law accounts for about 100%. For a given $\nu_{1\uparrow}$, the scattering rate τ_{ee}^{-1} is more suppressed at larger electron concentration, because of the larger disbalance in the LL spacing.

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