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$U(N)$-MONOPOLES ON KERR BLACK HOLES

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We describe $U(N)$-monopoles ($N > 1$) on Kerr black holes by the parameters of the moduli space of holomorphic vector $U(N)$-bundles over S^2 with the help of the Grothendieck splitting theorem. For $N = 2, 3$ we estimate the corresponding monopole masses.

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1. The present paper is a natural continuation of our previous work [1] and one of the motivations of writing [1] was in the Kerr black hole case to realize the program performed in Refs. [2, 3] for the Schwarzschild and Reissner – Nordström black holes, namely, to try finding the additional quantum numbers (nonclassical hair) characterizing Kerr black holes that might help in building a statistical ensemble necessary to generate the Kerr black hole entropy.

The paper of Ref.[1] obtained some description of $U(1)$(Dirac)-monopoles on Kerr black holes. The given paper is devoted to the extension of the constructions of Ref.[1] to the $U(N)$-monopoles ($N > 1$) on Kerr black holes. In the present paper, however, we shall use a gauge somewhat different from the gauge employed in Ref. [1] to avoid unnecessary complications.

In the Kerr black hole case we use the ordinary set of the local Boyer – Lindquist coordinates t, r, ϑ, φ covering the standard topology $R^2 \times S^2$ of the 4D black hole spacetimes except for a set of the zero measure. At this the surface $t = \text{const}$, $r = \text{const}$ is an oblate ellipsoid with topology S^2 and the focal distance a while $0 \le \vartheta < \pi, 0 \le \varphi < 2\pi$. Under the circumstances we write down the Kerr metric in the form

$$ds^2 = g_{\mu\nu}dx^\mu \otimes dx^\nu \equiv (1 - 2Mr/\Sigma)dt^2 - \frac{\Sigma}{\Delta}dr^2 - \Sigma d\vartheta^2 -$$

$$-[(r^2 + a^2)^2 - \Delta a^2 \sin^2\vartheta]\frac{\sin^2\vartheta}{\Sigma}d\varphi^2 + \frac{4Mra\sin^2\vartheta}{\Sigma}dtd\varphi, \qquad (1)$$

with $\Sigma = r^2 + a^2\cos^2\vartheta$, $\Delta = r^2 - 2Mr + a^2$, $a = J/M$, where J, M are, respectively, a black hole mass and an angular moment. The components of metric $g^{\mu\nu}$ in the cotangent bundle of manifold $R^2 \times S^2$ with the metric (1) (in tangent bundle) needed in calculations below can be found in Ref.[1]. Besides we have $\delta = |\det(g_{\mu\nu})| = (\Sigma\sin\vartheta)^2$, $r_\pm = M \pm \sqrt{M^2 - a^2}$, so $r_+ \le r < \infty$, $0 \le \vartheta < \pi$, $0 \le \varphi < 2\pi$.

Throughout the paper we employ the system of units with $\hbar = c = G = 1$, unless explicitly stated. Finally, we shall denote $L_2(F)$ the set of the modulo square integrable complex functions on any manifold F furnished with an integration measure.

2. In order to obtain the infinite families of $U(N)$-monopoles for $N > 1$, we should use the Grothendieck splitting theorem [4, 5] which asserts that any complex vector bundle over S^2 (and, as a consequence, over $R^2 \times S^2$) of rank $N > 1$ is a direct sum of N suitable complex line bundles over S^2. As is known [6], there exists the countable number of nontrivial complex vector bundles of any rank $N > 1$ over $R^2 \times S^2$. The sections of such bundles can be qualified as topologically inequivalent configurations (TICs) of N-dimensional (massless) complex scalar field. The standard classification confronts some $n \in Z$ with each $U(N)$-bundle over $R^2 \times S^2$-topology. In what follows we shall call it the Chern number of the corresponding bundle. TIC with $n = 0$ can be called *untwisted* one while the rest of the TICs with $n \neq 0$ should be referrred to as *twisted.*

So far we tacitly implied that the $U(N)$-bundles were supposed to be differentiable. Really, they admit holomorphic structures and since each differentiable complex line bundle over S^2 admits only one holomorphic structure then the Grothendieck splitting theorem in fact gives a description of the moduli space M_N of N-dimensional holomorphic complex vector bundles over S^2. Namely, each N-dimensional holomorphic complex vector bundle over S^2 is defined by the only N-plet of integers $(k_1, k_2, \ldots, k_N) \in Z^N$, $k_1 \geq k_2 \geq \ldots \geq k_N$. Two of such N-plets (k_i) and (k'_i) define the same differentiable N-dimensional bundle if and only if $\sum\limits_i k_i = \sum\limits_i k'_i$.

As was shown in Ref. [1], each complex line bundle (with the Chern number k_i, $i = 1, 2, ..., N$) over $R^2 \times S^2$ with the metric (1) has a complete set of sections in $L_2(R^2 \times S^2)$, so using the fact that all the U(N)-bundles over $R^2 \times S^2$ can be trivialized over the bundle chart of local coordinates $(t, r, \vartheta, \varphi)$ covering almost the whole manifold $R^2 \times S^2$, the mentioned set can be written on the given chart in the form

$$f^{a\omega_i}_{k_i l_i m_i} = \frac{1}{\sqrt{r^2 + a^2}} e^{i\omega_i t} R^{a\omega_i}_{k_i l_i m_i}(r) Y_{k_i l_i m_i}(a\omega_i, \vartheta, \varphi), \quad l_i = |k_i|, |k_i|+1, \ldots, |m_i| \leq l_i\,, \tag{2}$$

where some properties of both the *monopole oblate spheroidal harmonics* $Y_{k_i l_i m_i}(a\omega_i, \vartheta, \varphi)$ and the conforming eigenvalues $\lambda_i = \lambda_{k_i l_i m_i}(a\omega_i)$ can be found in [1], but we shall not need them further. As to the functions $R^{a\omega_i}_{k_i l_i m_i}(r) = R$ then, in the gauge under discussion, they obey the equation

$$\frac{d}{dr}\Delta\frac{d}{dr}\left(\frac{R}{\sqrt{r^2+a^2}}\right) + \frac{(r^2+a^2)^2\omega_i^2 - 4Mm_i ra\omega_i + m_i^2 a^2}{\Delta}\frac{R}{\sqrt{r^2+a^2}} = \tag{3}$$

$$= -(\lambda_i + k_i^2)\frac{R}{\sqrt{r^2+a^2}}\,, \text{with } l_i = |k_i|, |k_i|+1, \ldots, |m_i| \leq l_i\,.$$

Now, in accordance with the Grothendieck splitting theorem, any section of N-dimensional complex bundle ξ_n over $R^2 \times S^2$ with the Chern number $n \in Z$ can be represented by a N-plet $(\phi_1, \ldots, \phi_N)$ of complex scalar fields ϕ_i, where each ϕ_i is a section of a complex line bundle over $R^2 \times S^2$. According to the above, we can consider ϕ_i the section of complex line bundle with the Chern number $k_i \in Z$, where the numbers k_i are subject to the conditions

$$k_1 \geq k_2 \geq \ldots \geq k_N\,, \quad k_1 + k_2 + \cdots + k_N = n\,. \tag{4}$$

As a consequence, we can require the N-plets $(f^{a\omega_1}_{k_1 l_1 m_1}, \ldots, f^{a\omega_N}_{k_N l_N m_N})$ to form the basis in $[L_2(R^2 \times S^2)]^N$ for the sections of ξ_n, $l_i = |k_i|, |k_i| + 1, \ldots, |m_i| \leq l_i$, and this will define the wave equation for a section $\phi = (\phi_1, \ldots, \phi_N)$ of ξ_n with respect to the metric (1)

$$\Bigg[I_N\Box - \frac{1}{\Sigma^2 \sin^2\vartheta} \times$$

$$\left.\begin{pmatrix} 2ik_1\cos\vartheta(a\sin^2\vartheta\partial_t + \partial_\varphi) - k_1^2\cos^2\vartheta & 0 & \ldots & 0 \\ 0 & 2ik_2\cos\vartheta(a\sin^2\vartheta\partial_t + \partial_\varphi) - k_2^2\cos^2\vartheta & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 2ik_N\cos\vartheta(a\sin^2\vartheta\partial_t + \partial_\varphi) - k_N^2\cos^2\vartheta \end{pmatrix}\right] \times$$

$$\times \begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_N \end{pmatrix} = 0\,; \tag{5}$$

where I_N is the unit matrix $N \times N$, $\Box = (\delta)^{-1/2}\partial_\mu(g^{\mu\nu}(\delta)^{1/2}\partial_\nu)$ — the conventional wave operator conforming to metric (1).

The Eq. (5) will, in turn, correspond to the lagrangian

$$\mathcal{L} = \delta^{1/2} g^{\mu\nu}\overline{\mathcal{D}_\mu\phi}\mathcal{D}_\nu\phi\,, \tag{6}$$

with $\phi = (\phi_i)$ and a covariant derivative $\mathcal{D}_\mu = \partial_\mu - igA^a_\mu T_a$ on sections of the bundle ξ_n, while the overbar in (6) signifies hermitian conjugation and the matrices T_a will form a basis of the Lie algebra of $U(N)$ in N-dimensional space (we, as is accepted in physics, consider the matrices T^a hermitian), $a = 1, \dots, N^2$, g is a gauge coupling constant, i. e., we come to a theory describing the interaction of a N-dimensional twisted complex scalar field with the gravitational field described by metric (1). The coefficients A^a_μ will represent a connection in the given bundle ξ_n and will describe some nonabelian $U(N)$-monopole.

As can be seen, the Eq.(5) has the form $\mathcal{D}^\mu\mathcal{D}_\mu\phi = 0$, where $\mathcal{D}^\mu$ is a formal adjoint to $\mathcal{D}_\mu$ with regards to the scalar product induced by metric (1) in $[L_2(R^2 \times S^2)]^N$. That is, the operator $\mathcal{D}^\mu$ acts on the differential forms $a_\mu dx^\mu$ with coefficients in the bundle ξ_n in accordance with the rule

$$\mathcal{D}^\mu(a_\nu dx^\nu) = -\frac{1}{\sqrt{\delta}}\partial_\mu(g^{\mu\nu}\sqrt{\delta}a_\nu) + ig\overline{A_\mu}g^{\mu\nu}a_\nu\,, \tag{7}$$

with $A_\mu = A^a_\mu T_a$.

As a result, the equation $\mathcal{D}^\mu\mathcal{D}_\mu\phi = 0$ takes the form

$$I_N\Box\phi - \frac{ig}{\sqrt{\delta}}\partial_\mu(g^{\mu\nu}\sqrt{\delta}A_\nu\phi) - (ig\overline{A_\mu}g^{\mu\nu}\partial_\nu + g^2 g^{\mu\nu}\overline{A_\mu}A_\nu)\phi = 0\,. \tag{8}$$

Comparing (5) with (8) gives

$$A^a_r T_a = A^a_\vartheta T_a = 0\,, \tag{9}$$

$$A^a_t T_a = \frac{a\cos\vartheta}{g\Sigma}\begin{pmatrix} k_1 & 0 & \dots & 0 \\ 0 & k_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & k_N \end{pmatrix}, \tag{10}$$

$$A^a_\varphi T_a = -\frac{(r^2+a^2)\cos\vartheta}{g\Sigma}\begin{pmatrix} k_1 & 0 & \dots & 0 \\ 0 & k_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & k_N \end{pmatrix}. \tag{11}$$

Under the circumstances the connection in the bundle ξ_n is $A = A^a_\mu T_a dx^\mu = A^a_t(r,\vartheta)T_a dt + A^a_\varphi(r,\vartheta)T_a d\varphi$ which yields the curvature matrix $F = dA + A \wedge A$ for ξ_n-bundle in the form

$$F = F^a_{\mu\nu}T_a dx^\mu \wedge dx^\nu = -\partial_r(A^a_t T_a)dt \wedge dr - \partial_\vartheta(A^a_t T_a)dt \wedge d\vartheta + \\ +\partial_r(A^a_\varphi T_a)dr \wedge d\varphi + \partial_\vartheta(A^a_\varphi T_a)d\vartheta \wedge d\varphi\,, \tag{12}$$

because the exterior differential $d = \partial_t dt + \partial_r dr + \partial_\vartheta d\vartheta + \partial_\varphi d\varphi$ in coordinates t, r, ϑ, φ. From here it follows that the first Chern class $c_1(\xi_n)$ of the bundle ξ_n can be chosen in the form

$$c_1(\xi_n) = \frac{g}{4\pi}\mathrm{Tr}(F)\,, \tag{13}$$

so that, when integrating $c_1(\xi_n)$ over any surface $t = \mathrm{const}$, $r = $ const, we shall have with using (4) and (11):

$$\int_{S^2} c_1(\xi_n) = \frac{g}{4\pi}\int_{S^2} \mathrm{Tr}[\partial_\vartheta(A^a_\varphi T_a)]d\vartheta \wedge d\varphi =$$

$$= -\frac{n}{4\pi}\int_{S^2} \frac{(r^2+a^2)(a^2\cos^2\vartheta - r^2)}{\Sigma^2}\sin\vartheta d\vartheta \wedge d\varphi = n\,, \qquad (14)$$

which is equivalent to the conventional Dirac charge quantization condition

$$qg = 4\pi n, \qquad (15)$$

with (nonabelian) magnetic charge

$$q = \int_{S^2} \mathrm{Tr}(F)\,. \qquad (16)$$

Introducing the Hodge star operator $*$ conforming metric (1) on 2-forms $F = F^a_{\mu\nu}T_a dx^\mu \wedge$ $\wedge dx^\nu$ with the values in the Lie algebra of $U(N)$ by the relation (see, e. g., Refs. [7])

$$(F^a_{\mu\nu}dx^\mu \wedge dx^\nu)\wedge(*F^a_{\alpha\beta}dx^\alpha \wedge dx^\beta) = (g^{\mu\alpha}g^{\nu\beta} - g^{\mu\beta}g^{\nu\alpha})F^a_{\mu\nu}F^a_{\alpha\beta}\sqrt{\delta}\,dx^0\wedge\cdots\wedge dx^3\,, \qquad (17)$$

written in local coordinates x^μ [there is no summation over a in (17)], in coordinates t, r, ϑ, φ we have for F of (12)

$$*F = *F^a_{\mu\nu}T_a dx^\mu \wedge dx^\nu =$$

$$= (g^{t\varphi}g^{\vartheta\vartheta}\frac{\partial A_t}{\partial\vartheta} + g^{\vartheta\vartheta}g^{\varphi\varphi}\frac{\partial A_\varphi}{\partial\vartheta})\sqrt{|g|}\,dt\wedge dr - (g^{\varphi t}g^{rr}\frac{\partial A_t}{\partial r} + g^{rr}g^{\varphi\varphi}\frac{\partial A_\varphi}{\partial r})\sqrt{|g|}\,dt\wedge d\vartheta +$$

$$+(g^{tt}g^{\vartheta\vartheta}\frac{\partial A_t}{\partial\vartheta} + g^{\vartheta\vartheta}g^{t\varphi}\frac{\partial A_\varphi}{\partial\vartheta})\sqrt{|g|}\,dr\wedge d\varphi - (g^{tt}g^{rr}\frac{\partial A_t}{\partial r} + g^{rr}g^{t\varphi}\frac{\partial A_\varphi}{\partial r})\sqrt{|g|}\,d\vartheta\wedge d\varphi, \qquad (18)$$

with $A_t = A^a_t T_a$ and $A_\varphi = A^a_\varphi T_a$ of (10)–(11). We can now consider the Yang – Mills equations

$$dF = F\wedge A - A\wedge F\,, \qquad (19)$$

$$d*F = *F\wedge A - A\wedge *F\,. \qquad (20)$$

It is clear that (19) is identically satisfied by the above A, F — this is just the Bianchi identity holding true for any connection [7].

As for the Eq. (20), then, it is easy to check with the help of (10), (11) and (18) that $*F\wedge A = A\wedge *F$. Under this situation, from (18) it follows that the condition $d*F = 0$ is equivalent to the equations

$$\frac{\partial}{\partial r}\left[\sqrt{|g|}\left(g^{rr}g^{\varphi t}\frac{\partial A_t}{\partial r} + g^{rr}g^{\varphi\varphi}\frac{\partial A_\varphi}{\partial r}\right)\right] + \frac{\partial}{\partial\vartheta}\left[\sqrt{|g|}\left(g^{t\varphi}g^{\vartheta\vartheta}\frac{\partial A_t}{\partial\vartheta} + g^{\vartheta\vartheta}g^{\varphi\varphi}\frac{\partial A_\varphi}{\partial\vartheta}\right)\right] = 0\,, \qquad (21)$$

$$\frac{\partial}{\partial r}\left[\sqrt{|g|}\left(g^{tt}g^{rr}\frac{\partial A_t}{\partial r} + g^{rr}g^{t\varphi}\frac{\partial A_\varphi}{\partial r}\right)\right] + \frac{\partial}{\partial\vartheta}\left[\sqrt{|g|}\left(g^{tt}g^{\vartheta\vartheta}\frac{\partial A_t}{\partial\vartheta} + g^{\vartheta\vartheta}g^{t\varphi}\frac{\partial A_\varphi}{\partial\vartheta}\right)\right] = 0\,. \qquad (22)$$

The direct evaluation with the aid of (10), (11) shows that (21), (22) are satisfied. As a consequence, the Eq. (20) is fulfilled.

One can notice, moreover, that

$$Q_e = \int\limits_{S^2} \mathrm{Tr}(*F) = -\int\limits_{S^2} g^{rr}\mathrm{Tr}\left(g^{tt}\frac{\partial A_t}{\partial r} + g^{t\varphi}\frac{\partial A_\varphi}{\partial r}\right)\sqrt{|g|}d\vartheta \wedge d\varphi = -\frac{4\pi anr}{e}\int\limits_{-1}^{1}\frac{x\,dx}{\Sigma^2} = 0\,, \tag{23}$$

where $x = \cos\vartheta$. As a result, an external observer does not see any (internal) nonabelian electric charge Q_e of the Kerr black hole for any given N. Besides it should be emphasized that the total (internal) nonabelian magnetic charge Q_m of black hole which should be considered as the one summed up over all the $U(N)$-monopoles for any given N remains equal to zero because

$$Q_m = \frac{4\pi}{g}\sum_{n\in \mathbf{Z}} n = 0\,, \tag{24}$$

so the external observer does not see any nonabelian magnetic charge of the Kerr black hole either though $U(N)$-monopoles are present on black hole in the sense described above.

To estimate the monopole masses we should use the T_{00}-component of the energy-momentum tensor

$$T_{\mu\nu} = \frac{1}{4\pi}(-F^a_{\mu\alpha}F^a_{\nu\beta}g^{\alpha\beta} + \frac{1}{4}F^a_{\beta\gamma}F^a_{\alpha\delta}\, g^{\alpha\beta}g^{\gamma\delta}g_{\mu\nu})\,. \tag{25}$$

Since we are in the asymptotically flat spacetime, we can calculate the sought masses according to

$$m_{mon}(k_1,\ldots,k_N) = \int\limits_{t=\mathrm{const}} T_{00}\sqrt{\gamma}\,dr\wedge d\vartheta\wedge d\varphi\,, \tag{26}$$

where

$$\sqrt{\gamma} = \sqrt{\det(\gamma_{ij})} = \sqrt{\Sigma/\Delta}\sin\vartheta\sqrt{(r^2+a^2)^2 - \Delta a^2\sin^2\vartheta} \tag{27}$$

for the metric $d\sigma^2 = \gamma_{ij}dx^i\otimes dx^j$ on the hypersurface $t = $const, while T_{00} is computed at the given $U(N)$-monopole. Under the circumstances it is not complicated to check that the leading term in asymptotic of $T_{00}\sqrt{\gamma}$ at $r\to\infty$ will be defined by the addend $g^{\vartheta\vartheta}g^{\varphi\varphi}(F^a_{\vartheta\varphi})^2$ of (25), so one should solve the equation

$$F^a_{\vartheta\varphi}T_a = \partial_\vartheta(A^a_\varphi T_a)\,, \tag{28}$$

with $A^a_\varphi T_a$ of (11). Let us concretize it for $N = 2,3$.

3. At $N = 2$ we can take $T_1 = I_2$, $T_a = \sigma_{a-1}$ at $a = 2,3,4$, where σ_{a-1} are the ordinary Pauli matrices. Then the Eq. (28) gives $F^2_{\vartheta\varphi} = F^3_{\vartheta\varphi} = 0$, while

$$F^1_{\vartheta\varphi} = \frac{1}{2}(k_1+k_2)f(r,\vartheta)\,,\quad F^4_{\vartheta\varphi} = \frac{1}{2}(k_1-k_2)f(r,\vartheta) \tag{29}$$

with

$$f(r,\vartheta) = -\partial_\vartheta\left[\frac{(r^2+a^2)\cos\vartheta}{g\Sigma}\right]\,. \tag{30}$$

This yields at $r\to\infty$

$$T_{00}\sqrt{\gamma} \sim \frac{\sin\vartheta}{64\pi g^2r^2}[(k_1+k_2)^2 + (k_1-k_2)^2]\,. \tag{31}$$

As a result, we can estimate (in usual units) according to (26)

$$m_{mon}(k_1,k_2) \sim \left(\frac{\hbar^2c^2}{G}\right)\frac{(k_1+k_2)^2+(k_1-k_2)^2}{16g^2}\int\limits_{r_+}^{\infty}\frac{dr}{r^2} =$$

$$= \frac{(k_1+k_2)^2+(k_1-k_2)^2}{16g^2r_+}\left(\frac{\hbar^2c^2}{G}\right). \qquad (32)$$

At $N = 3$ we can take $T_1 = I_3$, $T_a = \lambda_{a-1}$ at $a = 2,\ldots,9$, where λ_{a-1} are the Gell-Mann matrices. From (28) this yields $F^2_{\vartheta\varphi} = F^3_{\vartheta\varphi} = F^5_{\vartheta\varphi} = F^6_{\vartheta\varphi} = F^7_{\vartheta\varphi} = F^8_{\vartheta\varphi} = 0$, while

$$F^1_{\vartheta\varphi} = \frac{1}{3}(k_1+k_2+k_3)f(r,\vartheta),\ \ F^4_{\vartheta\varphi} = \frac{1}{2}(k_1-k_2)f(r,\vartheta),\ \ F^9_{\vartheta\varphi} = \frac{\sqrt{3}}{6}(k_1+k_2-2k_3)f(r,\vartheta) \qquad (33)$$

with $f(r,\vartheta)$ of (30). This gives

$$m_{mon}(k_1,k_2,k_3) \sim [(k_1+k_2+k_3)^2+\frac{9}{4}(k_1-k_2)^2+\frac{3}{4}(k_1+k_2-2k_3)^2]\frac{1}{36g^2r_+}\left(\frac{\hbar^2c^2}{G}\right). \quad (34)$$

It is clear that the case of arbitrary N can be treated analogously but we shall not dwell upon it here. One can only noticed that the important case is the one of $U(4)$-monopoles because 4-dimensional complex vector bundles could describe TICs of both spinors and vector charged fields, i. e. these TICs physically could arise due to interaction with $U(4)$-monopoles. But this task requires its separate consideration.

Under the circumstances, evaluating the corresponding Compton wavelength $\lambda_{mon}(k_i) = \hbar/m_{mon}(k_i)c$, we can see that at any $n \neq 0, N \geq 1$, $\lambda_{mon}(k_i) \ll r_g$, where $r_g = r_+G/c^2$ is a gravitational radius of Kerr black hole, if $g^2/\hbar c \ll 1$. As a consequence, we come to the conclusion that under certain conditions $U(N)$-monopoles might reside in Kerr black holes as quantum objects and, e. g., to markedly modify the Hawking radiation [8, 9] or to generate the black hole entropy [3].

So, we can see that the masses of $U(N)$-monopoles really depend on the parameters of the moduli space M_N of holomorphic vector bundles over S^2.

4. The results of both the present paper and Refs. [1 − 3, 8, 9] show that the 4D black hole physics can have a rich fine structure connected with the topology $R^2 \times S^2$ underlying the 4D black hole spacetime manifolds. It seems to be quite probable that this fine structure is tied with the moduli spaces M_N of N-dimensional holomorphic vector bundles over S^2 and could manifest itself in solving the whole number of problems within the 4D black hole physics, so that one should seemingly thoroughly study the arising possibilities, in particular, also in the Kerr – Newman metric case as a natural charged generalization of Kerr metric. We hope to keep on studying elsewhere.

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