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SKYRMION MASS AND A NEW KIND OF THE CYCLOTRON RESONANCE FOR 2D ELECTRON GAS

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The skyrmionic mass is calculated using a gradient expansion method. A special cyclotron resonance is predicted, with a frequency determined by the exchange energy. The possibility of an extra bound electron is discussed.

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The skyrmion energy and charge for 2d electron systems at high magnetic fields were calculated in Refs. [1–4] and [5,6]. The difference in the expressions for the energy in those studies stems from the fact that the kinetic energy was treated as a constant cyclotron energy in [2–4] and as a differential operator in [5,6]. For experimental investigations of the existence of skyrmions it is useful to find specific properties which can be checked by physical measurements. I discuss here the problem of the skyrmion motion as a whole, which is not directly connected with its internal energy. For the calculations I shall use the approach developed in [6,7].

This approach is based upon the transformation $\psi = U\chi$ of the electron spinors ψ to new spinors χ by means of a nonsingular rotation matrix $U(\mathbf{r})$. In this manner one gets a Hartree–Fock equation for spinor χ. For the extremely simple case of local exchange, considered in this paper, it takes the form

$$i\frac{\partial\chi}{\partial t} = \frac{1}{2m}(-i\nabla_k - A_{0k} + \Omega^l_k\sigma_l)^2\chi - \gamma\sigma_z\chi + \Omega^l_t\sigma_l\chi\,.$$

Here σ_l are Pauli matrices and $-iU^+\partial_k U = \Omega^l_k\sigma_l$, with

$$\vec{\Omega}^z = \frac{1}{2}(\nabla\alpha + \cos\beta\nabla\alpha),$$

$$\vec{\Omega}^x = \frac{1}{2}(\sin\beta\cos\alpha\nabla\alpha - \sin\alpha\nabla\beta), \tag{1}$$

$$\vec{\Omega}^y = \frac{1}{2}(\sin\beta\sin\alpha\nabla\alpha + \cos\alpha\nabla\beta),$$

and γ is the exchange constant. We assume that Euler angles are α, β, α, with two equal angles to avoid the singularity in Ω^l for nontrivial degree of mappings Q, and suppose that $\cos\beta = -1$ at the singular point of $\alpha(\mathbf{r})$ ([6,7]). This equation for the electrons in the field of rotation matrix U is fully equivalent to the nonrotated Hartree–Fock equation with $\Omega^l = 0$ but with a nonuniform exchange term $-\gamma\mathbf{n}(\mathbf{r})\sigma$. Here $\mathbf{n}(\mathbf{r})$ is the unit vector in the direction of the mean spin. I use the system of units with $\hbar = B = l_B = 1$, where B is the external magnetic field, l_B is the magnetic length.

We shall assume that the rotation matrix which adjust the spin direction to a given mean spin direction at any point of the 2D plane depends on the position of the skyrmion center $U = U(\mathbf{r} - \mathbf{X})$ and calculate the proper term in the skyrmion action due to the time dependence of $\mathbf{X}(t)$. For the calculation we need to find the electron action for the ground state in the field of the matrix U. In the proper electron Hamiltonian we have the additional perturbation term [6,7]

$$H_1 = iU^+\nabla U\mathbf{X}_t = -\vec{\Omega}^l\sigma_l\mathbf{X}_t\,.$$

In order to find the proper term in the skyrmionic action we must perform perturbation theory calculations in H_1.

Due to the isotropy of the system there is no linear term in $\mathbf{X}_t$, and we must find the second-order term in the action $S = i\mathrm{Tr}\ln G$ where G is the electronic Green function. The second-order term in the action is

$$\delta S = \frac{i}{2}\mathrm{Tr}H_1G_0H_1G_0 \tag{2}$$

where G_0 is the unperturbed Green function for the Hartree-Fock equation with $\Omega^l = 0$

$$G_0(\mathbf{r},\mathbf{r}',t-t') = \sum_{s,p}\int\frac{d\omega}{2\pi}e^{i\omega(t'-t)}g_s(\omega)\Phi_{s,p}(\mathbf{r})\Phi^+_{s,p}(\mathbf{r}'). \tag{3}$$

Here $\Phi_{s,p}$ are normalized Landau wave functions in the Landau gauge and the summation is over all s and p. The matrices $g_s(\omega)$ correspond to the complete filling of the lowest spin sublevel for $s = 0$,

$$g_0(\omega) = \frac{1+\sigma_z}{2}\frac{1}{\omega-\omega_c/2+\gamma-i\delta} + \frac{1-\sigma_z}{2}\frac{1}{\omega-\omega_c/2-\gamma+i\delta}, \tag{4}$$

while all the other states are empty,

$$g_s(\omega) = \frac{1}{\omega-\omega_c(s+1/2)+\gamma\sigma_z+i\delta}, \tag{5}$$

where $\delta \to (+0)$.

The main term in (2), with no derivatives of Ω^l and $\mathbf{X}_t$, corresponds only to $s = 0$. Also, only the cross terms are important, with poles in ω above and below the real axis:

$$\delta S = \frac{1}{2}\int\mathrm{Tr}(\vec{\Omega}^l\mathbf{X}_t\sigma_l)g_0(\omega)(\vec{\Omega}^{l'}\mathbf{X}_t\sigma_{l'})g_0(\omega)e^{i\delta\omega}\frac{d\omega}{2\pi}\frac{d^2r}{2\pi}dt. \tag{6}$$

Here we perform the integration over intermediate space coordinates and the summation over p. It is easy to see that only the terms with $l = l' \neq z$ give a nonzero contribution. Using the isotropy and performing simple integration over ω, we get

$$\delta S = \frac{1}{2\gamma}\sum_{l\neq z}\int\frac{(\Omega^l_x)^2+(\Omega^l_y)^2}{2}\dot{X}^2\frac{d^2r}{2\pi}dt = \int\frac{\dot{X}^2}{16\gamma}(\frac{\partial n_i}{\partial r_k})^2\frac{d^2r}{2\pi}dt. \tag{7}$$

Here we use the expressions (1) for Ω^l and introduce the unit vector

$$\mathbf{n} = (\cos\beta, \sin\beta\cos\alpha, \sin\beta\sin\alpha).$$

It is known that for the state with minimal skyrmion energy for the given degree of mapping Q the value of the space integral [8] is

$$\frac{1}{2}\int(\frac{\partial n_i}{\partial r_k})^2 d^2r = 4\pi|Q|.$$

Therefore the kinetic energy term in the Lagrangian is $E_{kin} = m\dot{X}^2/2$, where $m_s = |Q|/2\gamma$ or in ordinary units,

$$m_s = eB|Q|/2c\gamma\,,$$

where B is the external magnetic field. As has been obtained in a number of papers (see, e.g., [3–5]) the skyrmion has a charge eQ. For a charged skyrmion there are also terms linear in $\mathbf{X}_t$ in the Lagrangian, corresponding to the product of the skyrmion current and the vector-potential of the external magnetic field $\mathbf{B}$, namely $Q\mathbf{X}_t\mathbf{A}_0$. This term can also be calculated by differentiation of the proper additional phase of the wave function obtained by translation of the skyrmion charge eQ. Therefore the full Lagrangian for the motion of the skyrmion as a whole is (in ordinary units)

$$L = \frac{m_s\dot{X}^2}{2} + \frac{e}{c}Q\dot{\mathbf{X}}\mathbf{A}_0\,.$$

The Hamiltonian momentum conjugate to $\mathbf{X}$ is $P_i = \partial L/\partial \dot{X}_i$ which can be considered as a quantum operator with the usual commutation relations $[P_i X_i] = i\hbar$. Therefore one has the cyclotron energy for the motion of the skyrmion as a whole

$$\hbar\omega_s = eB/m_s c = 2\gamma.$$

The minimal energy of such motion is

$$\hbar\omega_s/2 = \gamma$$

and must be added to the internal energy of the skyrmion. In experiments with a sufficient number of charged skyrmions one should observe a cyclotron resonance at a frequency determined by the exchange energy per electron:

$$\omega_s = \frac{1}{\hbar}2\gamma = \frac{e^2}{\hbar l_B}\sqrt{2\pi}$$

where $l_B = \sqrt{c\hbar/eB}$. The final expression is obtained from the expression for the exchange energy for a completely filled Landau level.

The preceding considerations have some important consequences. The thermodynamic energy of a system with a given chemical potential is the quantum average $\langle H - \mu N\rangle$, where H is the Hamiltonian, $\mu = \hbar\omega_c/2$ is the chemical potential, and N is the particle number. The change of the total thermodynamic energy due to the formation of the charged skyrmion is $E_{tot} = E_{int} + \gamma$ where E_{int} is the internal energy of the skyrmion, not including that due to its motion as a whole in the external magnetic field. If one puts an extra electron in the skyrmion core, its energy will consist of two main parts. One part is the increase in the exchange energy γ because the added electron must have the reverse spin direction, according to Pauli principle (all lower states are filled). The other part is the negative Coulomb energy due to the electron interaction with the skyrmion charge, $\sim -e^2Q/L_c$, where L_c is the skyrmion core size. All other terms in the electron energy are comparatively small $\sim 1/L_c^2$ and can be neglected for the large size of the core. The added electron make the total skyrmion-electron complex neutral. Therefore there is no correction to its energy connected with the motion of the complex as a whole in the external magnetic field. The lowest energy of the complex is $E_{compl} = E_{int} + \gamma - \text{const} \cdot e^2Q/L_c$ which is lower than the energy of the charged skyrmion for positive Q and $\mu = \hbar\omega_c/2$. One sees that a skyrmion with a large core size and positive charge must bind an electron and become neutral. The spin of this electron is reversed with respect to the direction of the average spin in the middle of the core, i.e., it coincides with the direction of the mean spin at large distances from the core. The results of [5, 6] give a negative thermodynamic energy E_{int} because of the strong reduction of the kinetic energy by $-\hbar\omega_c/2$ for $\mu = \hbar\omega_c/2$. Therefore such neutral skyrmions must be spontaneously created.

The calculations reported here use the assumption of a large size of skyrmion core; otherwise the perturbation theory in Ω_l is invalid. A large core size requires a small enough g-factor (see, e.g., [6]). The conclusions of this study must be numerically checked using the actual values of the g-factor and magnetic field.

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