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Monopole creation operator in presence of matter

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The monopole creation operator proposed recently by Fröhlich and Marchetti is investigated in the Abelian Higgs model with the compact gauge field. We show numerically that the creation operator detects the condensation of monopoles in the presence of the dynamical matter field.

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The value of the deconfinement temperature is one of the most important predictions of the lattice QCD. To study the temperature phase transition we have to investigate the order parameter. For full QCD when dynamical quarks are taken into account, the string tension and the expectation value of the Polyakov line are not the order parameters. On the other hand in the dual superconductor model of QCD vacuum [1] we have the natural order parameter for confinement–deconfinement phase transition. This is the value of the monopole condensate. It should be nonzero in the confinement phase (the monopoles are condensed as Cooper pairs in ordinary superconductor) and zero in the deconfinement phase. To extract monopole from vacuum of non-Abelian fields we have to perform the Abelian projection [2], and after that we can evaluate the value of the monopole condensate using the monopole creation operator.

Originally the gauge invariant monopole creation operator was proposed by Fröhlich and Marchetti for compact $U(1)$ gauge theory [3]. The construction is analogous to the Dirac creation operator [4] for a charged particle. The monopole operator was numerically studied in compact Abelian gauge model [5] and in the pure $SU(2)$ gauge theory in the usual [6] and the spatial [7] Maximal Abelian gauges. It was found that the expectation value of this operator behaves as an order parameter for confinement–deconfinement phase transition: the expectation value is non-zero in the confinement phase and zero in the deconfinement phase. The similar conclusions were made for another types of the monopole creation operators [8]. These results confirm the dual superconductor hypothesis [1] for gluodynamics vacuum.

However, the monopole operator discussed in Ref. [3] exhibits some inconsistency in the presence of charged matter fields, namely the Dirac string becomes visible. To get rid of the Dirac string dependence a new monopole operator was proposed recently [9]. Note that even the pure gluodynamics contains electrically charged fields in the Abelian projection: the off-diagonal gluons are (doubly) charged with respect to the diagonal gluon fields. Thus the newly proposed operator [9] is more suitable for the investigation of confinement in $SU(N)$ gauge theories then the older one [3]. The purpose of this paper is to check numerically whether the new monopole creation operator is the order parameter in theories with matter fields. Below we study compact Abelian Higgs model in the London limit having in mind the further numerical investigation of the new monopole creation operator in non-Abelian gauge theories.

The original version of the gauge invariant monopole creation operator [3] in compact $U(1)$ gauge theory is based on the duality of this model to the Abelian Higgs model. The Higgs field ϕ is associated with the monopole field and the non–compact dual gauge field B_μ represents the dual photon. The gauge invariant operator which creates the monopole in the point x, can be written as the Dirac operator [4] in the dual model:

$$\Phi^{\mathrm{mon}}_x(H) = \phi_x \, e^{i(B,H_x)}, \qquad (1)$$

where the magnetic field of the monopole, $\mathbf{H}$, is defined in the 3D time slice which includes the point x. By definition, the magnetic monopole field satisfies the Maxwell equation, $\mathrm{div}\mathbf{H} = \delta^3(x)$ which guarantees the invariance of the operator Φ under the dual gauge transformation:

$$\phi \to \phi e^{i\alpha}\,, \quad B \to B + \mathrm{d}\alpha\,. \qquad (2)$$

The monopole creation operator (1) can be rewritten in the original representation in terms of the compact field θ. In lattice notations the expectation value of this operator is [3]:

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$$\langle\Phi^{\mathrm{mon}}\rangle = \frac{1}{\mathcal{Z}} \int\limits_{-\pi}^{\pi} \mathcal{D}\theta \, \exp\{-S(d\theta + W)\}\,,$$
$$\mathcal{Z} = \int\limits_{-\pi}^{\pi} \mathcal{D}\theta \, \exp\{-S(d\theta)\}. \qquad (3)$$

For compact lattice electrodynamics the general type of the action satisfies the relation: $S(d\theta + 2\pi n) = S(d\theta)$, $n \in \mathbb{Z}$. Besides the Coulomb monopole field H the tensor form $W = 2\pi\delta\Delta^{-1}(H_x - \omega_x)$ depends on the Dirac string ω which ends at the monopole position, $\delta^*\omega_x = {}^*\delta_x$, and is not restricted to the 3D time–slice.

The operator (1) is well defined for the theories without dynamical matter fields. However, if an electrically charged matter is added, then the creation operator (1) depends on the position of the Dirac string. To see this fact we note that in the presence of the dynamical matter the dual gauge field B becomes compact. Indeed, as we mentioned the pure compact gauge model is dual to the non–compact $U(1)$ with matter fields (referred above as the (dual) Abelian Higgs model). Reading this relation backwards we conclude that the presence of the matter field leads to the compactification of the dual gauge field B.

The compactness of the dual gauge field implies that the gauge transformation (2) must be modified:

$$\phi \to \phi e^{i\alpha}\,, \quad B \to B + \mathrm{d}\alpha + 2\pi k, \qquad (4)$$

where the compactness of the gauge field, $B \in (-\pi, \pi]$, is supported by the integer–valued vector field $k = k(B,\alpha)$. The role of the field k is to change the shape of the dual Dirac string attached to the magnetic charge in the dual theory. One can easily check that the operator (1) is not invariant under the compact gauge transformations (4):

$$\Phi^{\mathrm{mon}}_x(H) \to \Phi^{\mathrm{mon}}_x(H)\, e^{2\pi i (k, H_x)}. \qquad (5)$$

This fact was discussed in Ref. [9]. According to eq.(5) if the field H is integer–valued then operator (1) is invariant under compact gauge transformations (4). This condition and the Maxwell equation require for the field H to have a form of a string attached to the monopole ("Mandelstam string"): $H_x \to j_x$, $j \in \mathbb{Z}$. The string must be defined in the 3D time–slice similarly to the magnetic field H. However, one can show that for a fixed string position the operator Φ creates a state with an infinite energy. This difficulty may be bypassed [9] by summation over all possible positions of the Mandelstam strings with some measure $\mu(j)$:

$$\Phi^{\mathrm{mon,new}}_x = \phi_x \sum_{\substack{{}^*j_x \in \mathbb{Z} \\ \delta^* j_x = \delta_x}} \mu(j_x)\, e^{i(B, j_x)}\,. \qquad (6)$$

If Higgs field ϕ is q–charged ($q \in \mathbb{Z}$), the summation in eq.(6) should be taken over q different strings each of which carries the magnetic flux $1/q$. The transformation of $\Phi^{\mathrm{mon,new}}_x$ to the original representation can be easily performed and we get the expression similar to eq. (3).

In this publication we present the results of our numerical investigation of the operator $\Phi^{\mathrm{mon,new}}_x$ (6) in the compact Abelian Higgs model with the action:

$$S = -\beta\cos(\mathrm{d}\theta) - \gamma\cos(\mathrm{d}\varphi + q\theta), \qquad (7)$$

where θ is the compact gauge field and φ is the phase of the Higgs field. For simplicity we considered the London limit of the model in which the radial part of the Higgs field is frozen. We calculated the (modified) effective constraint potential,

$$V_{\mathrm{eff}}(\Phi) = -\ln\Bigl(\langle\delta(\Phi - \Phi^{\mathrm{mon,new}})\rangle\Bigr). \qquad (8)$$

We simulated the 4D Abelian Higgs model on the $4^4, 6^4, 8^4$ lattice, with $\gamma = 0.3$. The larger charge, q, of the Higgs field, the easier the numerical calculation of $V_{\mathrm{eff}}(\Phi)$ is. We performed our calculations for $q = 7$. For each configuration of 4D fields we simulated 3D model to get the Mandelstam strings with the weight $\mu(j_x)$ which we specify below. We generated 60 statistically independent 4D field configurations, and for each of these configurations we generated 40 configurations of 3D Mandelstam strings. We imposed the anti-periodic boundary conditions in space.

To define the measure μ in eq.(6), we introduce the auxiliary 3D XY theory, "living" on the time slice x^0 with the action:

$$S(\chi, r) = \frac{\kappa}{2}\left\| \frac{d\chi - 2\pi B}{q} + 2\pi r \right\|^2, \qquad (9)$$

where χ is 0–form with value in $[-\pi q, \pi q]$ and r is $\mathbf{Z}$–valued 1–form. One can prove that

$$< e^{i\chi_x} e^{-i\chi_R} >_{R\to\infty} (B) \sim e^{i(B, H_x)}. \qquad (10)$$

In space dimension $d \geq 3$, for sufficiently large κ and sufficiently small B, $\langle e^{i\chi_x} e^{-i\chi_y}\rangle \to \mathrm{const}$ as $|x-y| \to \infty$. Moreover, two-point function $\langle e^{i\chi_x} e^{-i\chi_R}\rangle(B)$ is periodic in B, with period 1. Hence, it has the following Fourier representation: similar to (6):

$$\langle e^{i\chi_x} e^{-i\chi_R}\rangle(B) = \frac{1}{\mathcal{Z}} \sum_{\substack{j_x \in \frac{\mathbb{Z}}{q} \\ \delta j_x = \delta_x - \delta_R}} \mu(j_x)\, e^{i(B, j_x)}, \qquad (11)$$

where the measure μ is defined by:

$$\mu(j_x) = \exp\Bigl\{-\frac{1}{2\kappa}\|j_x\|^2\Bigr\}. \qquad (12)$$

Thus in the theory with the action (9) the two–point correlation function has the representation (10) analogous to the original representation (1) for the monopole creation operator and the dual representation (11) is analogous to the new representation (6) for the monopole creation operator. Therefore the measure in (6) should be defined by (12).

It is well known [10] that the 3D XY model in the Villain formulation has the phase transition for $\kappa_c(B=0) \approx 0.32$. According to suggestion of Fröhlich and Marchetti the expectation value of the operator (6) should be an order parameter in the $\kappa > \kappa_c$ phase, where the density of the Mandelstam strings ρ, is large enough. Our numerical observation has shown that in presence of the external field B, $\kappa_c(B) \approx 0.42$. In Fig.1 we present

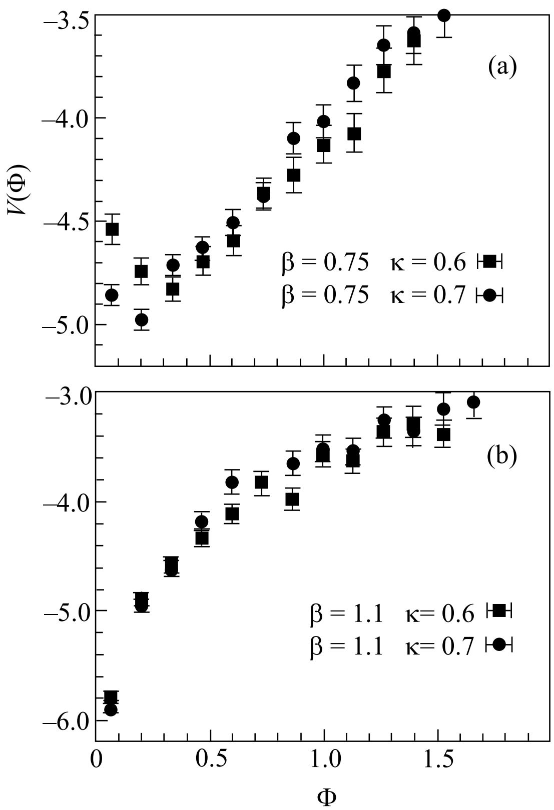

Fig.1. The effective monopole potential (8) in (a) confinement and (b) deconfinement phases

the effective potential (8) in the confinement ($\beta = 0.85$) and deconfinement ($\beta = 1.05$) phases for positive values of the monopole field. The potential is shown for two values of the 3D coupling constants $\kappa > \kappa_c$ corresponding to high densities of the Mandelstam strings. In the confinement phase, Fig.1a, the potential $V(\Phi)$ has a Higgs form signaling the monopole condensation. According to our numerical observations this statement does not depend on the lattice volume. In the deconfinement phase, Fig.1b, the potential has minimum at $\Phi = 0$ which indicates the absence of the monopole condensate.

For the small values of the 3D coupling constant κ (in the phase where Mandelstam strings j_x are not condensed), it was observed (Fig.2) that the potential $V(\Phi)$

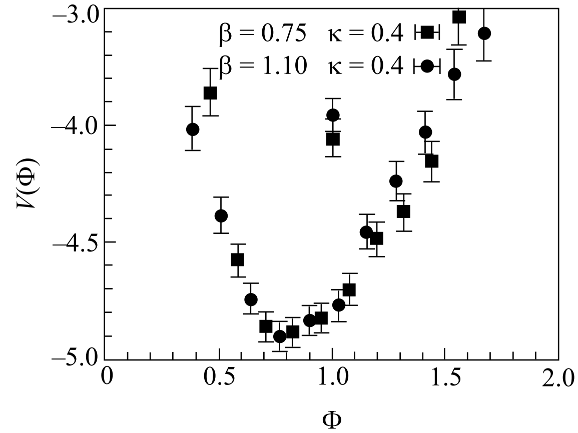

Fig.2. The effective monopole potential (8) in the low-κ region of the 3D model

has the same behaviour for the both phases of 4D model. Thus the operator (6) serves as the order parameter for the deconfinement phase transition, if the density of the Mandelstam strings is high, i.e. κ should be larger than $\kappa_c(B)$.

Summarizing, the new operator can be used as a test of the monopole condensation in the theories with electrically charged matter fields. Our calculations indicate that the operator should be defined in the phase where the Mandelstam strings are condensed as it was suggested by Fröhlich and Marchetti. The minimum of the potential, corresponding to the value of the monopole condensate is zero in deconfinement phase and non zero in the confinement phase.

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