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DYNAMICS OF SELF-SIMILAR DISPERSION-MANAGED SOLITON PRESENTED IN THE BASIS OF CHIRPED GAUSS – HERMITE FUNCTIONS

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Applying chirped Gauss – Hermite orthogonal functions we present an analytical description of the breathing dynamics of the chirped dispersion-managed soliton. Theory describes both self-similar evolution of the central, energy-containing core and accompanying nonstationary oscillations of the far-field tails of an optical pulse propagating in a fiber line with arbitrary dispersion map.

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Dispersion management (the variation of the chromatic dispersion along the line) is an attractive technique that allows to enhance the performance of fiber communication links both for soliton and non-soliton transmission (see e.g [1 – 6]). Recent developments in optical fiber communications have demonstrated that the dispersion management makes the features of the soliton transmission to be close to those of non-soliton one [4 – 6]. Dispersion-managed (DM) soliton is a new type of the information carrier with properties [1 – 17] rather different from that of traditional fundamental soliton (soliton solution of the integrable nonlinear Schroedinger equation (NLSE) [18]). In particular, during propagation along the fiber line DM soliton undergoes rapid self-similar breathing-like oscillations of the width and power, it is chirped and it can propagate at the zero or even normal average dispersion Theory of DM soliton in the system with strong dispersion management has been presented in [17]. The first implementation of the commercial fiber optical networks based on dispersion-managed soliton has recently been reported in [6]. Though the basic theory of DM soliton has been already presented in [8, 7, 12 – 14, 17], due to a wide range of possible practical system configurations many interesting problems are still open. An intriguing open theoretical problem is the origin and structure of the oscillatory tails of the DM soliton [16, 11]. As it has been shown in [16] such tails manifest themselves as nonself-similar modulations of soliton profile during the compensation period, though their amplitudes are rather small compared with the main peak. In this paper we present a systematic method to describe the dynamics of the self-similar core and oscillatory tails of the DM soliton using an orthogonal set of chirped Gauss – Hermite functions. This approach can be very useful in numerical modeling of the dynamics of arbitrary initial signal in the dispersion-managed communication systems, including chirped return-to-zero and non-return-to-zero formats [4, 5].

Pulse dynamics in dispersion-managed optical transmission systems is governed by the NLSE with periodic coefficients:

$$iA_z + d(z)A_{tt} + c(z)|A|^2 A = 0, \quad d(z) = \lambda_0^2\, D(z)\, L\,/(4\pi c_l t_0^2), \quad c(z) = P_0 L\sigma \exp(-2L\gamma z). \tag{1}$$

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Here the propagation distance z is normalized by the dispersion compensation period L, time t is normalized by the parameter t_0, the envelope of the electric field is scaled by the power parameter P_0. Periodic function $c(z)$ accounts for power decay due to fiber loss and periodic amplification, σ is the nonlinear coefficient, γ describes fibre losses. Lumped action of the amplifiers is accounted through the transformation of the pulse power at junctions corresponding to locations of amplifiers. Normalized chromatic dispersion $d(z) = \widetilde{d}(z) + \langle d \rangle$ represents the sum of a periodic, rapidly varying (over one compensation period) high local dispersion $(\widetilde{d})$ and a constant residual dispersion $(\langle d \rangle \sim \langle c \rangle \sim \widetilde{c} \ll \widetilde{d}\ )$. Here λ_0 is the operating wavelength, c_l is the speed of light, D is the local dispersion coefficient. The amplification distance in general can be different from the compensation period, but without loss of generality we assume here that c and d have the same period L. Nonlinearity comes into play on the scale $Z_{NL} \sim 1/(P_0\sigma) \gg L$ and the physical problem that we consider is how to describe asymptotic solution that realizes the balance between effects of nonlinearity, average dispersion and averaged effects resulting from the rapid variations of the dispersion $d(z)$ and power $c(z)$. The mathematical formulation of this problem is how to average Eq. (1) if the periodic oscillations of $d(z)$ are large. A small parameter in the problem is $L/Z_{NL} \ll 1$. Note that due to large variations of d, direct approaches like the averaging method known for the so-called Kapitza pendulum problem [19] does not work here, because rapidly varying field is of the same order as the averaged (slow varying) part. Basing on the results of numerical simulations, qualitatively, solution of Eq. (1) presents the central peak oscillating with z in self-similar manner, and the tails that are not self-similar, have smaller power compared with the main peak, but that are responsible for soliton-soliton interaction. Important feature of the DM soliton is the rapid oscillation of the phase (quadratic in time near the center) during the period. The typical shape of the DM soliton found numerically is shown in Fig. 1.

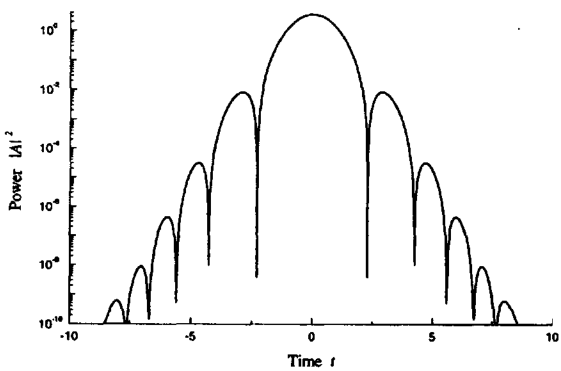

Fig. 1. Shape of the dispersion-managed soliton (periodic solution of Eq. (1)) is shown at the beginning (end) of the periodic cell

As a first step, we remove from Eq. (1) a fast phase dynamics due to the large variations of d. To describe a rapid self-similar dynamics of the main peak let us consider following [17] exact transformation of the function $A(z,t)$ that is, as a matter of fact, modification of the Talanov transform known in the self-focusing theory [20]:

$$A(t,z) = N\ \exp\left[i\frac{M(z)}{T(z)}t^2\right]\frac{Q(x,z)}{\sqrt{T(z)}}, \quad x = \frac{t}{T(z)}, \tag{2}$$

here T and M are periodic solutions of the following equations first obtained in [8, 13]

$$\frac{dT}{dz} = 4d(z)M; \quad \frac{dM}{dz} = \frac{d(z)}{T^3} - \frac{c(z)N^2}{T^2}. \tag{3}$$

The coefficient N in (3) is determined by the requirement that T and M are periodic solution to Eq. (3). We obtain then a partial differential equation for $Q(x,z)$:

$$i\frac{\partial Q}{\partial z} = \frac{\delta H}{\delta Q^*} = -\frac{d}{T^2}(Q_{xx} - x^2 Q) - \beta(z)(|Q|^2 Q + x^2 Q),$$

$$H = \int [\frac{d}{T^2}(|Q_x|^2 + x^2|Q|^2) - \beta(z)(\frac{1}{2}|Q|^4 + x^2|Q|^2)]dt. \tag{4}$$

Here $\beta(z) = c(z)N^2/T$. Next, we expand $Q(x,z)$ using complete set of the orthogonal normalized Gauss–Hermite functions $Q(x,z) = \sum_n b_n(z) f_n(x)$ with

$$(f_n)_{xx} - x^2 f_n = \lambda_n f_n, \ \ \lambda_n = -1 - 2n, \ \ f_n(x) = \frac{1}{\sqrt{2^n n! \sqrt{\pi}}} \exp(-\frac{x^2}{2}) H_n(x). \tag{5}$$

Here $H_n(x)$ is the n-th-order Hermite polynomial and coefficients b_n are given by the scalar product in $\mathcal{L}^2$ with f_n: $b_n = \langle f_n | Q \rangle$. Inserting this expansion into (4), after straightforward calculations we obtain a system of ordinary differential equations for the coefficients b_n:

$$i\frac{db_n}{dz} = \frac{\delta H_b}{\delta b_n^*} = -\frac{d}{T^2} b_n \lambda_n - \beta(z)\sum_m b_m S_{n,m} - \beta(z) \sum_{m,l,k} b_m b_l b_k^* R_{m,l,k,n} = 0. \tag{6}$$

the Hamiltonian H_b is

$$H_b = -\frac{d}{T^2} \sum_{n=0}^{n=\infty} \lambda_n |b_n|^2 - \beta(z) \sum_{n,m} S_{n,m} b_m b_n^* - \frac{\beta(z)}{2} \sum_{n,m,l,k} R_{n,m,l,k} b_l b_m b_n^* b_k^*. \tag{7}$$

Here we introduce notations

$$S_{n,m} = \langle f_m | x^2 f_n \rangle = \int_{-\infty}^{+\infty} f_m(x) x^2 f_n(x) dx,$$

$$R_{n,m,l,k} = \langle f_m | f_n f_l f_k \rangle = \int_{-\infty}^{+\infty} f_n(x) f_m(x) f_l(x) f_k(x) dx. \tag{8}$$

Since integrals of the form $\int x^n e^{-\alpha x^2}$ can be calculated analytically, it is possible to determine any $S_{n,m}$ and $R_{n,m,l,k}$. As a next step we remove rapid oscillations by the simple transform $b_n = B_n e^{iR(z)\lambda_n}$ with $dR/dz = d(z)/T(z)^2 - \langle d/T^2 \rangle$. In Fig. 2 it is shown typical dynamics of T, M and R for specific practical dispersion map d. Equation for the B_n reads:

$$i\frac{dB_n}{dz} + \langle \frac{d}{T^2} \rangle \lambda_n B_m + \beta(z) \sum_m e^{i(\lambda_m - \lambda_n)R(z)} S_{n,m} B_m +$$

$$+\beta(z)\sum_{m,l,k} e^{i(\lambda_m+\lambda_l-\lambda_k-\lambda_n)R(z)} B_m B_l B_k^* R_{m,l,k,n} = 0. \quad (9)$$

The averaging procedure based on the Lie-transform [21] can be applied directly to the Eq. (9). Field B_n is then composed from slowly varying (U_n) and rapidly varying (η_n, $d\eta_n/dz \gg \eta_n$) parts: $B_n = U_n + \eta_n + ...$, where $\eta_n \ll U_n$. This procedure is a natural generalization of the result obtained in [17] for the case of strong dispersion management. Averaging over one period gives

$$i\frac{dU_n}{dz} = \frac{\delta H}{\delta U_n^*} = -\langle\frac{d}{T^2}\rangle\lambda_n U_n - \sum_m \langle\beta(z)e^{i(\lambda_m-\lambda_n)R(z)}\rangle \times$$

$$\times S_{n,m}U_m - \sum_{m,l,k}\langle\beta(z)e^{i(\lambda_m+\lambda_l-\lambda_k-\lambda_n)R(z)}\rangle U_m U_l U_k^* R_{m,l,k,n} = 0. \quad (10)$$

Hamiltonian H reads

$$H = \langle\frac{d}{T^2}\rangle \sum_{n=0}^{n=\infty}(1+2n)|U_n|^2 - \sum_{n,m}\langle\beta(z)e^{2i(n-m)R(z)}\rangle S_{n,m}U_m U_n^* -$$

$$-\frac{1}{2}\sum_{n,m,l,k}\langle\beta(z)e^{2i(n+k-l-m)R(z)}\rangle R_{n,m,l,k}U_m U_l U_n^* U_k^*. \quad (11)$$

Averaged Eq. (10) is the main result of the present paper. Solution of Eq. (10) of the form $U_n = F_n \exp(ikz)$ (with F_n non-dependent on z) presents DM soliton for any given dispersion map. Obtained equation permits to describe in a rigorous mathematical way the properties of DM solitons and more generally the propagation of any input signal for arbitrary dispersion map. Important observation from Eq. (11) is that the sign of the average effective dispersion $\langle d/T^2\rangle$ plays a crucial role in the dynamics of dispersion-managed pulse. The basic condition $\langle d\rangle > 0$ that provides the existence of the traditional fundamental soliton is replaced by the requirement $\langle d/T^2\rangle > 0$ for the DM soliton. This explains a possibility to transmit DM soliton at zero or normal average dispersion observed in [16]. We present an expansion of the DM soliton in terms of chirped Gauss – Hermite functions. Similar approach has been used in [22] to describe the propagation of linear chirped pulse. Dynamics of any initial bell-shaped pulse (bell-shape provides rapid convergence of the expansion) can be effectively described using the method developed here. Any well-localized pulse will be presented by a limited number of terms in the expansion. This makes the considered basis very useful in different practical applications and numerical simulations of dispersion-managed transmission systems. Developed expansion plays in the considered problem a role that the Fourier-transform plays in the linear problems.

Solution of the Eq. (1) in the general case can be expanded using complete set of chirped Gauss – Hermite functions

$$A(z,t) = N\,\frac{\exp(i\frac{M}{T}t^2)}{\sqrt{T}}\sum_{n=0}^{n=\infty} B_n(z) f_n[\frac{t}{T(z)}]\exp[i\lambda_n R(z)], \quad (12)$$

here

$$B_n(z) = \exp[-i\lambda_n R(z)]\int_{-\infty}^{\infty} dx\, f_n(x)\exp(-i\,M\,T\,x^2)\,A[z, x\,T(z)].$$

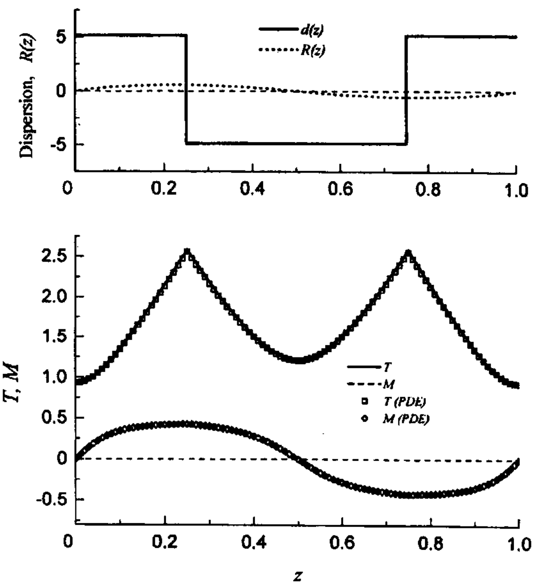

Fig.2. The dynamics of T (solid line - solution of Eqs. (3) and squares - solution of Eq. (1)), M (long-dashed line - solution of Eqs. (3) and rhombus - solution of Eq. (1)) and R (short-dashed line) over one period is presented for the dispersion map (above) $d(z) = \pm d + \langle d \rangle$, with $d = 5$ and $\langle d \rangle = 0.15$. A lossless model with $c(z) = 1$ is considered.

In Fig. 3 it is plotted evolution over one period of few first $|B_n|^2$ calculated from expansion of true DM soliton. It is seen that B_n^2 decay with increasing of n, though due to smallness of B_2 this decay is not exactly monotonic. Power of the solution in the general case is expressed as ($B_n = |B_n| \exp(i\Phi_n)$)

$$|A(z,t)|^2 = \frac{N^2}{T(z)} \sum_n |B_n(z)|^2 |f_n[\frac{t}{T(z)}]|^2 +$$

$$+\frac{N^2}{T(z)} \sum_{n,m} f_n[\frac{t}{T(z)}] f_m[\frac{t}{T(z)}] |B_n||B_m| \cos[2(n-m)R(z) + \Phi_m - \Phi_n]. \qquad (13)$$

First term in Eq. (13) presents a self-similar core of the solution and the last term is

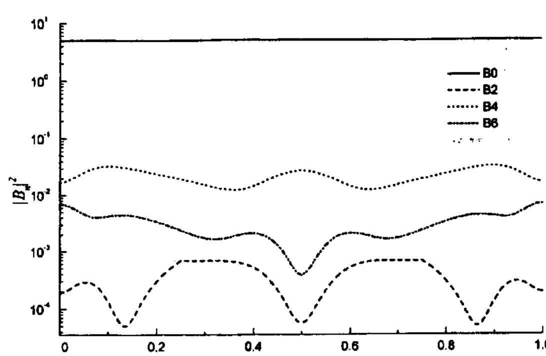

Fig. 3. Evolution over one period of the first coefficients $|B_n|^2$ in the expansion of dispersion-managed soliton (found numerically by solving Eq. (1))

responsible for non-self-similar oscillations of the tails inside compensation cell. It should be pointed out that the oscillating tails is an inherent part of the DM soliton, but not long-living radiative term as for the NLSE soliton [23].

To conclude, we have derived average equation describing the shape of dispersion-managed soliton for an arbitrary dispersion map. Using chirped Gauss – Hermite functions we have described both the self-similar structure of the main peak and the oscillating tails of the dispersion-managed pulse. Complete set of orthogonal chirped Gauss – Hermite functions is very useful in numerical simulations of the evolution of an arbitrary bell-shaped initial signal down the dispersion-managed fiber system.

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