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Bound states of three and four resonantly interacting particles

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We present an exact diagrammatic approach for the problem of dimer-dimer scattering in 3D for dimers being a resonance bound state of two fermions in a spin-singlet state, with corresponding scattering length a_F. Applying this approach to the calculation of the dimer-dimer scattering length a_B, we recover exactly the already known result $a_B = 0.6\, a_F$. We use the developed approach to obtain new results in 2D for fermions as well as for bosons. Namely, we calculate bound state energies for three *bbb* and four *bbbb* resonantly interacting bosons in 2D. For the case of resonance interaction between fermions and bosons we calculate exactly bound state energies of the following complexes: two bosons plus one fermion *bbf*, two bosons plus two fermions $bf_\uparrow bf_\downarrow$, and three bosons plus one fermion *bbbf*.

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1. Introduction. The physics of ultracold Fermi gases is the subject of intensive investigations for the last years. In particular, the possibility of experimental observation of the crossover from BCS to BEC limit due to a Feshbach resonance is actively discussed nowadays. In the vicinity of the resonance the scattering length is very large, being positive on one side of the resonance and negative on the other side. In the limit of the positive scattering length for two fermions, the formation of weakly bound dimers consisting of two different fermions becomes energetically favorable. Far from the resonance on the positive side, a weakly interacting gas of these composite bosons exists. In this paper we present a diagrammatic approach to an exact solution for the dimer-dimer elastic scattering, assuming that the (positive) scattering length greatly exceeds the characteristic radius r_0 of interaction between atoms (so-called resonance approximation). As it was firstly shown by Skorniakov and Ter-Martirosian [1], in the case of the 3-body fermionic problem a scattering length of a fermion on a weakly bound dimer is determined by a single parameter, the two-body scattering length a_F, and is equal to $1.18a_F$ in the zero-range approximation for the interatomic potential. The same situation takes place in the case of fermionic 4-body problem, where a dimer-dimer scattering amplitude is fully determined by the value of a_F.

In the first study of the crossover problem by Haussmann [2] the scattering length of composite bosons a_B was found in the lowest order (Born approximation) and equals to $2a_F$. Later on Pieri and Strinati [3] using diagrammatic approach greatly improved this result and found that in the ladder approximation the scattering length of composite bosons approximately equals to $0.75a_F$. However, the ladder approximation, strictly speaking, is not valid, because it misses an infinite number of diagrams which give a contribution of the same order of magnitude as those taken into account. Recently, Petrov, Salomon, and Shlyapnikov [4] have found the exact value of the scattering length of composite bosons $a_B = 0.6a_F$. They solved the Schrödinger equation using the well-known method of pseudopotentials. Below we show an exact solution of the scattering problem of two weakly bound dimers using the diagrammatic approach in the resonance approximation.

We use the developed approach to get the new results for two different systems in a 2D case. Namely we consider first a system of resonantly interacting bosons. We calculate exactly the three boson *bbb* and four boson *bbbb* bound state energies in this case. We also apply our approach to the 2D system of bosons resonantly interacting with fermions. Here we calculate exactly the bound state energies for the following complexes: two bosons plus one fermion *bbf*, two bosons plus two fermions $bf_\uparrow bf_\downarrow$, and three bosons plus one fermion *bbbf*.

This paper is a natural continuation of our previous results where we predicted the possibility of two fermion *ff* [5, 6] and two boson *bb* [7] pairing, as well as a composite fermion creation *fb* [8] in resonantly interacting

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($a \gg r_0$) 2D Fermi-Bose gases and Fermi-Bose mixtures.

2. Three particles scattering. As a preliminary exercise we will rederive the result of Skorniakov and Ter-Martirosian for dimer-fermion scattering length using the diagrammatic method [9].

Following Skorniakov and Ter-Martirosian in the presence of the weakly bound resonance level $-|E_b|$ in a two – particle cross section we can limit ourselves to the zero-range interaction potential between fermions. A two-fermion vertex can be approximated by a simple one-pole structure, which reflects the presence of the s-wave resonace level in a spin-singlet state:

$$\begin{gathered} T_{2\,\alpha\beta;\,\gamma\delta}(P) = T_2(P) \times (\delta_{\alpha,\,\gamma}\delta_{\beta,\,\delta} - \delta_{\alpha,\,\delta}\delta_{\beta,\,\gamma}) = \\ = T_2(P)\chi(\alpha,\beta)\chi(\gamma,\delta), \\ T_2(P) = \frac{4\pi}{m^{3/2}}\,\frac{\sqrt{|E_B|} + \sqrt{\mathbf{P}^2/4m - E}}{E - \mathbf{P}^2/4m + |E_B|}, \end{gathered} \tag{1}$$

where $P = \{E, \mathbf{P}\}$, E is the total energy and $\mathbf{P}$ is the total momentum of incoming particles, m is a fermionic mass, $|E_B| = 1/ma_F^2$. Indices α,β and γ,δ denote spin states of incoming and outgoing particles. The function $\chi(\alpha,\beta)$ stands for the spin singlet state, $\chi(\alpha,\beta) = \delta_{\alpha,\uparrow}\delta_{\beta,\downarrow} - \delta_{\alpha,\downarrow}\delta_{\beta,\uparrow}$.

The simplest process that contributes to dimer-fermion interaction is an exchange of a fermion. We will denote it as Δ_3. Its analytical expression reads

$$\Delta_{3\,\alpha,\beta}(p_1,p_2;P) = -\delta_{\alpha,\beta}\,G(P - p_1 - p_2), \tag{2}$$

where $G(p) = 1/\left(\omega - \mathbf{p}^2/2m + i0\right)$ is a bare fermion Green function. The minus sign in the right hand side of Eq.(2) comes from permutation of two fermions. In order to obtain a full dimer-fermion scattering vertex T_3 we need to build a ladder from Δ_3 blocks. One can easily verify that a spin projection is conserved in every order of T_3 and thus $T_{3\,\alpha,\beta} = \delta_{\alpha,\beta}T_3$. An equation for

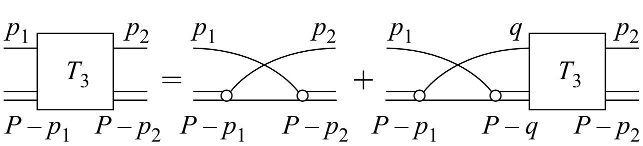

Fig 1. The graphic representation of the equation for the full dimer-fermion scattering vertex T_3

T_3 will have the diagrammatic representation, shown on Fig.1, and in analytical form it reads

$$\begin{gathered} T_3(p_1,p_2;P) = -G(P-p_1-p_2) - \\ - i\sum_q G(P-p_1-q)G(q)\,T_2(P-q)\,T_3(q,p_2;P), \end{gathered} \tag{3}$$

where $\sum_q \equiv \int d^3q d\Omega/(2\pi)^4$. We can integrate out the frequency Ω in Eq.(3) by closing the integration contour in the lower half-plane, since both $T_2(P-q)$ and $T_3(q,p_2;P)$ are analytical functions in this region. Moreover, if we are interested in the low-energy s-wave dimer-fermion scattering length a_3, we can safely put $P = \{E, \mathbf{P}\} = \{-|E_b|, 0\}$ and $p_2 = 0$. The full vertex T_3 is connected with a_3 by the following relation:

$$\left(\frac{8\pi}{m^2 a_F}\right) T_3\left(0,0;-|E_b|\right) = \frac{3\pi}{m}\,a_3. \tag{4}$$

Introducing a new function $a_3(\mathbf{k})$ according to the formula

$$\left(\frac{8\pi}{m^2 a_F}\right) T_3\left(\{k^2/2m,\mathbf{k}\},0;-|E_b|\right) = \frac{3\pi}{m}\,a_3(\mathbf{k}). \tag{5}$$

and substituting it in the Eq.(3), we will obtain Skorniakov–Ter-Martirosian equation for the scattering amplitude:

$$\begin{gathered} \frac{3/4\,a_3(\mathbf{k})}{\sqrt{m|E_B|} + \sqrt{3k^2/4 + m|E_B|}} = \frac{1}{k^2 + m|E_b|} - \\ - 4\pi \int \frac{a_3(\mathbf{q})}{q^2\left(k^2 + q^2 + \mathbf{kq} + m|E_b|\right)}\frac{d^3q}{(2\pi)^3}. \end{gathered} \tag{6}$$

Solving this equation one obtains the well known result for dimer-fermion scattering length $a_3 = a_3(0) = 1.18a_F$.

3. Dimer-dimer scattering. By now we can proceed to the problem of dimer-dimer scattering. This problem was previously solved by Petrov et al. [4] via studying Shrödinger equation for a 4-fermion wave function.

Inspired by the work of Petrov et al. [4] we are looking for a special vertex which describes an interaction of two fermions constituting first dimer with a second dimer as a single object. An obvious candidate for this vertex would be a sum of all diagrams with two fermionic and one dimer incoming lines. It would be naturally to suppose that these diagrams should have the same set of outgoing – two fermionic and one dimer – lines. However in this case there will be a whole set of disconnected diagrams contributing to our sum that describes interaction of a dimer with only one fermion. As it was pointed out by Weinberg [10], one can construct a good integral equation of Lippmann-Schwinger type only for connected class of diagrams. Thus far we are forced to pay our attention to the vertex $\Phi_{\alpha\beta}(q_1,q_2;p_2,P)$ corresponding to the sum of all diagrams with one incoming dimer line, two incoming fermionic lines and two outgoing dimer lines (see Fig.2). This vertex $\Phi_{\alpha\beta}(q_1,q_2;p_2,P)$ is

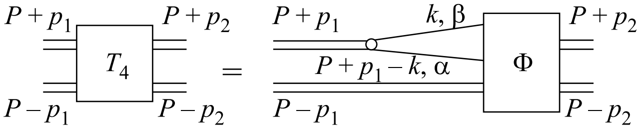

Fig 2. The vertex Φ represents the full dimer-dimer scattering matrix T_4 with one dimer line being cut

rather straightforward ly related to the standard dimer-dimer scattering vertex $T_4(p_1, p_2; P)$:

$$T_4(p_1, p_2; P) = \frac{i}{2} \sum_{k;\,\alpha,\beta} \chi(\alpha, \beta) G(P + p_1 - k) G(k) \times \\ \times \Phi_{\alpha\beta}(P + p_1 - k, k; p_2, P). \qquad (7)$$

Note that by definition in any order of interaction Φ contains only connected diagrams.

A spin part of the vertex $\Phi_{\alpha,\beta}$ has a simple form $\Phi_{\alpha,\beta}(q_1, q_2; P, p_2) = \chi(\alpha, \beta)\Phi(q_1, q_2; P, p_2)$. A diagrammatic representation of the equation on Φ is given on the Fig.3. One can assign some "physical meaning" to the processes described by these diagrams. The diagram of Fig.3a represents the simplest exchange process in dimer-dimer interaction. The diagram of Fig.3b accounts for more complicated nature of a "bare" (irreducible by two dimer lines) dimer-dimer interaction. Finally the diagram of Fig.3c allows for a multiple dimer-dimer scattering via a "bare" interaction. The last term on the Fig.3 means that we should add another three diagrams analogous to Fig.3a, b, c but with two incoming fermions (q_1 and q_2) exchanged. An analytical equation for the vertex Φ reads:

$$\Phi(q_1, q_2; p_2, P) = -G(P - q_1 + p_2) G(P - q_2 - p_2) - \\ - i \sum_k G(k) G(2P - q_1 - q_2 - k) T_2(2P - q_1 - k) \times \\ \times \Phi(q_1, k; p_2, P) + \frac{1}{2} \sum_{Q,k} G(Q - q_1) G(2P - Q - q_2) \times \\ \times T_2(2P - Q) T_2(Q) G(k) G(Q - k) \Phi(k, Q - k; p_2, P) + \\ + (q_1 \leftrightarrow q_2). \qquad (8)$$

Since we are looking for an s-wave scattering length we can put $p_2 = 0$ and $P = \{0, -|E_B|\}$. At this point we have a single closed equation on the vertex Φ in momentum representation, which, we believe, is analogous to Petrov et al. equation in coordinate representation. To make this analogy more prominent we have to exclude frequencies from equation. However this exclusion is rather cumbersome and we leave it for a more extended publication.

The dimer-dimer scattering length is proportional to the full symmetrized vertex $T_4(p_1, p_2; P)$:

$$\left(\frac{8\pi}{m^2 a_F}\right)^2 T_4(0, 0; -2|E_b|, 0) = \frac{2\pi(2a_B)}{m}. \qquad (9)$$

If one skips the second term in Eq. (8), i.e. omits diagram of Fig.3b, he will arrive to the ladder approximation of Pieri and Strinati. The exact equation (8) corresponds to the summation of all diagrams. We have calculated the scattering length in the ladder approximation and the scattering length derived from the exact equation and obtained $0.78a_F$ and $0.6a_F$ respectively. Thus, our results in the ladder approximation are in agreement with the results of Pieri et al. [3] and in the general form with the results of Petrov et al. [4]. Note also that our approach allows one to find dimer-dimer scattering length in the 2D case (this problem was previously solved by Petrov et al. [11]).

Finally we would like to mention that our results allow one to find a fermionic Green function, chemical potential and sound velocity as a function of a_F in the case of the dilute superfluid bose gas of dimers at low temperatures. The problem of dilute superfluid bose gas of di-fermionic molecules was solved by Popov [12], and later deeply investigated by Keldysh and Kozlov [13]. Those authors managed to reduce the gas problem to a dimer-dimer scattering problem in vacuum, but were unable to express the dimer-dimer scattering amplitude in a single two-fermion parameter. A direct combination of our results with those ones of Popov, Keldysh and Kozlov allows one to get all the thermodynamic values of a dilute superfluid resonance gas of composite bosons. However a more interesting subject for the application of our results will be a high-temperature expansion for the thermodynamic potential and sound velocity in the temperature region $T \sim T_* \sim |E_B|$, where the composite bosons start to appear.

4. New results in a 2D case. As it was first shown by Danilov [14] (see also the paper of Minlos and Fadeev [15]) in the 3D case a problem of three resonantly interacting bosons can not be solved in the resonance approximation. This statement stems from the fact that in the case of identical bosons the homogeneous part of Skorniakov-Ter-Martirosian equation (6) has a non-zero solution at any negative energies. The physical meaning of this mathematical artefact was elucidated by Efimov, who showed that a two-particle interaction leads to the appearance of an attractive $1/r^2$ interaction in a three body system. Since in the attractive $1/r^2$ potential in 3D a particle falls onto the center, the short range physics is important and one can not replace the exact pair interaction by its resonance approximation.

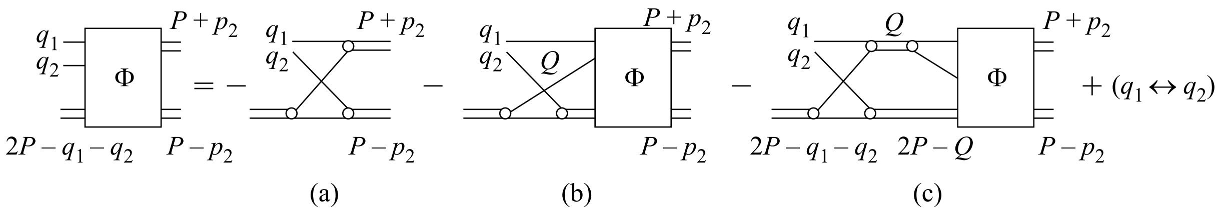

Fig 3. The graphic representation of the equation on function Φ describing dimer-dimer scattering

On the contrary in the case of the 2D problem the phenomena of the particle fall onto the center is absent and one can utilize the resonance approximation [16]. Therefore it is possible to describe three- and four-particle processes in terms of the two-particle binding energy $|E_B| = 1/ma^2$ only (below, for simplicity we will assume that all particles under consideration have the same mass m). We will omit the problem of composite particles scattering and will mainly concentrate on the problem of the binding energies of the complexes of three and four particles.

As well as in the 3D case, the cornerstone in the diagrammatic technique is the two-particle resonance scattering vertex T_2. For two resonantly interacting particles with total mass $2m$ it reads in 2D:

$$T_2(P) = -\frac{4\pi}{m} \frac{\alpha}{\ln\left(\{\mathbf{P}^2/4m - E\}/|E_B|\right)}, \qquad (10)$$

where we introduce factor $\alpha = \{1, 2\}$ in order to take into account whether two particles are indistinguishable or not. That is $\alpha = 2$ for the case of resonance interaction between identical bosons, while $\alpha = 1$ for the case of resonance interaction between fermion and boson, or for the case of two distinguishable bosons.

4.1. *Three particles in 2D.* We start with a system of three resonantly interacting identical bosons – *bbb* – in 2D. An equation for the dimer-boson scattering vertex T_3, which describes interaction of three bosons, has the same diagrammatic form as it is shown on the Fig.1, however the rules of its analytical notation are changed. It reads:

$$T_3(p_1, p_2; P) = G(P - p_1 - p_2) + i \sum_q G(P - p_1 - q) \times$$
$$\times\, G(q) T_2(P - q)\, T_3(q, p_2; P), \qquad (11)$$

where $\sum\limits_q \equiv \int d^2q d\Omega/(2\pi)^3$, $P = \{0, E\}$, and one should put $\alpha = 2$ for the two-particle vertex T_2 in Eq.(10). The opposite signs in Eq. (3) for fermions and Eq. (11) for bosons are due to the permutational properties of particles involved: an exchange of fermions results in a minus sign, while an analogous exchange of bosons brings no extra minus. Finally we note that three-particle s-wave (s-wave channel of a boson-dimer scattering) binding energies E_3 correspond to the poles in $T_3(0, 0; -|E_3|)$ and, consequently, at energies $E = E_3$ the homogeneous part of Eq. (11) has non-trivial solution. Solving Eq.(11) we find that a complex of three identical bosons has two s-wave bound states $E_3 = 16.52 E_B$ and $E_3 = 1.267 E_B$ in accordance with the previous results of Bruch and Tjon [16, 17].

Let us now consider a complex – *fbb* – consisting of one fermion and two bosons. As noted above we consider bosons and fermions with equal masses $m_b = m_f = m$. We assume that a fermion-boson interaction U_{fb}, characterized by the radius of interaction r_{fb}, yields a resonance two-body bound state with an energy $E = -|E_B|$. In the same time a boson-boson interaction U_{bb}, characterized by the interaction radius r_{bb}, does not yield a resonance. Hence if we are interested in the low-energy physics the only relevant interaction is U_{fb} and we can ignore the boson-boson interaction U_{bb}, the latter would give small corrections of the order $|E_B| m r_{bb}^2 \ll 1$ at low energies. In order to determine three-particle bound states one has to find poles in the dimer-boson scattering vertex T_3. Since we neglect boson-boson interaction U_{bb} the vertex T_3 is described by the same diagrammatic equation of Fig.1 as in the problems of three bosons. The analytical form of this equation also coincides with Eq.(11), with minor correction that the resonance scattering vertex T_2 now corresponds to the interaction between a boson and a fermion and therefore we should put $\alpha = 1$ in the Eq. (10) for T_2. Solving the equation for T_3 we find that *fbb* complex has only one s-wave bound state with the energy $E_3 = 2.39 E_B$.

Note that a complex – *bff* – consisting of a boson and two spinless identical fermions (or a complex of a boson and spin $\uparrow$ and spin $\downarrow$ fermions $b f_\uparrow f_\downarrow$) with the resonance interaction U_{fb} does not have any three-particle bound states.

4.2. *Four particles in 2D.* After solving the above three-particle problems we may proceed to the complexes consisting of four particles. At first we will consider four identical resonantly interacting bosons

Table 1. Bound states of resonantly interacting particles in 2D

System	Relative [1)] interaction	Number of bound states	Energy (in E_B) [2)]	α [3)]
bbb	U_{bb}	2	1.267, 16.52	2
fbb	U_{fb}	1	2.39	1
$fbbb$	U_{fb}	1	4.10	1
$bf_\uparrow bf_\downarrow$	U_{fb}	2	2.84, 10.64	1
$bbbb$	U_{bb}	2	24, 194	2

[1)]Interaction that yields resonance scattering. All other interactions are negligible.

[2)]$m = m_b = m_f$.

[3)]The indistinguishability parameter in Eq. (10).

$bbbb$ [18]. Any two bosons would form a stable dimer with a binding energy $E = -|E_B|$. We are going to find a four-particle binding energy as an energy of an s-wave bound state of two dimers. Generally speaking bound states could emerge in channels with larger orbital momenta, however this question will be a subject of further investigations. To find a binding energy we should examine an analytical structure of the dimer-dimer scattering vertex T_4 and find its poles. A set of equation for T_4 has the same diagrammatic structure as those shown on Fig.2 and Fig.3. An analytical expression of the first equation reads:

$$T_4(p_1,p_2;P) = \frac{i}{\alpha}\sum_k G(P+p_1-k)G(k) \times \\ \times \Phi(P+p_1-k,k;p_2,P), \quad (12)$$

and the equation for the vertex Φ reads:

$$\Phi(q_1,q_2;p_2,P) = G(P-q_1+p_2)G(P-q_2-p_2) + \\ + i\sum_k G(k)G(2P-q_1-q_2-k)T_2(2P-q_1-k) \times \\ \times \Phi(q_1,k;p_2,P) - \frac{1}{2\alpha}\sum_{Q,k} G(Q-q_1)G(2P-Q-q_2) \times \\ \times T_2(2P-Q)T_2(Q)G(k)G(Q-k)\Phi(k,Q-k;p_2,P) + \\ + (q_1 \leftrightarrow q_2), \quad (13)$$

where T_2 should be taken from Eq.(10) and one should put $\alpha = 2$ for the case of identical resonantly interacting bosons. Solving the above equations for the poles of T_4 as a function of the variable $P = \{0, E\}$, we found 2 bound states for $bbbb$ complex (see Table 1). Certainly for the validity of our approximation we should have $|E_4| \ll 1/mr_0^2$. For the case of four bosons $bbbb$ it means that $194|E_b| \ll 1/mr_0^2$ and hence $a/r_0 \gg \sqrt{194}$. This case still can be realized in the Feshbach resonance scheme.

The case of a four-particle complex – $bf_\uparrow bf_\downarrow$ – consisting of resonantly interacting bosons and fermions is described by the same equations (12,13) but with parameter $\alpha = 1$. In this case we found 2 bound states and they are also listed in Table 1.

In order to obtain bound states of the $fbbb$ complex one has to find energies $P = \{0, E\}$ corresponding to nontrivial solutions of the following homogeneous equation

$$\Phi(q_1,q_2;p_2,P) = i\sum_k G(k)G(2P-q_1-q_2-k) \times \\ \times T_2(2P-q_1-k)\Phi(q_1,k;p_2,P) + (q_1 \leftrightarrow q_2). \quad (14)$$

This equation corresponds to the diagrams of Fig.3b. We found a bound state of the $fbbb$ complex with an energy $E_4 = 4.10E_B$.

Finally we summarize the results concerning binding energies of three and four resonantly interacting particles in 2D in Table 1. Note that all our calculations correspond to the case of particles with equal masses $m_f = m_b = m$ though they can be easily generalized to the case of different masses.

5. Conclusions. For the problem of resonantly interacting fermions in 3D we developed an exact diagrammatic approach that allows to find a dimer-dimer scattering length. We apply the developed approach to get the new results in the 2D case. Namely, we calculate exactly the binding energies of the following complexes: three bosons bbb, two bosons plus one fermion bbf, three bosons plus one fermion $bbbf$, two bosons plus two fermions $bf_\uparrow bf_\downarrow$, and four bosons $bbbb$.

Our investigations enrich the phase-diagram of ultracold Fermi-Bose gases with resonance interaction. They serve as an important step for future calculations of the thermodynamic properties and the spectrum of collective excitations in different temperature and density regimes. Note that in purely bosonic models in 2D or in the Fermi-Bose mixtures in the case of prevailing density of bosons $n_B > n_F$ creation of larger complexes consisting of 5, 6 and so on particles is also possible. In fact here we are dealing with the macroscopic phase separation (with the creation of large droplets). The radius of this droplet R_N for N bosons in 2D is estimated in [18] on the basis of a variational approach. Note that already for $N = 5$ the exact calculation of the bound state energies requires huge computational capability and that is why it was not performed by us.

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