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HOT-ELECTRON-PUMPED K-SHELL X-RAY LASER

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Amplified spontaneous inner shell emission produced via an ultrafast burst of high energy electrons from a femtosecond laser-produced plasma is proposed as a novel electron-pumped x-ray laser. In this scheme, a population inversion of the upper-laser level is created via impact ionization of atomic inner shells by electron bombardment. Based on the requirement of a positive gain coefficient for amplifying spontaneous K-α line emission, a simple pumping threshold is found for the incident electron flux and assessments are made of the feasibility of the scheme for a range of low-Z elements.

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It is well-known that bombardment of solids by energetic electrons can produce characteristic x-rays via ionization of atoms in the target material [1]. Selective ejection of electrons from inner shells can in principle create a population inversion, providing the conditions for lasing at X-ray wavelengths. However, owing to the high electron flux requirements, short pumping times and superior ionizing selectivity of photons compared to electrons, recent research has concentrated on photo-pumping by soft X-ray pulses from femtosecond laser-produced plasmas (LPPs) [2, 3]. The attraction of inner shell X-ray laser schemes is that they offer the possibility of much shorter wavelengths than either traditional collisional and recombination schemes or techniques based on optical field ionization [4] using sub-100 fs lasers [5].

In this Letter we present a simple analysis of an innershell ionization X-ray laser pumped by a hot-electron flux from a laser-produced plasma and investigate its feasibility for the K-α line emission of different materials, namely: carbon, fluorine, sodium and titanium as gain media. We believe that no such kind of assessments for this type of active lasing system has been earlier reported. Related experimental aspects for high power fs-laser systems are discussed.

A prerequisite to achieving lasing in an active medium is to create an innershell population inversion of the upper laser level determining the positive gain-length product. Underneath we follow the time-integrated pumping approach first proposed in [3] that is reasonable from the physical viewpoint and e.g., does not depend on any assumptions to construct appropriate rate equations to describe the evolution of the level populations.

Assuming that the number of vacancies (K-shell holes) per unit volume $N_K^{(vac)}$ is much smaller than the atomic density N_m of the medium: $N_K^{(vac)} \ll N_m$, then the number-vacancy distribution $n_K^{(vac)}$ pumped through an hot-electron K-shell ionization of ground state atoms as a function of the depth x into the medium is given by (cf. [3])

$$n_K^{(vac)}/dx = \sigma_K^{(ion)}(U) N_m \tau S \Phi(U), \qquad (1)$$

where $\tau\Phi(U)$ is the time-integrated pump electron flux per cm^2 bombarding a solid sample at depth x from a laser-plasma source; τ is the laser pulse duration and S is the illuminated area of the medium in [cm^2]. We assume that the duration of the electron burst $\tau_e \simeq \tau$ and further, that the K-shell lifetime τ_K is typically shorter than τ. $U(x, U_0)$ is the electron-impact energy at depth $x > 0$, and we define $U_0 \equiv U(x = 0)$ is the initial energy of an impact electron at the depth $x = 0$. The electron-impact K-shell ionization cross section $\sigma_K^{(ion)}[U(x, U_0)]$ is given for a large range of elements in Ref.[6]. The energy $U(x, U_0)$ at depth x can be found from the Bethe formula valid over the range $U_0 \simeq$ 10–100 keV [7]. Note that Eq.(1) takes into account the inversion population of the *upper* laser level related to the number of K-shell vacancies n_K only. The eventual saturation of this value due to other processes detrimental to the lasing effect – in particular the L-shell vacancy production – will be discussed in detail later. For simplicity, we neglect these additional destructive effects in order to obtain the gain threshold.

In Eq.(1) a monoenergetic electron beam has been assumed. In fact, electrons from a LPP generally have an energy distribution $f(U_0)$. It is then necessary to modify Eq.(1) substituting

$$\sigma_K^{(ion)}(U)\Phi(U) \rightarrow \overline{\sigma_K^{(ion)}(x)\Phi(x)} \equiv \int (dU_0 \;\; f(U_0) \times \sigma_K^{(ion)}(U)\Phi(U)).$$

For a monodirectional Maxwellian $f_H(U_0)$ has a normalized form $f_H(U_0) = U_0 T_H^{-2} \exp(-U_0/T_H)$, where T_H is the characteristic hot-electron temperature.

Let us designate the average gain coefficient of amplified spontaneous emission (ASE) as

$$\overline{G}_K \equiv X_H^{-1} \int_0^{X_H} G_K(x)dx,$$

where X_H is the hot-electron penetration depth, equal to the effective thickness of the gain medium. Then

$$\overline{G}_K = \sigma_K^{(stim)} \frac{N_m \tau_K}{X_H} \int_0^{X_H} dx \overline{\sigma_K^{(ion)}(x)\Phi(x)} - \mu_K, \tag{2}$$

where μ_K is the absorption coefficient of the K-α line radiation obtained (see, Ref.[8]). X_H can be estimated from Bethe's formula

$$X_H = \frac{4\pi}{e^4 N_m Z_m}[0.5(1 - \ln 2) + \ln(T_H/J)]^{-1}, \tag{3}$$

where Z_m is the atomic number of the medium, and $J = 13.6 Z_m$ eV is the average ionization potential. Setting $\overline{G}_K = 0$ in Eq.(2) we obtain an estimate for the time-integrated pump electron flux threshold over the effective lifetime τ_K

$$\Phi_H^{(thres)} \tau_K \approx \mu_K / \left(\sigma_K^{(stim)} N_m \sigma_K^{(ion)}(T_H)\right), \tag{4}$$

required to produce single-pass self-terminating ASE over the length of a gain medium equal to the lateral size of the pump hot-electron beam.

In the case under consideration here, the X-ray K-α line width mainly depends on a broadening of τ_K, in which case the cross section for stimulated emission $\sigma_K^{(stim)}$ is given by [9]

$$\sigma_K^{(stim)} = \lambda_K^2 \tau_K / 4\tau_K^{(rad)}, \tag{5}$$

where $1/\tau_K^{(rad)}$ is the probability of spontaneous K-α line emission. Note that the K-α transitions of the materials considered here have typical effective lifetimes τ_K of the order of few femtoseconds, which are much shorter than a laser pulse duration τ. Thus the effective pumping flux is reduced by a factor $\tau_K/\tau < 1$.

In order to evaluate Eq.(4), we need values of the electron impact ionization cross section for the production of K-shell vacancies in a variety of candidate media for lasing. The direct calculations taking into account corrections for weakly relativistic energies for the K-shell cross sections $\sigma_K^{(ion)}$ of carbon, fluorine, sodium and titanium definitely yields that of the four media investigated, carbon has the maximum cross section $\sigma_K^{(ion)} = = 10^{-19}$ cm^2 in the electron impact-energy range, extending well above the minimum energy threshold required for ejection of a K-shell electron to continuum. Using the numerical values of the $\sigma_K^{(ion)}$ obtained together with Eqs.(2)–(5), the resulting hot-electron flux threshold $\Phi_H^{(thres)}\tau_K$ $[10^{16}\text{cm}^{-2}]$ for the K-α lines of carbon, fluorine, sodium and titanium are equal to 0.163, 0.401, 1.099 and 21.035, respectively.

On the other hand, the corresponding values of $\sigma_L^{(ion)}$ are at least one order of magnitude larger than $\sigma_K^{(ion)}$, which at first sight appears to support the original conclusion made in Ref.[2] that electrons are not as well suited as photons for selectively removing electrons from inner shells. However, it will be shown later that L-shell ionization sets an effective upper flux limit which for low Z is sufficiently far above the K-shell threshold to allow a flux window in which positive gain is possible. We therefore initially consider the elements above as candidates for selective K-shell X-ray stimulated emission.

Before we can determine the X-ray gain which can be obtained with this type of scheme, we need to know the hot electron flux available from contemporary fs-laser-produced plasmas. Although a number of experimental studies of hot-electron generation with short pulse lasers now exist, there are considerable variations in the absorption levels and hot-electron temperatures found (see Ref.[10] for a recent review). To determine the hot-electron temperature T_H and number flux $\Phi[T_H]$, we have performed particle-in-cell (PIC) simulations for a range of intensities relevant to the electron-pumped X-ray laser scheme. By performing a series of these simulations for different laser intensity, we obtain the scaling law for the hot-electron temperatures T_H:

$$T_H = 7.5(I_{16}\lambda_{\mu m}^2)^{1/3}, \tag{6}$$

where I_{16} is the intensity in 10^{16}Wcm^{-2} and $\lambda_{\mu m}$ is the wavelength is microns. The hot-electron energy flux can be obtained from the energy balance equation

$$Q_H = \int dU_0 U_0 f_H(U_0) = \eta_H I\tau\cos\theta,$$

where η_H is the fraction of laser energy absorbed by the hot electrons and θ is the angle of incidence to the target normal. Assuming we can approximate the energy spectrum of hot electrons by a Maxwellian, we may write $Q_H = 2T_H\Phi_H\tau$, so that:

$$\Phi_H\tau = \eta_H I\tau\cos\theta/2T_H. \tag{7}$$

Both η_H and T_H are to be determined by simulation. In general, if $f_H(U_0)$ is highly non-Maxwellian, then Φ_H must also be determined numerically.

For the purposes of this study however, it suffices to use the temperature from the simulations given by Eq.(6), which on substituting into Eq.(7), gives the following expression for the time-integrated hot-electron flux:

$$\Phi_H \tau [10^{16}\text{cm}^{-2}] \simeq 0.9\eta_H I_{16}^{2/3} \lambda_{\mu m}^{-2/3} \tau_{fs} \cos\theta, \tag{8}$$

where τ_{fs} is the pulse length in femtoseconds. For example, the hot-electron number flux expected for a 10 fs Ti-Sa-laser at an intensity of 10^{16}Wcm^{-2} is $1.1 \cdot 10^{16}\text{cm}^{-2}$ with a temperature of 6 keV, whereas at 10^{18}Wcm^{-2} we would have $\Phi_H = 2.4 \cdot 10^{17}\text{cm}^{-2}$ with $T_H = 30$ keV. This assumes an average laser absorption of $\eta_H = 0.15$, which was roughly constant (± 2 %) over the range of parameters considered here, and no attempt was made to maximise it according to the plasma conditions (in fact, one would expect 50-60 % absorption at grazing incidence angles [11]). Note that Eq.(8) is not universal and does not take into account differences in target material, bulk plasma temperature, or lateral spreading of the electrons due to scattering.

For typical laser parameters considered here ($I \sim 10^{16} - 10^{17}\text{Wcm}^{-2}$, $\tau \sim 10$ fs), one finds that the ratio of the number of K-shell vacancies per unit volume to the atomic density $N_K^{(vac)}/N_m \sim 10^{-3}$–$10^{-2}$ for elements with $Z < 10$. However, for the L shell the ratio is of the order of unity, implying that the L-shell vacancy population equation should really be taken into account when calculating the phenomenological damping factor. Increasing the either flux or Z will increase the ratios $N_K^{(vac)}/N_m$, $N_L^{(vac)}/N_m$ etc., reducing and eventually reversing the gain.

A quantitative assessment of the threshold beyond which L-vacancy generation becomes dominant can be made within the scope of the present model by introducing a correction factor into Eq.(1). We assume that this correction has two principle contributions: on the one hand, the L-shell vacancies 'poison' the ground state of neutral atoms N_m, and on the other hand, they enhance the pumping hot-electron flux by adding a alightly 'colder' secondary electron flux. The pumping flux and density of neutral atoms are therefore modified as follows:

$$\Phi^{cor}\tau_K = \Phi\tau_K(1 + \tau_K(2U/m_e)^{1/2}\sigma_L N_m), \tag{9}$$

$$N_m^{cor} = N_m(1 - \sigma_L \Phi^{cor}\tau_K). \tag{10}$$

Combining Eq.(1) with Eqs.(9),(10), with $v_H = (2U/m_e)^{1/2}$, the correction factor to the K-vacancy production rate can be written:

$$A \equiv \frac{N_m^{cor}\Phi^{cor}}{N_m\Phi} = \Big[1 - \sigma_L\Phi\tau_K(1 + \tau_K(2U/m_e)^{1/2}\sigma_L N_m)^2 + \\ + \tau_K(2U/m_e)^{1/2}\sigma_L N_m\Big]. \tag{11}$$

The second term on the RHS of Eq.(11) diminishes the population of the upper laser level due to L-vacancy generation, so reducing the gain. In contrast, the third term actually enhances the number of K-shell vacancies owing to the flux of secondary electrons produced by the L-shell ionization. This leads to an enhancement of the inversion and gain. To give a numerical example, the correction factor for a Carbon target with $N_m = 10^{23}\text{cm}^{-3}$, $\sigma_L = 2 \cdot 10^{-18}\text{cm}^2$, $\tau_K = 10^{-14}$s, $v_H = 3 \cdot 10^9$cm/s ($2U \sim T_H = 10$keV) and $\Phi = 3 \cdot 10^{15}\text{cm}^{-2}$ is $A \simeq 1 - 0.2 + 6 = 6.8$, whereas for a $\Phi = 3 \cdot 10^{17}\text{cm}^{-2}$, we have

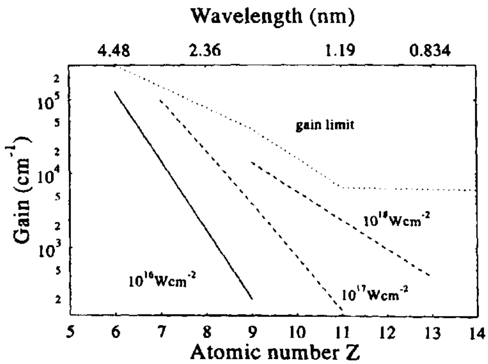
$A \simeq 1 - 22 + 6 = -15$. In other words, for a given element there should be an operating 'window' of flux in which we can expect amplification of the K_α line, the upper limit of which is determined by setting Eq.(10) to zero.

Combining Eqs.(2) and (11), averaging over the cross-section integral $\overline{\sigma}_K \simeq \simeq X_H^{-1} \int_0^{X_H} dx \sigma_K$, with $\Phi = \overline{\Phi} = \text{const.}$, we finally arrive at an expression for the gain including the effect of secondary electrons:

$$\overline{G}_K = \sigma_K^{(stim)} N_m \tau_K \overline{\Phi} \overline{\sigma}_K \Big[1 - \sigma_L \Phi \tau_K (1 + \tau_K (2U/m_e)^{1/2} \sigma_L N_m)^2 + + \tau_K (2U/m_e)^{1/2} \sigma_L N_m \Big] - \mu_K \qquad (12)$$

This expression is evaluated for different Z using values for τ_K and μ_K from Ref.[12].

We are now in a position to estimate the gain coefficient according to Eq.(12) by substituting Eq.(8) for the electron flux. This is done in Figure, which shows best-fit curves for different elements and laser intensities, assuming a constant pulse length of 10 fs.



Scaling of gain coefficients with Z for electron pumping at different laser intensities and wavelength of 0.8μm. The top x-axis shows the corresponding K_α wavelength. The 'gain limit' curve is obtained by setting $d\overline{G}_K/d\Phi = 0$ according to Eq.(12)

The gain-length product will of course depend on the lateral extent of the hot electrons, or the laser focal spot size. For example, a 1 J, 10 fs Ti-Sa laser focused to a spot size of $100\,\mu$m and intensity $3 \cdot 10^{17}$ Wcm^{-2} would yield $G_K L \simeq 130$ for fluorine. The maximum gain can be found by setting $d\overline{G}_K/d\Phi = 0$ and is shown as the upper dashed curve in Figure.

One could argue that the L-shell and secondary processes will always dominate because the absolute value of the 'correction' factor in Eq.(11) is substantially larger than unity. Nevertheless, we neglect all additional processes (production of tertiary electrons, effects of plasma radiation etc) on the grounds that these will mainly occur *after* the lasing lifetime τ_K.

The presence of satellite lines arising from transitions in multiply-ionized inner shells on the high-energy side of the K- and L-shell emission (see, *e.g.*, [13]) could also reduce the gain coefficients depicted in Figure. On the other hand, by including the fluorescence yields and impact cross sections for the production of satellites, the model presented here could be extended to *multimode* soft X-ray ASE pumping schemes by LPP hot electrons.

In conclusion, we have developed a semi-analytical model to assess the hot-electron pumping requirements for a transverse ASE positive gain on the K shell transitions of light elements. The scheme appears to be best suited to laser pulse lengths around 10 fs and elements with $Z \leq 15$, and offers a potential means of extending the operational regime of X-ray lasers down to sub-nanometer wavelengths.

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