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Waves and instabilities in dark interstellar molecular clouds containing ferromagnetic dust grains

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The propagation characteristics of magnetization waves as well as the instabilities of sound waves in a self-gravitating dark interstellar molecular cloud containing ferromagnetic dust grains and baryonic gas clouds have been theoretically investigated by including the dynamics of both ferromagnetic dust grains and baryonic gases. It has been shown that there exist two types of subsonic or supersonic (depending on the field stength of the magnetization) tranverse magnetization waves, which can be regarded as counterparts of Alfvén waves (for the parallel propagation) and of magnetosonic waves (for the perpendicular propagation) in a magnetoactive plasma. It has also been found that in addition to the usual Jeans instability, the sound waves suffer a new type of instability which is due to the combined effects of the baryonic gas dynamics and self-gravitational field in both weakly and highly collisional regimes.

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Waves and instabilities in molecular clouds have become outstanding and challenging topics in space science and modern astrophysics because of their crucial role in understanding collapse, formation and evolution of interstellar molecular clouds, star formation, galactic structure and its evolution, etc [1 – 10]. The instability of self-gravitating large gas clouds was first predicted by Jeans [1] about a hundred years ago. It has been later rigorously investigated by a number of other distinguished physicists including Eddington [2], Chandrasekhar [3], Friedman and Polyachenko [4], Mestel and Spitzer [5], Spitzer [6], etc.

Theoretical studies [5 – 10] suggested that magnetic fields play a vital role in the evolution of interstellar clouds into self-gravitating star-forming regions. Mestel and Spitzer [5] as well as Spitzer [6] recognized the importance of ambipolar diffusion (a process by which the magnetic field carried by the ions diffuse through the neutral gas). Strittmatter [7] estimated the critical mass for gravitational collapse perpendicular to a magnetic field. Mouschovias [8, 9] rigorously investigated self-gravitating magnetic clouds by numerical simulations. Mouschovias and Spitzer [10] obtained an expression for the critical mass-to-magnetic flux ratio from the numerical studies of Mouschovias [8, 9].

On the other hand, Jones and Spitzer [11] provided a model for the existence of gas-dust interstellar mediums with a highly pronounced property of magnetic polarizability. This can be assumed due to a superparamagnetic dispersion of the fine ferromagnetic grains suspended in a gaseous cloud of molecular hydrogen. The regular galactic magnetic field threading such a medium introduces anisotropy in the orientation of permanently magnetized solid particles tending to align their magnetic moments. The alignment of magnetic grains can be accompanied by filamentary agglomeration of dust particles (presumably by means of dipole-dipole interaction between magnetic moments of ferromagnetic grains) in the form of long-range magnetic chains extending along the direction of the regular magnetic field. Based on this model of Jones and Spitzer [11] and motivated by recent measurements of magnetic fields toward cores in magnetically supported dark interstellar clouds [12], Yang and Bastrukov [13] reported an alternative mechanism of large scale wave motion in a one component ferromagnetic neutral fluid. They [13] suggested that supersonic line-widths (inferred from the recent measurements of magnetic fields toward cores in magnetically supported dark interstellar clouds [12]) may be due to transverse waves of magnetization propagating in such a one-component ferromagnetic neutral fluid.

The limitations of the analysis of Ref. [13] are that the ferromagnetic dust particles and baryonic gas molecules are assumed be to identical with a constant mass density [i.e. $m_d = m_b$ and $n_d = n_b =$ constant are assumed, where m_d (m_b) is the mass of the ferromagnetic dust particle (baryonic gas molecule) and n_d (n_b) is the ferromagnetic dust particle (baryonic gas molecule) number

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density], and the effects of self-gravitational field, collisions of dust particles with baryonic gas molecules, and motions of ferromagnetic dust particles parallel to the direction of the magnetic moment are neglected. However, in interstellar molecular clouds [14–20] $m_d \neq m_b$ and $n_d \neq n_b \neq$ constant, and the effects of self-gravitational field, collisions of dust particles with baryonic gas molecules, etc. cannot be neglected [14–20]. Thus, in order to study waves and instabilities in interstellar molecular clouds containing ferromagnetic dust particles [14–20], one must consider a two-component neutral fluid (ferromagnetic dust particles and baryonic gas molecules) and include the effects of the mass density perturbations of the ferromagnetic dust fluid and the baryonic gas cloud, self-gravitational field, collisions of dust particles with baryonic gas molecules, etc. In this letter, we generalize the model of Yang Bastrukov [13] for a two-component (ferromagnetic massive dust fluid and baryonic gas cloud), self-gravitating, compressible fluid system including collisions. We have found that there exist two types of subsonic or supersonic (depending on the field stength of the magnetization) tranverse magnetization waves, and in addition to the usual Jeans instability, the sound waves are subjected to a new type of instability which is due to the combined effects of the baryonic gas dynamics and self-gravitational field acting on massive dust particles.

We consider a model of a self-gravitating interstellar molecular cloud containing ferromagnetic dust particles and baryonic gas cloud. These two fluids (ferromagnetic dust fluid and baryonic gas cloud) are assumed to be inter-coupled via the gravitational interaction and dust-baryonic gas molecule collisions. For our purposes, we have the linearized basic equations describing the massive ferromagnetic dust fluid [13]:

$$\partial\rho_d/\partial t + \rho_{d0}\nabla\cdot\mathbf{u_d} = 0, \tag{1}$$

$$\frac{\partial \mathbf{u_d}}{\partial t} = -\nabla\Psi + \frac{2V_m^2}{m_0}\nabla\times(\mathbf{m}\times\hat{\mathbf{z}}) - \nu_{db}(\mathbf{u_d}-\mathbf{u_b}), \tag{2}$$

$$\frac{\partial \mathbf{m}}{\partial t} = \frac{m_0}{2}(\nabla\times\mathbf{u_d})\times\hat{\mathbf{z}}, \tag{3}$$

the baryonic gas cloud [14]:

$$\partial\rho_b/\partial t + \rho_{b0}\nabla\cdot\mathbf{u_b} = 0, \tag{4}$$

$$\frac{\partial \mathbf{u_b}}{\partial t} = -\nabla\Psi - \frac{C_b^2}{\rho_{b0}}\nabla\rho_b, \tag{5}$$

and the gravitational potential Ψ:

$$\nabla^2\Psi = 4\pi G(\rho_d + \rho_b), \tag{6}$$

where $\mathbf{u_d}$ ($\mathbf{u_b}$) is the dust fluid (baryonic gas cloud) velocity, ρ_d (ρ_{d0}) is the perturbed (equilibrium) dust fluid mass density, ρ_b (ρ_{b0}) is the perturbed (equilibrium) baryonic gas mass density, $\mathbf{m}$ ($\mathbf{m_0} = \hat{\mathbf{z}}m_0$) is the perturbed (equilibrium) field of magnetization (magnetic moment per unit volume), $V_m = \sqrt{2\pi m_0^2/3\rho_{d0}}$ represents the speed of the magnetization wave [13], C_b represents the sound speed for the baryonic gas, ν_{db} is the collision frequency of ferromagnetic dust particles with baryonic gas molecules, and G is the universal gravitational constant.

To derive the dispersion relation for the perturbation waves, we assume that all perturbed quantities ρ_d, $\mathbf{u_d}$, $\mathbf{m}$, ρ_b, $\mathbf{u_b}$, and Ψ are proportional to $\exp(-i\omega t + \mathbf{k}\cdot\mathbf{r})$, where ω is the frequency and $\mathbf{k}$ is the wave propagation vector. Thus, substituting $\partial/\partial t = -i\omega$ and $\nabla = i\mathbf{k}$ into (1)–(6) we have

$$(\Omega^2 - k_z^2V_m^2)\mathbf{u_d} = -[V_\alpha^2(\mathbf{k}\cdot\mathbf{u_d}) + V_m^2k_zu_z]\mathbf{k} + V_m^2[k^2u_{dz} - k_z(\mathbf{k}\cdot\mathbf{u_d})]\hat{\mathbf{z}}, \tag{7}$$

where $\Omega^2 = \omega^2 + i\nu_{db}\omega$, $V_\alpha^2 = \alpha_b\omega_{Jd}^2/k^2$, $\alpha_b = 1 - (\omega_{Jb}^2 - i\nu_{db}\omega)/(\omega^2 - k^2C_b^2 + \omega_{Jb}^2)$, $\omega_{Jd} = \sqrt{4\pi G\rho_{d0}}$, and $\omega_{Jb} = \sqrt{4\pi G\rho_{b0}}$. From (7) we can express x, y, and z-components of $\mathbf{u_d}$ as

$$(\Omega^2 - k_z^2V_m^2 + k_x^2V_\alpha^2)u_{dx} + k_xk_yV_\alpha^2u_{dy} + k_xk_z(V_\alpha^2 + V_m^2)u_{dz} = 0, \tag{8}$$

$$k_yk_xV_\alpha^2u_{dx} + (\Omega^2 - k_z^2V_m^2 + k_y^2V_\alpha^2)u_{dy} + k_yk_z(V_\alpha^2 + V_m^2)u_{dz} = 0, \tag{9}$$

$$k_zk_x(V_\alpha^2 + V_m^2)u_{dx} + k_zk_y(V_\alpha^2 + V_m^2)u_{dy} + [\Omega^2 - (k_x^2 + k_y^2)V_m^2 + k_z^2V_\alpha^2]u_{dz} = 0. \tag{10}$$

Equations (8)–(10) then readily give a general dispersion relation

$$\begin{vmatrix} \Omega^2 - k_z^2V_m^2 + k_x^2V_\alpha^2 & k_xk_yV_\alpha^2 & k_xk_z(V_\alpha^2+V_m^2) \\ k_yk_xV_\alpha^2 & \Omega^2 - k_z^2V_m^2 + k_y^2V_\alpha^2 & k_yk_z(V_\alpha^2+V_m^2) \\ k_zk_x(V_\alpha^2+V_m^2) & k_zk_y(V_\alpha^2+V_m^2) & \Omega^2 - (k_x^2+k_y^2)V_m^2 + k_z^2V_\alpha^2 \end{vmatrix} = 0. \tag{11}$$

We now assume that $\mathbf{k}$ lies in the $y-z$ plane, i.e. $k_x = 0$. Thus, the dispersion relation (11) can be simplified as

$$(\Omega^2 - k_z^2 V_m^2)(\Omega^2 - k^2 V_m^2)(\Omega^2 + k^2 V_\alpha^2) = 0. \qquad (12)$$

Equation (12) predicts that there are three possible modes, namely $\Omega^2 - k_z^2 V_m^2 = 0$, $\Omega^2 - k^2 V_m^2 = 0$, and $\Omega^2 + k^2 V_\alpha^2 = 0$. These three modes can be interpreted as follows.

A. The mode $\Omega^2 - k_z^2 V_m^2 = 0$: Substituting Ω we can express this mode as

$$\omega^2 + i\nu_{db}\omega = k_z^2 V_m^2. \qquad (13)$$

When $\nu_{db} \ll \omega$, we have a stable transverse mode of magnetization ($\omega^2 = k_z^2 V_m^2$) considered by Yang and Bastrukov [13]. On the other hand, when we consider $\nu_{db} \neq 0$ and assume $\omega = \omega_r + i\omega_i$, we have $\omega_r^2 = k_z^2 V_m^2 - \omega_i^2$ and $\omega_i = -\nu_{db}/2$. Accordingly, we have a damped mode with a damping rate $\nu_{db}/2$.

B. The mode $\Omega^2 - k^2 V_m^2 = 0$: Substituting Ω we can express this mode as

$$\omega^2 + i\nu_{db}\omega = k^2 V_m^2. \qquad (14)$$

The difference between the mode (14) and the mode (13) is that the mode (14) contains the extra-term $k_y^2 V_m^2$. The mode (14) represents a more general form of the dispersion relation for the obliquely propagating magnetization waves. We note that one cannot consider the perpendicular propagation of the magnetization waves from the dispersion relation (13) or from the dispersion relation for the magnetization waves derived by Yang and Bastrukov [13]. This is because of their taking curl of the ferromagnetic dust fluid velocity $\mathbf{u_d}$. We now compare the magnetization wave speed $V_m = \sqrt{2\pi m_0^2/3\rho_{d0}}$ with the Alfveń speed $V_A = \sqrt{B_0^2/4\pi\rho_{d0}}$ and with the isothermal sound speed $C_b = \sqrt{k_B T_b/m_b}$. Using $m_0 = 3B_0/8\pi$ we have $V_m/V_A \simeq 0.61$ and

$$\frac{V_m}{C_b} = \sqrt{\frac{3B_0^2 m_b}{32\pi n_{d0} m_d k_B T_b}}. \qquad (15)$$

It is clear from $V_m/V_A \simeq 0.61$ that the magnetization wave motions are sub-Alfvénic. Taking typical parameters for interstellar molecular clouds, i.e. considering one micron sized ferromagnetic dust grains of the number density [15–18] $n_{d0} \simeq 10^{-7}$ cm^{-3}, H_2 cloud [15-20] of the temperature $T_b \simeq 10°$ K and the number density $n_{b0} \simeq 10^3$ cm^{-3}, we have $V_m/C_b \simeq 0.4012$ for $B_0 \simeq 10$ μG and $V_m/C_b \simeq 1.003$ for $B_0 \simeq 25$ μG. This means that the magnetization wave motions are subsonic for $B_0 < 25$ μG and supersonic for $B_0 \geq 25$ μG. However, Yang and Bastrukov [13] showd that the magnetization wave motions are supersonic ($V_m/C_b \simeq 1.47$) for $B_0 \simeq 10$ μG. This discrepancy is due to the fact that Yang and Bastrukov [13] used $m_d = m_b = 3.9 \cdot 10^{-13}$ gm and $n_{d0} = n_{b0} = 10^3$ cm^{-3}, whereas we have used $n_{d0} = 10^{-7}$ cm^{-3}, $n_{b0} = 10^3$ cm^{-3}, $m_d = 5 \cdot 10^{-13}$ gm (corresponding to one micron sized dust grains), and $m_b = 3.9 \cdot 10^{-24}$ gm. For these parameters ($n_{d0} = 10^{-7}$ cm^{-3}, $n_{b0} = 10^3$ cm^{-3}, $m_d = 5 \cdot 10^{-13}$ gm, and $m_b = 3.9 \cdot 10^{-24}$ gm) we have also numerically estimated the Jeans frequency corresponding to the dust particles and the baryonic gas molecules. These are $\omega_{Jd} \simeq 2.05 \cdot 10^{-13}$ s^{-1} and $\omega_{Jb} \simeq 5.72 \cdot 10^{-14}$ s^{-1}, respectively.

C. The mode $\Omega^2 + k^2 V_\alpha^2 = 0$: Substituting Ω and V_α we can express this mode as

$$(\omega^2 + i\nu_{db}\omega + \omega_{Jd}^2)(\omega^2 - k^2 C_b^2 + \omega_{Jb}^2) =$$
$$= \omega_{Jd}^2(\omega_{Jb}^2 - i\nu_{db}\omega). \qquad (16)$$

Equation (16) represents the sound waves associated with the baryonic gas molecules coupled by their collisions with dust particles or/and the self-gravitation field acting on dust particles and baryonic gas molecules. To explain it theoretically, we now consider two cases:

(i) Weakly collisional case: We consider weakly collisional case, i.e. $\nu_{db} \ll \omega_{Jd}, \omega_{Jb}$, which allows to express (16) as

$$\omega^4 + (\omega_J^2 - k^2 C_b^2)\omega^2 - \omega_{Jd}^2 k^2 C_b^2 = 0, \qquad (17)$$

where $\omega_J = \sqrt{\omega_{Jb}^2 + \omega_{Jd}^2}$. When $\omega \gg kC_b$, we have from (17)

$$\omega^2 = k^2 C_b^2 - \omega_J^2. \qquad (18)$$

This clearly represents a purely growing mode (since $\omega_J > kC_b$ in order to satisfy $\omega \gg kC_b$), where C_b plays a stabilizing role. This instability is just the usual Jeans instability and is well understood since Jeans predicted the instability of self-gravitating large gas clouds [1].

On the other hand, when $\omega^2 \ll |\omega_J^2 - k^2 C_b^2|$, we have from (17)

$$\omega^2 = \frac{\omega_{Jd}^2 k^2 C_b^2}{(\omega_J^2 - k^2 C_b^2)}. \qquad (19)$$

Equation (19) represents a new mode which is due to the combined effects of the baryonic gas dynamics and the self-gravitational field. This mode disappears if we neglect the baryonic gas dynamics or the gravitational field acting on the dust grains. The important characteristics of this mode is that it is stable for $\omega_J > kC_b$, but is unstable (purely growing) for $\omega_J < kC_b$, which is opposite to the criterion for the Jeans instability.

(ii) Highly collisional case: We consider a very low-frequency mode in a highly collisional case for which we can take $\nu_{db} \gg \omega$ and $|\omega_J^2 - k^2 C_b^2| \gg \omega^2$. These approximations allow to express (16) as

$$\omega = i\frac{\omega_{Jd}^2 k^2 C_b^2}{\nu_{db}(k^2 C_b^2 - \omega_J^2)}. \tag{20}$$

This clearly indicates that this particular mode is purely damped for $\omega_J > kC_b$, but is purely growing (unstable) for $\omega_J < kC_b$. The instability is due to the combined effects of the baryonic gas dynamics and the self-gravitational field in a highly collisional regime.

To summerize, we have considered a two component neutral fluid (one is a massive ferromagnetic dust fluid and the other is a baryonic gas cloud) and have investigated the properties of obliquely propagating sub-Alfvénic magnetization waves as well as sound waves by including the effects of collisions, self-gravitational field, and the dynamics of both ferromagnetic dust particles and baryonic gas molecules. We have shown that two types of transverse magnetization waves, which can be regarded as counterparts of Alfvén waves (for the parallel propagation i.e. for $k_y = 0$) and of magnetosonic waves (for the perpendicular propagation i.e. for $k_z = 0$) in a magnetoactive plasma. We have found that for typical interstellar molecular cloud parameters [15-20], viz. $T_b \simeq 10\,\mathrm{K}$, $n_{b0} \simeq 10^3$ cm^{-3}, $m_b \simeq 3.9 \cdot 10^{-24}$ gm (mass of the hydrogen molecule), $n_{d0} \simeq 10^{-7}$ cm^{-3}, $m_d \simeq 5 \cdot 10^{-13}$ gm (corresponding to one micron sized dust grains), the magnetization wave motions are supersonic for $B_0 \geq 25$ μG.

We have also investigated the sound waves propagating in a self-gravitating gas-dust medium containing tiny (micron sized) dust grains suspended in a cold gas cloud of molecular hydrogen. We have shown that in addition to the usual Jeans instability, the sound waves satisfying $\omega < kC_b < \omega_J$ and $\omega^2 \ll |k^2C_b^2 - \omega_J^2|$ suffer a new type of instability which is due to the combined effects of the baryonic gas dynamics and self-gravitational field in both weakly collisional (cf. Eq.(19)) and highly collisional (cf. Eq. (20)) limits.

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