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SUPERFLUIDITY OF INDIRECT MAGNETOEXCITONS IN COUPLED QUANTUM WELLS

Yu.E.Lozovik[1)], *O.L.Berman*, *V.G.Tsvetus*

Institute of Spectroscopy
142092 Troitsk, Moscow region, Russia

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The temperature T_c of the Kosterlitz–Thouless transition to a superfluid state for a system of magnetoexcitons with spatially separated electrons e and holes h in coupled quantum wells is obtained as a function of magnetic field H and interlayer separation D. It is found that T_c *decreases as* a function of H and D at fixed exciton density n_{ex} as a result of an increase in the exciton magnetic mass. The highest Kosterlitz–Thouless transition temperature as a function of H *increases* (at small D) on account of an increase in the maximum magnetoexciton density n_{ex} versus magnetic field, where n_{ex} is determined by a competition between the magnetoexciton energy and the sum of the activation energies of incompressible Laughlin fluids of electrons and holes.

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Systems of excitons with spatially separated electrons e and holes h (indirect excitons) in coupled or double quantum wells (CQWs) in magnetic fields H are now under intensive experimental investigation [1 – 3]. They are of interest, in particular, in connection with the possibility of superfluidity of indirect excitons or e–h pairs, which would manifest itself in the CQWs as persistent electrical currents in each well [4], and also in connection with curious quasi-Josephson phenomena [5]. In high magnetic fields two-dimensional (2D) excitons exist in a substantially wider temperature region, as the exciton binding energies increase with increasing magnetic field [6, 7]. In addition, 2D e–h systems in high fields H are of interest because of the existence, under some conditions, of unique exact solutions of the many-body problem and nontrivial kinetic properties [8 – 10].

Attempts at experimental investigation of magnetoexciton superfluidity in CQWs [1] make it essential to study the magnetic-field dependence of the temperature of the phase transition to the superfluid state in systems of indirect excitons and to analyze the density of the superfluid component. This is the subject of this paper. It will be shown below that increasing the magnetic field at a fixed magnetoexciton density leads to a *lowering* of the Kosterlitz–Thouless transition temperature T_c on account of the increase of the exciton magnetic mass as a function of H. But it turns out that the highest possible Kosterlitz–Thouless transition temperature *increases* with increasing H (at small D) due to the rise in the maximum density of magnetoexcitons *versus* H.

To estimate the superfluid density let us start by obtaining the spectrum of collective excitations of a system of indirect magnetoexcitons.

For an isolated magnetoexciton there exists a conserved quantity [11] (the exciton magnetic momentum $\mathbf{P}$) connected with the invariance of the system upon a simultaneous translation of e and h and a gauge transformation (see [7, 12]). Here $\hat{\mathbf{P}} = -i\nabla_{\mathbf{R}} - \frac{e}{2}[\mathbf{H},\mathbf{r}]$, where $\mathbf{R} = \frac{1}{2}(\mathbf{r}_e + \mathbf{r}_h)$ are the coordinates of the center

[1)] e-mail: lozovik@isan.troitsk.ru

of mass, $\mathbf{r} = \mathbf{r}_e - \mathbf{r}_h$ are the internal exciton coordinates, and the cylindrical gauge for the vector potential is used: $\mathbf{A}_{e,h} = \frac{1}{2}[\mathbf{H}, \mathbf{r}_{e,h}]$ $(c = \hbar = 1)$.

The dispersion relation $\varepsilon(P)$ for an isolated magnetoexciton at small $\mathbf{P}$ is a quadratic function: $\varepsilon(P) \approx P^2/2m_H$, where m_H is the effective magnetic mass, which depends on H and the distance D between e layers and h layers (see [7]). For the magnetoexciton ground state $m_H > 0$.

The quadratic dispersion relation holds for small P at arbitrary H and follows from the fact that $P = 0$ is an extremal point of the dispersion relation $\varepsilon(P)$. The last statement may be proved by taking into account the regularity of the Hamiltonian $H_\mathbf{P}$ as function of the parameter $\mathbf{P}$ at $\mathbf{P} = 0$ and also the invariance of $H_\mathbf{P}$ upon simultaneous rotation of $\mathbf{r}$ and $\mathbf{P}$ in the plane of the CQW [12] ($H_\mathbf{P}$ is the effective Hamiltonian for eigenfunctions of $\mathbf{P}$ [6, 7, 11]).

For high magnetic fields $r_H \ll a_0^*$ and at $D \sim r_H$ the quadratic dispersion relation is valid at $P \ll 1/r_H$, but for $D \gg r_H$ it holds over a wider region: at least at $P \ll (1/r_H)(D/r_H)$ ($a_0^* = 1/2\mu e^2$ is the radius of a 2D exciton at $H = 0$; $\mu = m_e m_h/(m_e + m_h)$; $m_{e,h}$ are the effective masses of e and h; $r_H = (1/eH)^{1/2}$ is the magnetic length).

Using the quadratic dispersion relation for magnetoexcitons, one has at any H an expression for the magnetoexciton velocity analogous to that for the ordinary momentum, $\dot{\mathbf{R}} = \partial\varepsilon/\partial\mathbf{P} = \mathbf{P}/m_H$, and the following expression for the mass current of an isolated magnetoexciton:

$$\mathbf{J}(\mathbf{P}) = \frac{M}{m_H}\mathbf{P}, \tag{1}$$

where $M = m_e + m_h$.

The indirect magnetoexcitons interact as parallel dipoles if D is larger than the mean distance between an electron and hole along the quantum wells, $D \gg \langle \mathbf{r} \rangle$. But in high fields H one has $\langle \mathbf{r} \rangle \approx P r_H^2$ [6, 7]. Typical values of the magnetic momentum P for dilute magnetoexciton systems obey the inequality $P \ll \sqrt{n_{ex}}$ (due to the "large" logarithm $\ln(n_{ex})$ — see below). So the inequality $D \gg \langle \mathbf{r} \rangle$ is valid at $D \gg \sqrt{n_{ex}} r_H^2$.

The distinction between excitons and bosons is due to exchange effects [13]. But these effects for indirect excitons with spatially separated e and h in a dilute system $n_{ex}a^2(H, D) \ll 1$ are suppressed on account of the barrier associated with the dipole-dipole repulsion of the excitons [14] (e.g., in high fields H the small parameter is

$$\exp\left[-(2m_H)^{5/6} e^{5/3} D^{2/3} n_{ex}^{-1/12} \ln^{1/3}\left(1/8\pi n_{ex} m_H^2 e^4 D^4\right)\right];$$

$a(D, H)$ is the magnetoexciton radius along the quantum wells). Therefore exchange effects are negeligible and the system under consideration can be treated by the diagram technique for a boson system.

For a dilute 2D magnetoexciton system (at $n_{ex}a^2(D, H) \ll 1$) summation of the ladder diagrams is adequate. But in contrast with a 2D system without a magnetic field [15], some problems arise due to nonseparation of the relative motion of e and h and the magnetoexciton center-of-mass motion [6, 7, 11]. So the Green functions depend on both the external coordinates $\mathbf{R}, \mathbf{R}'$ and the internal coordinates $\mathbf{r}, \mathbf{r}'$. It is convenient to treat the problem in the representation of eigenfunctions $|nm\mathbf{P}\rangle$ of the Hamiltonian and magnetic momentum of an isolated magnetoexciton.

In high magnetic fields, when the typical interexciton interaction $D^2 n_{ex}^{-3/2} \ll \hbar\omega_c$ (where $\omega_c = eH/\mu$ is the cyclotron energy, and $\mu = m_e m_h/(m_e + m_h)$), one can ignore transitions between Landau levels and consider only the states on the lowest Landau level $m = n = 0$. Since r has a typical value of r_H and $P \ll 1/r_H$, the equation for the vertex Γ in the ladder approximation for a dilute magnetoexciton system $(n_{ex} a^2(H, D) \ll 1)$ in the $n, m, \mathbf{P}$ representation turns out to be analogous to that for a 2D boson system without a magnetic field, but with the magnetoexciton magnetic mass m_H (which depends on H and D) instead of the exciton mass $(m = m_e + m_h)$:

$$\Gamma(p, q; L) = U_F(\mathrm{p} - \mathrm{q}) + \int \frac{d\mathrm{l}}{(2\pi)^2} \frac{U_F(\mathrm{p}-\mathrm{l})\Gamma(l, q; L)}{\kappa^2/m_H + \Omega - L^2/4m_H - l^2/m_H + i\delta}, \qquad (2)$$

$$\mu = \kappa^2/2m_H = n_{ex}\Gamma = n_{ex}\Gamma(0, 0, 0).$$

Here μ is the chemical potential of the system (L is the sum and $2p$ the difference of the initial magnetic momenta of a pair of excitons, and $2q$ is the difference of the final magnetic momenta), and $U_F(P)$ is the Fourier transform of the potential energy $U(\mathbf{R}_1 - \mathbf{R}_2) = e^2 D^2/|\mathbf{R}_1 - \mathbf{R}_2|^3$.

At small magnetic momenta P the spectrum of collective excitations is $E(P) = c_s(H, D)P$, with the sound velocity $c_s = \sqrt{\mu/m_H}$.

A simple (analytical) solution for the chemical potential $\mu = \mu(H, D)$ can be obtained from Eq.(2), e.g., at $r_H \ll D \ll (r_H^4/n_{ex}^{1/2})^{1/5}$:

$$\mu = \kappa^2/2m_H = 8\pi n_{ex}/2m_H \ln\left(1/8\pi n_{ex} m_H^2 e^4 D^4\right).$$

So at fixed n_{ex} and in high magnetic fields the sound velocity in the magnetoexciton system (due to $m_H = m_H(H, D)$) falls off approximately as $H^{-1/2}$ at $D \ll r_H$ and as H^{-2} at $D \gg r_H$.

The temperature T_c of the Kosterlitz–Thouless transition [16] to the superfluid state in a 2D magnetoexciton system is determined by the equation $T_c = 0.5\pi\hbar^2 n_s(T_c)/k_B m_H$, where $n_s(T)$ is the superfluid density of the magnetoexciton system as a function of the temperature T, magnetic field H, and interlayer distance D, and k_B is Boltzmann's constant.

The function $n_s(T)$ can be found from the relation $n_s = n_{ex} - n_n$ (n_{ex} is the total density, n_n is the normal-component density). We determine the normal-component density by the usual procedure (see, e.g., [17]). Suppose that the magnetoexciton system moves with a velocity u. At $T \neq 0$ dissipating quasiparticles will appear in this system. Since their density is small at low T, one can assume that the gas of quasiparticles is an ideal Bose gas.

To calculate the superfluid-component density we start by finding the total current of quasiparticles in a frame in which the superfluid component is at rest. Using Eq.(1), we can see that the total current of the system is proportional to the total momentum, with a coefficient that depends on n_n. As a result, we have for the superfluid density

$$n_s = n_{ex} - n_n = n_{ex} - \frac{3\zeta(3)}{2\pi} \frac{T^3}{c_s^4 m_H}. \qquad (3)$$

Substituting the superfluid-component density n_s from Eq.(3) into the equation above for the Kosterlitz–Thouless temperature, we obtain:

$$T_c = \left[\left(1 + \sqrt{\frac{16}{(6\cdot 0.45)^3\pi^4}\left(\frac{m_H T_c^0}{n_{ex}}\right)^3 + 1}\right)^{1/3} + \left(1 - \sqrt{\frac{16}{(6\cdot 0.45)^3\pi^4}\left(\frac{m_H T_c^0}{n_{ex}}\right)^3 + 1}\right)^{1/3}\right]\frac{T_c^0}{(4\pi)^{1/3}}. \tag{4}$$

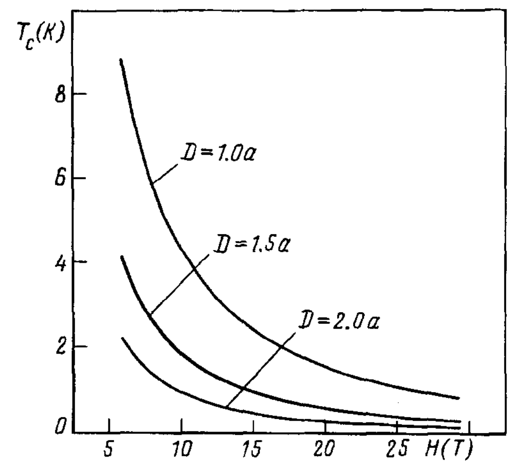
Here T_c^0 is an auxiliary quantity equal to the temperature T_c^0 at which the superfluid density vanishes in the mean field approximation; $n_s(T_c^0) = 0$:

$$T_c^0 = \left(\frac{2\pi n_{ex} c_s^4 m_H}{3\zeta(3)}\right)^{1/3}. \tag{5}$$

In high magnetic fields the Kosterlitz–Thouless temperature is:

$$T_c \approx \frac{T_c^0}{(4\pi)^{1/3}} \approx \left(\frac{32}{3\zeta(3)\ln^2(1/8\pi n_{ex} m_H^2 D^4)}\right)^{1/3} \frac{\pi n_{ex}}{m_H}. \tag{6}$$

In high fields H at small **P** the effective magnetic mass of an exciton on the lowest Landau level ($n = 0$) and with quantum number $m = 0$ is given by $m_H = 2^{3/2}/e^2 r_H\sqrt{\pi}$ at $D \ll r_H$ and by $m_H \approx D^3/e^2 r_H^4$ at $D \gg r_H$. At large D, i.e., for $D \gg a_0^*$ in weak fields ($r_H \gg a_0^*$) or for $D \gg r_H$ in high fields ($r_H \ll a_0^*$), one has $m_H = M + H^2 D^3/c^2$ [7, 12].



The Kosterlitz–Thouless transition temperature T_c as a function of the magnetic field H for different interwell separations D (for AsGa structures)

According to Eqs.(4) and (5) the temperature of the onset of superfluidity due to the Kosterlitz–Thouless transition at a fixed magnetoexciton density decreases as a function of magnetic field due to the increase in m_H as a function of H and D, while T_c decreases as $H^{-1/2}$ at $D \ll r_H$ or as H^{-2} at $D \gg r_H$, and n_s is a slowly decreasing function of D. The dependence of T_c on H is shown in Figure.

From Eqs.(4) and (5) one can see that the Kosterlitz–Thouless temperature of a dilute magnetoexciton system is proportional to the magnetoexciton density n_{ex}. At high magnetic fields the symmetry $\nu \to 1 - \nu$, $e \longleftrightarrow h$ obtains at the Landau level. Thus unoccupied states on Landau levels for spatially separated electrons and holes can bind to "antiexcitons", and superfluidity of antiexcitons may also take place at $1 - \nu \ll 1$. The Kosterlitz–Thouless temperature for superfluidity of antiexcitons as function of H and D is symmetrical to that for excitons. The top Kosterlitz–Thouless temperature at high magnetic fields corresponds to the "maximum" density n_{max} of magnetoexcitons at the Landau level $n_{max} = \nu_{max} 1/4\pi r_H^2 \sim H$, where $\nu_{max}(D)$ is the maximum filling of the Landau level for magnetoexcitons (for antiexcitons the corresponding critical value is $1 - \nu_{max}(D)$), which obeys $\nu_{max}(D) \leq 1/2$ (it is also possible to have an excitonic phase of the BCS type, originating from e–h pairing of composite fermions at $\nu = 1/2$ [12]). The excitonic phase is stable at $D < D_{cr}(H)$ when the magnetoexciton energy $E_{exc}(D, H)$ (calculated in [7]) is larger than the sum of the activation energies $E_L = ke^2/\epsilon r_H$ for incompressible Laughlin fluids of electrons or holes; $k = 0.06$ for $\nu = \frac{1}{3}$ etc. [18] (compare [9]). Since $k \ll 1$, the critical value $D_{cr} \gg r_H$. In this case one has

$$E_{exc} = \frac{e^2}{\epsilon D}(1 - \frac{r_H^2}{D^2})$$

for a magnetoexciton with quantum numbers $n = m = 0$. As a result we have $D_{cr} = r_H(1/2k - 2k)$. This estimate is correct for small ν when the interaction between magnetoexcitons is small in comparison with the magnetoexciton energy E_{exc}. For greater ν it gives an upper bound on D_{cr}. The coefficient k in the activation energy E_L may be represented as $k = k_0\sqrt{\nu}$. So from the relation between D_{cr} and r_H one has

$$\nu_{cr} = \frac{1}{k_0^2}\frac{r_H^2}{D^2}(1 - \frac{r_H^2}{8D^2}).$$

Hence, the maximum Kosterlitz–Thouless temperature at which superfluidity appears in the system is $T_c^{max} \sim n^{max}(H, D)/m_H \sim \sqrt{H}$ at $D \leq r_H$ or $T_c^{max} \sim H^{-1}$ at $D \gg r_H$ in high magnetic fields. It is of interest to compare this fact with experimental results on magnetoexciton systems. Note that if at a given density of e and h and a given magnetic field H several Landau levels are filled (but the high-field limit $r_H \ll a_0^*$ obtains) the superfluid phase can exist for magnetoexcitons on the highest nonfilled Landau level.

We have shown that at fixed exciton density n_{ex} the Kosterlitz–Thouless temperature T_c for the onset of superfluidity of magnetoexcitons *decreases* as a function of magnetic field as $H^{-\frac{1}{2}}$ (at $D \sim r_H$). But the maximum T_c (corresponding to the maximum magnetoexciton densities) *increases* with H in high magnetic fields as $T_c^{max}(H, D) \sim \sqrt{H}$ (at $D \sim r_H$). This fact needs to be compared in detail with the results of experimental studies of the collective properties of magnetoexcitons. The excitonic phase is more stable than the Laughlin states of electrons and holes (with negligible e–h correlations) at a given Landau filling ν if $D < D_{cr} = r_H(1/2k - 2k)$, where k is the coefficient in the Laughlin activation energy. Below the Kosterlitz–Thouless temperature one may observe the appearance of persistent currents in separate quantum wells. The interlayer resistance due to the drag of electrons and holes can also be a sensible indicator of the transition to the superfluid and other phases of the e–h system [12].

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