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QED CORRECTIONS TO DIS CROSS SECTION WITH TAGGED PHOTON

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We calculate the QED corrections to deep inelastic scattering cross section with tagged photon at HERA taking into account the leading and next-to-leading contributions. Different types for event selection are investigated.

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1. In order to investigate the proton structure functions $F_2(x, Q^2)$ and $F_L(x, Q^2)$ in a large region of its arguments it is necessary to measure the cross section of the process $e + p \rightarrow e + X$ with different values the center-of-mass energy. For this perpose one can employ a method suggested in [1], that utilizes the radiative events. This method takes advantage of a photon detector (PD) in forward direction along the 3-momentum of initial electron beam. Such a device is part of the luminosity monitors of H1 and ZEUS experiments at HERA.

The emission of photons in forward direction can be interpreted as a reduction of the effective beam energy which can be determined in every radiative event by means of tagged photon energy. For the high precision description of the corresponding cross section one has to consider the radiative corrections. In this letter we calculate the QED radiative corrections to deep inelastic scattering cross section provided at least one hard photon is recorded by PD. Our result includes leading and next-to-leading contributions for calorimeter and inclusive event selections. Our approach to the problem of the QED corrections calculation is based on the investigation of all essential Feynman diagrams which describe the observed cross section inside choosen approximation. The same approach was used recently for corresponding calculations in the case of small angle Bhabha scattering at LEP1 [2].

2. Our aim is to calculate the QED radiative correction to neutral current events with simultaneous detection of a hard photon emitted along initial electron with the energy ε in the process

$$e(p_1)+p(P)\to e(p_2)+\gamma(k)+X+(\gamma),$$

where ($\theta_1=\widehat{p_1k}\lesssim\theta_0\approx 5\cdot 10^{-4}$ rad). The cross-section under consideration in the lowest Born approximation has the following form:

$$z\frac{\mathrm{d}^3\sigma}{y\mathrm{d}x\mathrm{d}y\mathrm{d}z}=\frac{\alpha}{2\pi}P(z,L_0)\tilde{\Sigma},$$

$$\tilde{\Sigma}=\Sigma(\hat{x},\hat{y},\hat{Q}^2)=\frac{2\pi\alpha^2(-\hat{Q}^2)}{\hat{Q}^2\hat{x}\hat{y}^2}F_2(\hat{x},\hat{Q}^2)\left[2(1-\hat{y})-2\hat{x}^2\hat{y}^2\frac{M^2}{\hat{Q}^2}+\left(1+4\hat{x}^2\frac{M^2}{\hat{Q}^2}\right)\frac{\hat{y}^2}{1+R}\right],$$

$$R=\left(1+4\hat{x}^2\frac{M^2}{\hat{Q}^2}\right)\frac{F_2(\hat{x},\hat{Q}^2)}{2\hat{x}F_1(\hat{x},\hat{Q}^2)}-1,\quad P(z,L_0)=\frac{1+z^2}{1-z}L_0-\frac{2z}{1-z},\qquad (1)$$

$$\alpha(-\hat{Q}^2)=\frac{\alpha}{1-\Pi(-\hat{Q}^2)},\quad L_0=\ln\left(\frac{\varepsilon^2\theta_0^2}{m^2}\right),\quad z=\frac{\varepsilon-k^0}{\varepsilon}.$$

In the left side of Eq.(1) we use the standard Bjorken variables

$$x=\frac{Q^2}{2P(p_1-p_2)},\quad y=\frac{2P(p_1-p_2)}{V},\quad Q^2=2p_1p_2\ ,\qquad (2)$$

and in the right hand side – the *shifted* ones

$$\hat{Q}^2=zQ^2,\quad \hat{x}=\frac{zxy}{z+y-1},\quad \hat{y}=\frac{z+y-1}{z}.\qquad (3)$$

Quantities F_2 and F_1 are the proton structure functions. Here we intereste mainly with the events at small Q^2, therefore we neglect Z-boson exchange contribution.

The model independent QED correction to the Born cross section includes the contributions due to virtual and real (soft, with the energy less than $\Delta\varepsilon$, and hard, with the energy more than $\Delta\varepsilon$) photon emission. Virtual and soft photon correction can be derived by the help of Compton tensor given in [3]. The corresponding contribution to the observed cross section in the frame of our approximation reads

$$z\frac{\mathrm{d}^3\sigma^{SV}}{y\mathrm{d}x\mathrm{d}y\mathrm{d}z}=(\frac{\alpha}{2\pi})^2[P(z,L_0)\rho-T]\tilde{\Sigma}\ ,\qquad (4)$$

where

$$\rho=2(L_Q-1)\ln\frac{\Delta^2}{y_0}+3L_Q+3\ln z-\ln^2 y_0-\frac{\pi^2}{3}-\frac{9}{2}+L_{i2}\left(\frac{1+c}{2}\right)\ ,$$

$$T=\frac{1+z^2}{1-z}L_0[(2L_0L_Q-L_0-2\ln(1-z))\ln z+\ln^2 z--2L_{i2}(1-z)]-$$

$$- \frac{4z\ln z}{1-z} L_Q - \frac{1+2z-z^2}{2(1-z)} L_0, \tag{5}$$

$$L_Q = \ln \frac{Q^2}{m^2} \ , \quad y_0 = \frac{\varepsilon_2}{\varepsilon} \ , \quad L_{i2}(x) = - \int_0^x \frac{dt}{t} \ln(1-t) \ ,$$

ε_2 is the energy of scattered electron and $c = \cos\theta_s$ (θ_s is the electron scattering angle). The quantity y_0 can be expressed in terms of standard Bjorken variables as follows

$$y_0 = 1 - y + xyE/\varepsilon \ ,$$

where E is the proton energy.

As concerns the contribution due to additional hard photon emission we will divide it on three parts

$$z \frac{d^3\sigma^H}{y dx dy dz} = (\frac{\alpha}{2\pi})^2 (\Sigma_a + \Sigma_b + \Sigma_{sc}) \ . \tag{6}$$

The first one describes the case when the additional photon belongs to PD. The corresponding contribution can be derived using formula (П.6) in [4]. It reads

$$\Sigma_a = \frac{1}{2} L_0 \Bigg[L_0 \Bigg(P_\theta^{(2)}(z) + 2\frac{1+z^2}{1-z}(\ln z - \frac{3}{2} - 2\ln\Delta) \Bigg) +$$

$$+ 6(1-z) + \Bigg(-1 - z + 4\frac{1}{1-z} \Bigg) \ln^2 z - 4\frac{(1+z)^2}{1-z} \ln \frac{1-z}{\Delta} \Bigg] \tilde{\Sigma} \ , \tag{7}$$

where we use the notation $P_\theta^{(2)}(z)$ for the θ-part of the second order kernel of expansion of the nonsinglet electron structure function:

$$P_\theta^{(2)}(z) = 2[\frac{1+z^2}{1-z}(2\ln(1-z) - \ln z + \frac{3}{2}) + \frac{1}{2}(1+z)\ln z - 1 + z] \ .$$

The second term in the right hand side of Eq.(6) describes the case when additional hard photon moves along the final electron and belongs to small cone with the opening angle θ_0' centered along the electron momentum. The corresponding contribution depends on the manner of event selection. For inclusive event selection when only electron energy is detected in the final state the result can be derived using the formula (П.8) in [4]

$$\Sigma_b^{incl} = P(z, L_0) \int_{\Delta/y_0}^{y_2^{max}} \frac{dy_2}{1+y_2} \left[\frac{1 + (1+y_2)^2}{y_2} (\widetilde{L} - 1) + y_2 \right] \Sigma_s, \quad \Sigma_s = \Sigma(x_b, y_b, Q_b^2), \tag{8}$$

where

$$\widetilde{L} = \ln(\varepsilon\theta_0'/m)^2 + 2\ln y_0, \quad y_2 = \frac{x_2}{y_0}, \quad y_2^{max} = \frac{2z - y_0(1+c)}{y_0(1+c)} \ ,$$

$$x_b = \frac{zxy(1+y_2)}{z - (1-y)(1+y_2)} \ , \quad y_b = \frac{z - (1-y)(1+y_2)}{z} \ , \quad Q_b^2 = Q^2 z(1+y_2) \ .$$

More realistic is calorimeter event selection, when inside the small cone with opening angle $2\theta_0'$ along the momentum of the final electron the full energy of the scattered electron and photon is detected. If photon escapes this cone only the energy of the scattered electron is measured. In this case the result can be written as follows

$$\Sigma_b^{cal} = P(z,L_0)\int\limits_{\Delta/y_0}^{\infty}\frac{dy_2}{(1+y_2)^3}\left[\frac{1+(1+y_2)^2}{y_2}(\tilde{L}-1)+y_2\right]\tilde{\Sigma}. \tag{9}$$

We use in the last equation the relation

$$\Sigma_s = \frac{1}{(1+y_2)^2}\tilde{\Sigma}, \tag{10}$$

which is valid in the calorimetrical setup. Note that the quantity Σ_a does not depend on event selection.

The third term in the right hand side of Eq.(6) describes the contribution due to semicollinear kinematics, when the additional photon escapes both, the PD and the narrow cone along the 3-momentum of the scattered electron. We note that in this case the event selection by definition is inclusive. The result can be derived by the help of the quasireal electron method [5] and has the form

$$\Sigma_{sc} = P(z,L_0)\left[\int\limits_{\Delta}^{x_2^t}\frac{dx_2}{x_2}\frac{z^2+(z-x_2)^2}{z(z-x_2)}\ln\frac{2(1-c)}{\theta_0^2}\Sigma_t + \right.$$

$$\left. + \int\limits_{\Delta}^{y_2^{max}}\frac{dy_2}{y_2}\frac{1+(1+y_2)^2}{1+y_2}\ln\frac{2(1-c)}{\theta_0'^2}\Sigma_s + Z\right] \quad , \quad \Sigma_t = \Sigma(x_t,y_t,Q_t^2), \tag{11}$$

$$x_t = \frac{xy(z-x_2)}{z-x_2+y-1} \quad , \quad y_t = \frac{z-x_2+y-1}{z-x_2} \quad , \quad Q_t^2 = Q^2(z-x_2) \quad x_2^t = z - y_0(1+c)/2 \ ,$$

where the quantity Z represents the integral on the whole photon phase space of the function free from collinear and infrared singularities. It reads

$$Z = -\frac{4(1-c)}{Q^2}\int\limits_0^{\infty}\frac{du}{1+u^2}\left[\int\limits_{\eta}^{1}\frac{dt_1}{t_1|t_1-a|}\int\limits_0^{x_m}\frac{dx_2}{x_2}(\Phi(t_1,t(t_1,u)-\Phi(a,0))+\right.$$

$$\left. + \int\limits_{\eta}^{a}\frac{dt_1}{t_1 a}\int\limits_0^{x_m}\frac{dx_2}{x_2}(\Phi(a,0)-\Phi(0,a))\right]_{\eta\to 0} , \tag{12}$$

where

$$t(t_1,u) = \frac{(a-t_1)^2(1+u^2)}{y_+ + u^2 y_-} \ .$$

We put here the explicite expression for $\Phi(t_1,t_2)$:

$$\Phi(t_1,t_2) = \alpha^2(Q_{sc}^2)\left[\left(\frac{M^2 x_{sc}}{Q_{sc}^2}F_2(x_{sc},Q_{sc}^2) - F_1(x_{sc},Q_{sc}^2)\right)\frac{Q_{sc}^4 - 2st + z^2Q^4}{Q_{sc}^4} + \right.$$

$$+\frac{V^2}{Q^4_{sc}}F_2(x_{sc},Q^2_{sc})\left[x_{sc}(z^2+(1-y)^2)+xy(1-y)(z-\frac{s}{Q^2})-xyz(z-\frac{t}{Q^2})\right]\Bigg],$$

$$Q^2_{sc}=zQ^2-s-t,\quad s=4\epsilon^2 y_0x_2t_2,\quad t=-4\epsilon^2 zx_2t_1, \tag{13}$$

and x_m has a form:

$$x_m=\frac{z(e+p)-\Delta_m-y_0(e+z)-(p-z)y_0c}{z+e-y_0+(p-z)c_1+y_0c_2},\quad e=\frac{E_p}{\epsilon},$$

$$p=\frac{P_p}{\epsilon},\quad \Delta_m=\frac{(M+m_\pi)^2-M^2}{2\epsilon^2}, \tag{14}$$

where m_π is the pion mass.

3. The final result is the sum of (4) and (6). It is convenient to represent it as a sum of two terms

$$z\frac{d\sigma}{ydydx}=\left(\frac{\alpha}{2\pi}\right)^2(\Sigma_i+\Sigma_f)\ , \tag{15}$$

where the first one is universal, another words, does not depend on the manner of event selection. It can be written as follows

$$\Sigma_i=\Bigg[\frac{1}{2}L_0^2P_\theta^{(2)}(z)+P(z,L_0)\Bigg[\frac{1-16x-z^2}{2(1+z^2)}+(3-2\ln y_0+\frac{4z}{1+z^2}\frac{L_Q}{L_0})\ln z+$$

$$+\ln^2 y_0-2\mathrm{Li}_2(z)+2\mathrm{Li}_2(\frac{1+c}{2})-\frac{2(1+z)^2}{1+z^2}\ln(1-z)+\frac{1-z^2}{2(1+z^2)}\ln^2 z\Bigg]\Bigg]\tilde{\Sigma}+$$

$$+P(z,L_0)\tilde{\Sigma}\ln\frac{2(1-c)}{\theta_0^2}\left[\int\limits_0^{u_0}\frac{du}{u}(1+(1-u)^2)\left(\frac{\Sigma_t}{(1-u)\tilde{\Sigma}}-1\right)-\int\limits_{u_0}^{1}\frac{du}{u}(1+(1-u)^2)\right]+Z, \tag{16}$$

$$u=x_2/z\ ,\quad u_0=x_t/z\ .$$

Because the quantity θ_0 is the physical parameter which defines the condition for the tagged photon registration, it enters in the final result.

The second term in the right hand side of Eq.(15) just depens on event selection. It is defined with final-state radiation only. For inclusive event selection we have

$$\Sigma_f=\Sigma_f^{incl}=P(z,L_0)\int\limits_0^{x_2'/y_0}\Bigg[(\frac{1+(1+y_2)^2}{y_2}L+y_2)\frac{1}{1+y_2}\theta(y_2-\frac{\Delta}{y_0})+$$

$$+L\delta(y_2)(2\ln\frac{\Delta}{y_0}+\frac{3}{2})\Bigg]\Sigma_s dy_2\ ,\quad L=L_Q+\ln y_0-1\ . \tag{17}$$

In this case the parameter θ_0' play the role of auxiliary ones and does not enter the expression for the cross section.

In the more realistic, calorimetrical setup, the counter of events does not distinguish the events with bare electron and the ones when electron is accompanied with hard photon both emitted within small cone with the opening angle $2\theta_0'$ along the direction of the scattered electron. In this setup

$$\Sigma_f = \Sigma_f^{cal} = P(z, L_0)\left[\frac{1}{2}\tilde{\Sigma}+\ln\frac{2(1-c)}{\theta_0'^2}\int\limits_0^\infty \frac{dy_2}{y_2}\frac{1+(1+y_2)^2}{1+y_2}\left[\theta(y_2^s-y_2)\Sigma_s-\frac{1}{(1+y_2)^2}\tilde{\Sigma}\right]\right]. \tag{18}$$

In the calorimetrical manner of event selection the parameter θ_0' is the physical one and the final result depends on it. However the mass singularity, connected with the emission of the scattered electron is cancelled in accordance with the Lee-Nauenberg theorem [6].

The numerical calculations for some experimental setups at HERA will be given in next publications [7].

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