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HALL RESISTANCE ANOMALIES INDUCED BY A RADIATION FIELD

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A study is made of the four-terminal Hall junction under the influence of a radiation field. The frequency of the radiation field is tuned to a transition between the energy of a bound state below a conduction subband and the Fermi energy of the incident electrons. Radiation-field-induced resonant dips of the Hall resistance are exhibited at low magnetic fields.

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For several decades the transport of electrons in structures of low dimensionality and complicated geometry has been the focus of extensive theoretical and experimental study. Electrons can be confined to very narrow regions fabricated on an interface of an AlGaAs/GaAs heterostructure. Since the electrons in such regions can have high mobilities in the two dimensions available to them, such systems are called two-dimensional electron gases (2DEGs). The study of electronic transport properties of 2DEGs is of great current interest not only from the standpoint of the basic quantum effects involved but also for potential engineering applications. An idealized sample becomes an electron waveguide, wherein the quantum transport properties are solely determined by the geometry of the structure and the wavelike nature of the electrons. A remarkable manifestation of the successful achievement of quantum ballistic transport through a semiconductor nanostructure is the observation of quantized steps on the conductance through a narrow structure as the number of one-dimensional channels is successively varied [1, 2], the quenching of the Hall effect, and the last plateau and the negative bend resistance in the cross geometry [3, 4, 5].

Ford et al. [5] presented a systematic investigation of the influence of cross geometry on the Hall effect. They fabricated various differently shaped cross sections based on GaAs–$Al_xGa_{1-x}As$, which demonstrated that near zero magnetic field the Hall resistance can be quenched, enhanced over its classical value, or even negative. This effect has been considered in detail theoretically by Schult et al. [6] and Amemiya and Kawamura [7]. The aim of the present article is to demonstrate similar effects induced by a radiation field which is assumed to be resonant to the energy of a transition between a bound state of the cross section and the Fermi energy of the incident electron state. The radiation field mixes the localized bound state to the propagating wave functions and changes the quantum mechanical interference within the cross section. Therefore the application of a radiation field to a Hall junction is similar to a variation of the geometry of the junction.

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The bound states in a four-terminal junction of narrow wires have been found independently by Schult et al. [6] and Peeters [8] (see also [9, 10], and those in quantum wires with circular bends have been found by Exner and Šeba [11] (see also [12, 13, 14]). For a stationary energy-conserving process of electron transmission through a 2D structure, only quasibound states with energies within the conduction subbands are important [6, 7]. In particular, it was shown that the quasibound states in a Hall junction result in resonant dips of the resistance in high magnetic fields.

Although the bound states below the lowest subband threshold do not participate in the steady-state transmission, the possibility of observing of them, at least in principle, was shown by Berggren and Ji in the case of two intersecting electron waveguides with finite electrodes [15]. In that case the bound states can be probed by resonant tunneling through the electrodes below the subband. However, it is possible to include the bound states directly in the electron transmission through a Hall junction with infinite electrodes by applying a radiation field, provided that matrix elements between the bound state and propagating states are not equal to zero. Let E_0 be the energy of the bound state below the subband transmission energies, which for zero external magnetic field are equal to $E(k) = \frac{\hbar^2}{2m^* d^2}(k^2 + \pi^2 n^2)$, where d is the width of the electrodes, n is the number of the subband, and k is the wave number of incident electron. At a perturbation frequency $\hbar\omega = E(k) - E_0$ one can expect resonant anomalies in the electron transmission through a Hall junction.

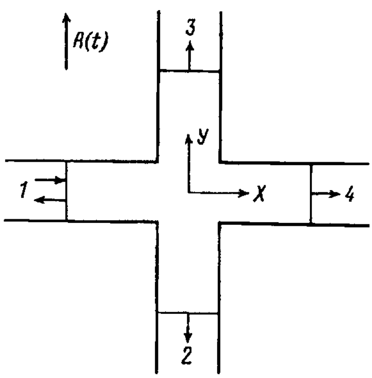

Fig.1. Schematic representation of a four-terminal Hall junction. A static magnetic field is applied normal to the plane of the junction. The input and output wave functions are shown by arrows. The solutions are to be matched across the boundaries shown by fine lines

Theoretically, the simplest device is the four-terminal junction (Fig. 1), for which the steady-state transport was studied in [6, 9, 7]. That junction is a combining element of the Hall structures. The Schrödinger equation for an electron of mass m^* subjected to a magnetic field B applied normal to the junction and to a radiation field $\mathbf{A}_1(t)$ directed in the plane of the junction (Fig. 1) can be written as follows

$$i\hbar \frac{\partial \psi(\mathbf{r},t)}{\partial t} = \frac{\hbar^2}{2m} \left(i\nabla + \frac{e}{\hbar c}(\mathbf{A}_0(\mathbf{r}) + \mathbf{A}_1 \cos\omega t) \right)^2 \psi(\mathbf{r},t). \quad (1)$$

Here we use the gauge $\mathbf{A}_0(\mathbf{r}) = (-By, 0, 0)$. The radiation field is considered in the long-wavelength approximation, in which the wavelength of the radiation field is much greater than the size of the junction. We use the following procedure of dimensionless transformations:

$$t \to \hbar t/2md^2, \quad \mathbf{r} \to \mathbf{r}/d, \quad \epsilon = 2md^2 E/\hbar^2,$$
$$\omega \to 2md^2\omega/\hbar, \quad \mathbf{a} = 2\pi d\mathbf{A}_1/\phi_0, \quad \tilde{B} = 2\pi d^2 B/\phi_0, \tag{2}$$

where $\phi_0 = ch/e$ is the magnetic flux quantum.

Since inside the Hall junction the processes of absorption/emission of photons give rise to satellite channels of transmission at quasienergies $E + n\hbar\omega$, we write the wave function of the electrodes as [17, 18]

$$\phi(\mathbf{r}, t) = \sum_n exp[-i(E + \hbar n\omega)t]\phi_{E+\hbar n\omega}(\mathbf{r}). \tag{3}$$

Substituting (3) into (1) with a gauge transformation of the wave function,

$$\psi(\mathbf{r}, t) = \exp\left(ie\frac{\mathbf{A}_1\mathbf{r}}{\hbar c}\cos\omega t\right)\phi(\mathbf{r}, t),$$

and using the dimensionless variables (2), we obtain the following equation for the satellite functions $\phi_{\epsilon+n\omega}(\mathbf{r}) = \phi_n(\mathbf{r})$:

$$(\epsilon + n\omega)\phi_n = (i\nabla + \mathbf{a}_0(\mathbf{r}))^2\phi_n + \frac{i\omega}{2}(\mathbf{a}\mathbf{r})(\phi_{n+1} - \phi_{n-1}), \tag{4}$$

where $\mathbf{a}_0(\mathbf{r}) = (-\tilde{B}y, 0, 0)$. In order to simplify the solutions inside the electrodes we assume that the perturbation is weak and not in resonance with transitions between subbands for a fixed energy of the incident electron in the electrodes. That allows us to neglect by the last term in (4) and write the solution inside electrodes 1 and 4 as

$$(1,4); \quad \chi_\epsilon(\mathbf{r}, t) = e^{i(kx-\epsilon t)}f_k(y), \tag{5}$$

where $f_k(y)$ satisfies the equation

$$\epsilon f_k(y) = -\frac{d^2}{dy^2}f_k(y) + (k + \tilde{B}y)^2 f_k(y). \tag{6}$$

In electrodes 2 and 3 it is convenient to use the Truscott transformation [19]

$$\phi(\mathbf{r}, t) = \varphi(x, t)\exp\left[-\frac{i}{2}a^2 t + \frac{2ika}{\omega}\sin\omega t + iy(k - a\cos\omega t) - \frac{ia^2}{4\omega}\sin 2\omega t\right]$$

and the following gauge for the magnetic field: $\mathbf{a}_0(\mathbf{r}) = (0, \tilde{B}x, 0)$. We then have

$$(2,3); \quad i\frac{\partial}{\partial t}\varphi(x, t) = \left[-\frac{\partial^2}{\partial x^2} + (k - \tilde{B}x)^2 + 2\tilde{B}ax\cos\omega t\right]\varphi(x, t). \tag{7}$$

Further we consider weak perturbations $\tilde{B}ad \ll 1$, for which the effect of the time-dependent term in (7) can be neglected. Actually, in computer calculations this restriction can be relaxed considerably. In that case Eq. (7) transforms to a form which is similar to Eq. (6). Therefore, in electrodes 2 and 3 one can write approximate solutions in the same form as in electrodes 1 and 4, except for the phase

$$\tilde{\chi}_\epsilon(\mathbf{r}, t) = \exp\left\{-i\left[\tilde{B}xy + \epsilon t - \frac{2ak}{\omega}\sin\omega t - y(k - a\cos\omega t)\right]\right\} f_{-k}(x). \tag{8}$$

Also, we have neglected the a^2 terms in (8).

Inside the Hall junction, with boundaries in each electrode (shown by fine lines in Fig. 1), is the scattering region, in which there are two bound states with energies ϵ_0 and ϵ_1, with $\epsilon_0 < \pi^2$ and $\pi^2 < \epsilon_1 < 4\pi^2$ [6]. The positions of the boundaries in the electrodes are choosen so that the contribution of evanescent modes at the boundaries in computer simulations is negligible.

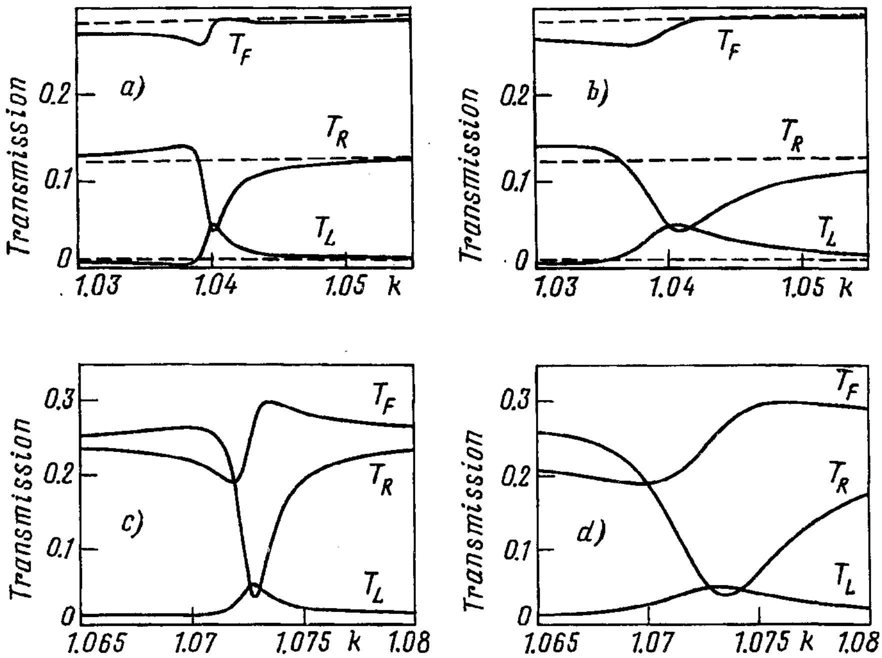

Fig.2. The energy (wave number) dependence of the transmission probabilities in the Hall junction. T_F, T_R, and T_L are the probabilities of transmission from the first electrode to the fourth, second, and third electrodes, respectively (see Fig. 1). The steady-state case for zero radiation field is shown by dashed lines. (*a*) $\tilde{B} = 1.0, a = 0.05$; (*b*) $\tilde{B} = 1.0, a = 0.1$; (*c*) $\tilde{B} = 2.0, a = 0.05$; (*d*) $\tilde{B} = 2.0, a = 0.1$

Since we have assumed that the radiation field is resonant to transitions between the ground bound state and a propagating state, for small perturbations we can restict ourselves to only two satellite states in Eq. (4), with the following equation for them:

$$-(i\nabla + \mathrm{a}(\mathrm{r}))^2\phi_0 + \epsilon\phi_0 = -\frac{i}{2}(\mathrm{ar})\phi_{-1},$$

$$-(i\nabla + \mathrm{a}(\mathrm{r}))^2\phi_{-1} + (\epsilon - \omega)\phi_{-1} = \frac{i}{2}(\mathrm{ar})\phi_0, \tag{9}$$

where the functions ϕ_0 and ϕ_{-1} correspond to the propagating and the bound states, respectively. In order to obtain a complete solution of the scattering problem in the Hall junction, the solutions of equations (9) are matched with propagating modes in each respective electrode [6]. The electrode 1 in which electron is incident we denote as In(1), while the electrodes in which electron is outgoing we denote as Out:

$$\mathrm{In}(1) = (\chi_\epsilon, 0); \quad \mathrm{Out}(1,4) = (\chi_\epsilon, \chi_{\epsilon-\omega}); \quad \mathrm{Out}(2,3) = (\tilde{\chi}_\epsilon, \tilde{\chi}_{\epsilon-\omega}).$$

These boundary conditions for the solutions of equations (9) can be expressed as Neumann boundary conditions for numerical solution of Eqs. (9) with the help of the MatLab PDE Toolbox.

Results of the calculations are given in Figs. 2 and 3. In Fig. 2 the transmission probabilities from the input electrode 1 to all the other electrodes 2–4 are shown as functions of the incident electron energy in the presence of a radiation field with the frequency $\omega \approx \epsilon(k, \tilde{B}) - \epsilon_0$. Also for comparison the transmission probabilities for the steady-state case are shown in Fig. 2. From this figure one can see that in the vicinity of exact resonance the radiation field induces significant anomalies in the electron transmission. In particular, the transmission probability to the left electrode can exceed the transmission probability to the right electrode. As a result, the Hall resistance becomes negative in the vicinity of the resonance. The total Hall resistance [18] in terms of $\hbar/e^2$, viz.,

$$R_H = \frac{2(T_R - T_L)}{(2T_F + T_R + T_L)^2 + (T_R - T_L)^2}, \tag{10}$$

is shown in Fig. 3 as a function of applied magnetic field $\tilde{B}$. Since the frequency of the radiation field and the Fermi energy of incident electron are fixed, only the external magnetic field, by slightly varying the electron energy, can fulfill condition of resonance at a specific value $\tilde{B}$. Therefore the field dependence of the Hall resistance under the influence of a radiation field has a resonant dip at the corresponding values of the magnetic field. This phenomenon is distinctly demonstrated in Fig. 3, where for two values of the Fermi energy of the incident electron one can see two corresponding resonant quenchings of the Hall resistance.

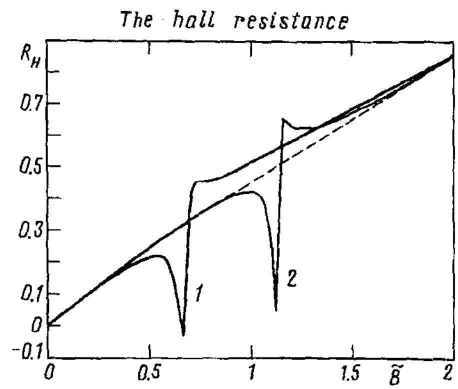

Fig.3. The magnetic-field dependence of the Hall resistance (formula (10)) for different values of the Fermi energies $\epsilon_F = \pi^2 + k_F^2$. Curve *1* corresponds to $k_F = 1.04$, and curve *2* to $k_F = 1.06$. The frequency and amplitude of the radiation field are $\omega = 4.3$ and $a = 0.05$, respectively. The dashed curve corresponds to the steady-state case $a = 0$

Hall resistance anomalies in the form of negative resistance have previously been observed theoretically and experimentally by special construction of the Hall junctions [6, 7, 20, 5]. The present study gives another possibility for observing this phenomenon, by applying a resonant radiation field without changing the geometry of the Hall junction. Note that in contrast to the steady-state quenching of the Hall resistance [6, 7], the radiation-field-induced anomalies take place at low values of the external magnetic field (Fig. 3).

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