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SELF-ENERGY CORRECTION TO THE HYPERFINE STRUCTURE SPLITTING OF THE $1s$ AND $2s$ STATES IN HYDROGENLIKE IONS

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The one-loop self-energy correction to the hyperfine structure splitting of the $1s$- and $2s$-states of hydrogenlike ions is calculated both for the point and finite nucleus. The results of the calculation are combined with other corrections to find the ground state hyperfine splitting in lithiumlike $^{209}Bi^{80+}$ and $^{165}Ho^{64+}$.

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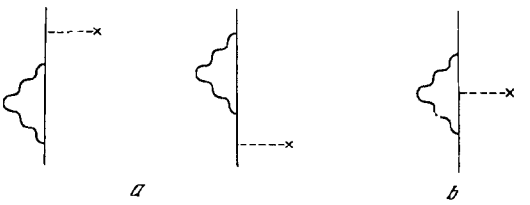
The recent experimental investigation of the ground-state hyperfine splitting of $^{209}Bi^{82+}$ [1] and $^{165}Ho^{66+}$ [2] shows that the present experimental accuracy is much higher than the accuracy of the corresponding theoretical values. At present, measurements of the ground state hyperfine splitting of lithium-like ions are designed. In this connection a necessity of an accurate calculation of the QED corrections to the hyperfine splitting of the $1s$- and $2s$-states of highly charged ions is obvious.

The one-loop self-energy correction to the first-order hyperfine interaction for the ground state of hydrogenlike ions in the case of an extended nucleus was calculated in [3] in a wide interval Z. In the case of $Z = 83$ and a point nucleus such a calculation was done in [4]. In the present work we recalculate the self-energy correction for the $1s$-state and present results for the $2s$-state. The calculation was made using the full-covariant scheme based on an expansion of the Dirac–Coulomb propagator in terms of interactions with the external potential [5, 6].



a *b*

Self energy-hyperfine interaction diagrams

The self energy contribution to the hyperfine splitting is defined by the diagrams shown in Figure where the dotted line denotes the hyperfine interaction. The formal expressions for these diagrams can easily be derived by the two-time Green function method [7]. The diagrams in Figure *a* are conveniently divided into irreducible and reducible parts. The reducible part is the part in which the

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intermediate-state energy (between the self energy and the hyperfine interaction line) coincides with the initial-state energy. The irreducible part is the remaining one. The irreducible part is calculated in the same way as the first order self-energy contribution. For a point nucleus the external wave function containing the hyperfine interaction line is calculated analytically by using the generalized virial relations for the Dirac–Coulomb problem [8]. For an extended nucleus a calculation of the external wave function was performed using the reduced Green function.

The reducible part is grouped with the vertex part presented in Figure *b*. According to the Ward identity the counterterms for the vertex and the reducible parts cancel each other and, so, the sum of these terms regularized in the same covariant way is ultraviolet finite. To cancel the ultraviolet divergences we separate free propagators from the bound electron lines and calculate them in the momentum representation. The remainder is ultraviolet finite but contains infrared divergences, which are explicitly separated and cancelled.

The calculations were carried out for both point and extended nucleus. In the last case the model of an uniformly charged shell with the radius $R = \frac{\sqrt{15}}{4}\langle r^2\rangle^{1/2}$ was used for the nuclear charge distribution. With high precision, this model is equivalent to the model of an uniformly charged sphere with the radius $R = \sqrt{\frac{5}{3}}\langle r^2\rangle^{1/2}$ if the first-order hyperfine structure splitting is calculated. Our test calculation shows a good agreement between these models for the self-energy correction to the hyperfine splitting too. The Green function expressed in terms of the Whittaker and Bessel functions [9] was used in the numerical calculation in the case of an extended nucleus. A part of the vertex term was calculated using the B-spline basis set method for the Dirac equation [10].

Table 1

Self-energy correction to the hyperfine splitting of the 1*s*-state in hydrogenlike ions

Z	$\langle r^2\rangle^{1/2}$	X_{point}	δX_{fin}	X_{total}	F_{total}
49	4.598	-1.057	0.042	-1.015	-2.629
59	4.892	-1.496	0.096	-1.400	-3.293
67	5.190	-1.995	0.192	-1.803	-3.856
75	5.351	-2.737	0.393	-2.344	-4.470
83	5.533	-3.940	0.850	-3.090	-5.141

The results of the calculation for the 1*s* state are listed in the table 1. The values of the root-mean-square nuclear charge radii given in the second column of the table are taken from [11]. The values X listed in the table are defined by the equation

$$\Delta E_{SE} = \alpha X \Delta E_{nr}\,, \tag{1}$$

where ΔE_{SE} is the self-energy correction to the hyperfine splitting, ΔE_{nr} is the non-relativistic value of the hyperfine splitting (Fermi energy), α is the fine structure constant. The results of the calculation for a point nucleus are listed in the third column of the table. The finite nuclear size contributions and the total self-energy corrections are given in the forth and fifth columns, respectively. In

the last column the values F defined by the equation

$$\Delta E_{SE} = \frac{\alpha}{\pi} F \Delta E_{rel} \tag{2}$$

are given. Here ΔE_{rel} is the relativistic value of the first-order hyperfine splitting including the finite nuclear size correction. The value F is more stable than X as respects to a variation of the nuclear parameters. In the table 2 the corresponding values for the $2s$-state of hydrogenlike ions are listed. The relative precision of the results is estimated to be not worse than $5 \cdot 10^{-3}$.

Table 2

Self-energy correction to the hyperfine splitting of the $2s$ state in hydrogenlike ions

Z	$\langle r^2 \rangle^{1/2}$	X_{point}	δX_{fin}	X_{total}	F_{total}
49	4.598	-1.073	0.050	-1.023	-2.437
59	4.892	-1.605	0.107	-1.498	-3.112
67	5.190	-2.267	0.232	-2.035	-3.696
75	5.351	-3.321	0.509	-2.812	-4.347
83	5.533	-5.157	1.185	-3.972	-5.076

As is mentioned above, the self-energy correction was calculated for the $1s$ state earlier in [3,4]. In [4] for $Z = 83$ and a point nucleus it was found $X = -3.8$, while the present calculation gives $X = -3.94$. This discrepancy results from a noncovariant regularization procedure using in [4], which, as it turned out, gives a small additional spurious term [2)] A comparison of the present results for an extended nucleus with the previous calculation of [3] reveals some discrepancy too. So, for $Z = 83$ in [3] it was obtained $F = 5.098$, while the present calculation gives $F = 5.141$ (a difference due to a discrepancy between the nuclear parameters is negligible). A detailed comparison of our calculation with one from [6] shows that this discrepancy results from a term in the vertex contribution omitted in [6].

Taking into account the present results for the self-energy correction and values of the other corrections (nuclear magnetization distribution, interelectronic interaction, and vacuum polarization) calculated in [12], we find that the wavelength of the hyperfine splitting transition for the $2s$-state in lithiumlike $^{209}Bi^{80+}$ and $^{165}Ho^{64+}$ is $\lambda = 1.548(9)\,\mu$m ($\mu = 4.1106(2)\mu_N$ [13]) and $\lambda = 4.059(13)\,\mu$m ($\mu = 4.132(5)\mu_N$ [14, 2]) respectively. The uncertainty of these values is mainly given by the nuclear magnetization distribution correction.

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