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QUASIPARTICLE DYNAMICS AND PHASE LOCKING IN A S–I–S MULTILAUER JOSEPHSON JUNCTION

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We obtain new dynamical equations describing the Josephson effect and nonequilibrium quasiparticle distribution in a multilayer Josephson tunnel structure at $T \sim T_c$ starting from the microscopic theory. It is shown that quasiparticle dynamics has a strong influence on the Josephson effect. We find novel regimes with giant charge imbalance oscillations. New type of hysteresis on the voltage-current characteristic is predicted.

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Dynamics of multilayer Josephson structures is an important subject of theoretical and experimental investigations during the last few years. Recent experiments on the Josephson effect in artificial Nb-AlO_x-Nb stacked junctions [1] and natural layered high-T_c superconductors [2] show that these structures have a similar dynamic behavior and can be considered on common basis [3]. In both of these systems interaction between Josephson junctions and their mutual phase locking are of great interest and importance. A theory of magnetic coupling in layered structures is developed in Ref. [4] and applied to the problem of the synchronization of the Josephson vortex motion. But in the case of thin superconducting layers some other mechanisms are to be taken into account, especially disequilibrium (of the electron-hole imbalance type) of quasiparticle distribution inside superconducting layers, which can be essential if the layer thickness d_0 is smaller than the characteristic length of the disequilibrium relaxation l_E (see [5, 6, 7] and references therein). This criterion is obviously fulfilled for structures with layers of atomic thickness (high-T_c superconductors). For artificial structures $l_E = \sqrt{2\hbar DT/\pi\Delta^2}\left(1 + 4\Delta^2 t_\epsilon^2/\hbar^2\right)^{1/4}$, where $t_\epsilon^{-1} = 14g\hbar^{-1}\Theta_D^{-2}T^3\zeta(3)$ is the inelastic electron-phonon scattering frequency, g is the electron-phonon interaction constant, Θ_D is the Debye temperature, $D = lv_F/3$ is the electron diffusion coefficient, Δ is the energy gap, l is the free path, v_F is the Fermi velocity. A typical value of l_E is about 1 μm, thus $d_0 \ll l_E$ can be fulfilled at least at $T \sim T_c$ when $\Delta \to 0$ and $l_E \to \infty$. Disequilibrium results in the so-called quasiparticle coupling, which is well known in S–N–S junction systems.

In this paper we consider the Josephson effect with quasipartical dynamics taken into account in a S–I–S multilayer Josephson tunnel structure with layer thickness $d_0 \ll l_E$, thus superconductors are in homogeneous nonequilibrium state. We assume also the dirty limit ($l \ll d_0$) and use the averaged-over-momentum-direction quasiparticle distribution function n_ϵ^i introduced by Eliashberg [8], which describes quasielectron (at $\epsilon > 0$) and quasihole (at $\epsilon < 0$) distributions over energies. In equilibrium $n_\epsilon^i = n_{-\epsilon}^i = n_\epsilon^{(0)} = 1/2(1 - \mathrm{th}(|\epsilon|/2T))$.

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The origin of the nonequilibrium Josephson effect in S−I−S systems is well established [9−11]. First of all, a tunnel junction is a source of disequilibrium in a nonstationary state due to injection of quasiparticles, thus n^i_ϵ changes in some way. We start from the kinetic equation for $n^i_\epsilon(t)$ in the i-th layer obtained for the considered system by Bulyzhenkov and Ivlev [9], Ivlev [10] and Gulyan and Zharkov [11]:

$$u_\epsilon \frac{\partial n^i_\epsilon}{\partial t} = Q_{i-1\,i}(n_\epsilon) + Q_{i+1\,i}(n_\epsilon) + I^{e-ph}(n^i_\epsilon), \tag{1}$$

$I^{e-ph}(n_\epsilon)$ is the electron-phonon collision integral and $Q(n_\epsilon)$ is the tunnel source of disequilibrium which for tunneling from S_i to S_j has the form (at $T \sim T_c$)

$$\begin{aligned} Q_{ij}(n_\epsilon) = \frac{\nu}{2}[(u_{\epsilon-v} + u_\epsilon u_{\epsilon-v})(\beta_{\epsilon-v} - \beta_\epsilon - \alpha_\epsilon) - (u_{\epsilon+v} - u_\epsilon u_{\epsilon+v})(\beta_{\epsilon+v} - \beta_\epsilon - \alpha_\epsilon) + \\ +(1+u_\epsilon)\alpha_{\epsilon-v} + (1-u_\epsilon)\alpha_{\epsilon+v}]\mathrm{sign}\epsilon, \end{aligned} \tag{2}$$

where we introduce the following notations in every layer:

$$\alpha_\epsilon = (n_\epsilon - n_{-\epsilon})\theta(\epsilon^2 - \Delta^2)\mathrm{sign}\epsilon, \qquad \beta_\epsilon = (n_\epsilon + n_{-\epsilon} - 1)\theta(\epsilon^2 - \Delta^2)\mathrm{sign}\epsilon, \qquad v = \frac{\hbar}{2}\frac{d\varphi_{ij}}{dt},$$

$$u_\epsilon = \frac{|\epsilon|\theta(\epsilon^2 - \Delta^2)}{\sqrt{\epsilon^2 - \Delta^2}}, \qquad v_\epsilon = \frac{\Delta\theta(\epsilon^2 - \Delta^2)\mathrm{sign}\epsilon}{\sqrt{\epsilon^2 - \Delta^2}}, \qquad w_\epsilon = \frac{\Delta\theta(\Delta^2 - \epsilon^2)}{\sqrt{\Delta^2 - \epsilon^2}},$$

$\varphi_{ij} = \theta_j - \theta_i$ is the Josephson phase difference, all functions with shifted arguments relate to the S_i-superconductor (injector) and all functions with unshifted arguments relate to the S_j-superconductor, $\nu = (4e^2 N(0) R S d_0)^{-1}$ is the "tunnel frequency", R is the normal resistivity of the tunnel junction, $V = Sd_0$ is the volume of the superconducting layer, $N(0) = mp_F/2\pi^2$.

Nonequilibrium n_ϵ results in generation of a nonzero invariant potential

$$\Phi = \phi + (\hbar/2e)(\partial\theta/\partial t)$$

in superconducting layers, where ϕ is the electrostatic potential and θ is the phase of superconducting condensate ($\Phi = 0$ in equilibrium state). $\mu_s = e\Phi$ equals the shift of the chemical potential of superconducting condensate from its equilibrium value and is determined by

$$e\Phi = \int_\Delta^\infty (n_\epsilon - n_{-\epsilon})d\epsilon = \int_\Delta^\infty \alpha_\epsilon d\epsilon, \tag{3}$$

which is a direct consequence of the quasineutrality condition [8]. From (3) one can see that Φ is proportional to the difference in the electron and hole distribution functions and thus is associated with the so-called "charge imbalance". Charge imbalance phenomena are extensively studied in tunnel structures beginning from the pioneer work of Tinkham and Clarke [12]. Besides a large variety of static states (e.g. at resistive N-S boundary), weak charge imbalance oscillations are investigated in the form of linear waves (see the review in [6]) and also in Josephson junctions [13]. In this work we show that new regimes with strong charge imbalance oscillations are possible in S−I−S multilayer structures.

Energy gap Δ is to be obtained from the nonequilibrium self-consistency equation

$$1 = g\int_{\Delta}^{\Theta_D} \frac{1 - n_\epsilon - n_{-\epsilon}}{(\epsilon^2 - \Delta^2)^{1/2}} d\epsilon = g\int_{\Delta}^{\Theta_D} \frac{-\beta_\epsilon}{(\epsilon^2 - \Delta^2)^{1/2}} d\epsilon. \tag{4}$$

The next important point to notice is that in the nonequilibrium regime an ordinary Josephson relation $(d\varphi/dt) = (2e/\hbar)V$ between the Josephson phase difference $\varphi_{ij} = \theta_j - \theta_i$ and voltage $V_{ij} = \phi_i - \phi_j$ is violated [5–7]. Instead, we have (from Φ definition)

$$\frac{d\varphi_{ij}}{dt} = \frac{2e}{\hbar} V_{ij} + \frac{2e}{\hbar}(\Phi_j - \Phi_i). \tag{5}$$

Tunnel current also is different from the equilibrium one. A correct expression at $d\varphi_{ij}/dt = const$ was obtained by Gulyan and Zharkov [11]

$$\begin{aligned} J_{ij} &= J_0 \sin(\varphi_{ij}) + J_1 \cos(\varphi_{ij}) + J_{qp}, \\ J_0 &= \frac{1}{2eR}\int_{-\infty}^{\infty} d\epsilon [v_\epsilon w_{\epsilon+v}\beta_\epsilon + v_{\epsilon+v} w_\epsilon \beta_{\epsilon+v}], \\ J_1 &= \frac{1}{2eR}\int_{-\infty}^{\infty} d\epsilon v_\epsilon v_{\epsilon+v}(\beta_{\epsilon+v} - \beta_\epsilon), \\ J_{qp} &= \frac{1}{2eR}\int_{-\infty}^{\infty} d\epsilon \{u_\epsilon u_{\epsilon+v}(\beta_\epsilon - \beta_{\epsilon+v}) + u_\epsilon \alpha_{\epsilon+v} - u_{\epsilon+v}\alpha_\epsilon\}. \end{aligned} \tag{6}$$

From (5) and (6) we see that disequilibrium modifies the interlayer Josephson effect, and a self-consistent description is necessary. The kinetic equation (1), (2) together with the Josephson relation (5), current expression (6) and self-consistency equations (3), (4) are the full set of equations to be solved.

In this paper we consider temperatures $T \sim T_c$ when analytical solution of the kinetic equations may be obtained. It means that we use a small parameter $\Delta/T \ll 1$ and the results have the same accuracy. Furthermore, a typical Josephson voltage is of order of $V_c = RI_c$, where $I_c = (\pi/2)(\Delta/eR)\text{th}(\Delta/2T)$ (Ambegaokar–Baratoff formula). At $T \sim T_c$ we obtain $V_c = (\pi/4)(\Delta/e)(\Delta/T)$ and thus $eV_c \ll \Delta \ll T$. Typical Josephson frequencies are $\hbar\omega_J \sim 2eV_c \ll \Delta, T$ and finally $e\Phi \ll \Delta$. As a result of these equalities, kinetic equation is linear and may be solved in adiabatic limit in which the microscopic expressions for the current (6) and tunnel source (2) are correct. We can also neglect the energy gap change in this case.

At $\Delta \ll T$ Φ is determined by quasiparticle distribution over a large energy range $\epsilon \sim T$ and all peculiarities at $\epsilon \sim \Delta$ can be neglected. Taking $v \ll T$ we obtain from (2)

$$Q_{ij}(n_\epsilon) = \nu \left[-\frac{d\beta_\epsilon^{(0)}}{d\epsilon} v - \alpha_\epsilon^j + \alpha_\epsilon^i \right] \text{sign}\epsilon.$$

This tunnel source is antisymmetric over energies and thus we can take $\beta_\epsilon = \beta_\epsilon^{(0)} = -\text{th}(\epsilon/2T)$ at $|\epsilon| > \Delta$ in the first approximation (and correspondingly equilibrium Δ) and for α_ϵ^i we obtain

$$\frac{d\alpha_\epsilon^i}{dt} = 2\nu \left[\frac{v_{i-1\,i} - v_{i\,i+1}}{2T\text{ch}^2(\epsilon/2T)} - 2\alpha_\epsilon^i + \alpha_\epsilon^{i-1} + \alpha_\epsilon^{i+1} \right] - \tau_q^{-1}\alpha_\epsilon^i, \tag{7}$$

where we take $I^{e-ph}(n_\epsilon)$ in the τ-approximation, which is correct in our case, and $\tau_q = (4T/\pi\Delta)t_\epsilon$ is a well-known charge-imbalance relaxation time.

Solution of (7) in all layers simultaneously can be found in the well known form [6]

$$\alpha_\epsilon^i = \frac{e\Phi_i(t)}{2T\mathrm{ch}^2(\epsilon/2T)}. \tag{8}$$

One can see that the self-consistency equation (3) is satisfied automatically and for $\Phi_i(t)$ we obtain

$$\tau_q \frac{d\Phi_i}{dt} + \Phi_i = \eta(V_{i-1\,i} - V_{i\,i+1}), \tag{9}$$

where

$$\eta = 2\nu\tau_q = \frac{8T}{\pi\Delta}\nu t_\epsilon \tag{10}$$

is the parameter of disequilibrium. At $\eta = 0$ we obtain $\Phi = 0$ and ordinary Josephson relations. In the same approximation from (6) we obtain the expression for current

$$J_{ij} = J_c \sin(\varphi_{ij}) + \frac{V_{ij}}{R}. \tag{11}$$

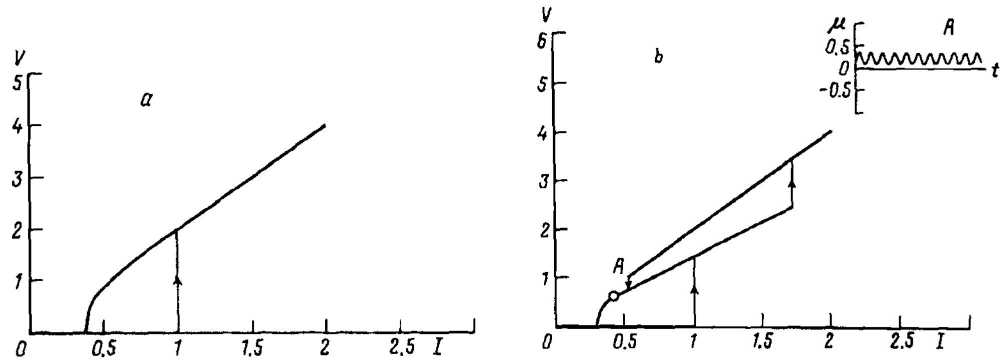
Taking (9), (11) and (5) we obtain the full set of dynamic equations for a S–I–S multilayer structure with nonequilibrium layers.

$$\begin{gathered}
\frac{d\varphi_{ij}}{dt} = \frac{2e}{\hbar}V_{ij} + \frac{2e}{\hbar}(\Phi_j - \Phi_i), \\
J_{ij} = J_c \sin(\varphi_{ij}) + \frac{V_{ij}}{R} + C\frac{dV_{ij}}{dt} = J(t), \\
\tau_q \frac{d\Phi_i}{dt} + \Phi_i = \eta(V_{i-1\,i} - V_{i\,i+1}) \qquad V(t) = \sum_i V_{i\,i+1}.
\end{gathered} \tag{12}$$

Here displacement current CdV/dt associated with junction capacitance C is added as in the usual tunnel theory, $J(t)$ and $V(t)$ are external current and voltage. This set of equations describes quasiparticle interaction between Josephson junctions. At $\eta = 0$ this system describes noninteracting junctions with independent phases $\varphi_{ij}(t)$. In the static limit our equations coincide with the equations obtained by Ivlev [10]. In the case of zero capacitance (C=0) they are similar to the system obtained by Artemenko and Volkov for S–N–S like structures [6]. One sees that at finit η a trivial solution $\varphi_{ij} = \varphi(t)$, $\mu = 0$ always exists. But we find that this solution can be unstable (due to parametric instability of charge imbalance) and strong $\Phi \sim V$ takes place in this case.

Below we consider (as an example) the simple S–I–S′–I–S Josephson tunnel structure with equilibrium banks (S) and thin middle layer (S′) at fixed current limit and in dimensionless form we obtain

$$\begin{gathered}
\beta\frac{d^2\varphi_1}{d\tau^2} + \frac{d\varphi_1}{d\tau} + \sin(\varphi_1) - \mu - \beta\frac{d\mu}{d\tau} = j, \\
\beta\frac{d^2\varphi_2}{d\tau^2} + \frac{d\varphi_2}{d\tau} + \sin(\varphi_2) + \mu + \beta\frac{d\mu}{d\tau} = j, \\
\alpha\frac{d\mu}{d\tau} + \mu = \tilde{\eta}\left(\frac{d\varphi_1}{d\tau} - \frac{d\varphi_2}{d\tau}\right),
\end{gathered} \tag{13}$$



Voltage-current characteristics of a S−I−S′−I−S Josephson structure at small coupling (a) and large coupling (b) between junctions

$$\beta = \frac{2eJ_cR^2C}{\hbar}, \qquad \alpha = \frac{\tau_q\omega_c}{1+2\eta}, \qquad \tilde{\eta} = \frac{\eta}{1+2\eta},$$
$$\mu(t) = \frac{2e}{\hbar\omega_c}\Phi(t), \qquad \omega_c = \frac{2eRJ_c}{\hbar}, \qquad \tau = \omega_c t.$$

Voltage-current characteristics of a large-β junction at various coupling parameters η ($\alpha = 0.01$, $\beta = 10$, $\tilde{\eta} = 0.05, 0.3$) are shown in Figure and the corresponding $\mu(t)$ dynamics is shown in one selected point. We see that at small coupling ($\tilde{\eta} = 0.05$, Figure a) the voltage-current characteristic is the same as for tow independent junctions, but junction phases are locked due to quasiparticle interaction. At large coupling ($\tilde{\eta} = 0.3$, Figure b) the "charge imbalance" regime at low currents is changed by the phase locked regime at high currents and novel type of hysteresis takes place.

Finally, it is shown that dynamics of a S−I−S multilayer Josephson structure is drastically changed by quasiparticle effects in the case of strong coupling.

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