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Low temperature kinetics of 2D exciton gas cooling in quantum well bilayer

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We study the kinetics of 2D Bose gas cooling provided Bose particles interact with 3D phonons. At low temperatures phonon emission is prohibited by the energy and the momentum conservation. We show that both particle-particle scattering and impurity scattering assist Bose-gas cooling. The temporal relaxation of temperature follows the law $T \sim 1/\sqrt{t}$ above the Berezinski-Kosterlitz-Thouless phase transition point and $T \sim 1/t$ after a Bose-Einstein 2D quasi-condensate develops.

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Exciton gas in GaAs bilayer quantum well represents a system where a 2D Bose-Einstein quasi-condensation (BEqC) is possible at low temperatures. Experimental efforts [1, 2] have been directed to produce such an exciton gas and to cool it down to BEqC temperature. A short laser photo-illumination pulse excites electron and hole pairs. The so-called indirect exciton technique is used where a perpendicular electric field drags electrons and holes apart into two spatially separated layers. Then an electron and a hole bind themselves into an indirect exciton particle. This experimental setup suppresses the electron-hole recombination giving rise to a relatively long exciton life time. During the initial photoexcitation pulse newly born excitons are hot and form a non-equilibrium state. In short time after the pulse ends the exciton gas reaches the equilibrium at some effective exciton temperature which is much higher than the lattice temperature of the cold GaAs crystal. Frequent exciton-exciton collisions ensures the exciton temperature to be uniform across the bilayer. Exciton gas then start to cool down slowly due to emission of phonons into a crystal away from the bilayer. This is the longest phase of the experiment limited only by a decay time of excitons due to the electron-hole recombination. In order to reach BEqC point one needs both low temperature and a high density of excitons. Hence a fast cooling is essential.

An important point is that the phonon emission gives the only way for exciton gas to cool. Otherwise it is a closed system with conserved energy. Recent calculation of energy losses in 2D ideal exciton system has predicted an extremely slow cooling at low temperatures with the temporal law $T(t) \sim 1/\log(t)$, where t is the time [3]. This fact is intimately related to the energy and momentum conservation which prohibits an emission of phonons by an exciton moving slower than the velocity of sound in GaAs crystal c. Thus, the exciton gas cooling appears to stop when the exciton temperature falls below a characteristic blocking temperature $T_b = mc^2/2$, where m is the mass of exciton, even if the crystal temperature is zero.

This kinetic bottleneck problem becomes especially acute when the exciton gas is subjected to a strong perpendicular magnetic field that quenches the motion of exciton to the lowest Landau level and, thus, is helping to bind electrons and holes into exciton pairs. In this case the effective mass of an exciton is determined by the Coulomb interaction and can be much larger than either the electron or hole mass [4]. This results in a higher blocking temperature T_b and makes it difficult to reach low temperatures in the end.

In this communication we supplement the analysis of exciton cooling of Ref.[3] by an addition of exciton-exciton collisions and scattering on impurities. Both events assist the phonon emission. We specialize to the case of exactly zero lattice temperature which allows us to neglect exciton-phonon scattering. Actually we are dealing with a general problem of 2D Bose gas cooling provided its particles interact with 3D phonons. The universal nature of 2D scattering at low energy of incoming particles makes these two assistance mechanisms to be robust to specific details of a particle-particle or impurity potential. The latter is only assumed to be short-ranged with the characteristic interaction radius r_0 being shorter than the DeBroigle wavelength. For example, the indirect exciton interacts with an impurity or another exciton via the electron-hole dipole moment $e\mathbf{d}$, directed along the normal to the bilayer. Hence, the radius of such a dipole interaction equals to the spacing between the electron and the hole layers $r_0 \sim d$.

The Hamiltonian of the particle phonon interaction can be written generally as

$$H_{x-ph} =$$

$$= \int\int \psi^+(\mathbf{r})\psi(\mathbf{r})\delta(z)\Gamma_i(\boldsymbol{\rho}-\boldsymbol{\rho'})u_i(\boldsymbol{\rho'})\,d^3\boldsymbol{\rho'}d^2\mathbf{r}dz, \quad (1)$$

where $\boldsymbol{\rho} = (\mathbf{r}, z)$, ψ^+ and ψ are the particle creation and annihilation operators and $\mathbf{u}$ is a crystal deformation induced by an acoustic phonon. Wavelengths of relevant phonons are much larger than the width of the bilayer d. The lattice deformation $\mathbf{u}$ can be expanded into the normal phonon modes as

$$u_i(\boldsymbol{\rho}) = \sum_{s,\mathbf{q}} \left(\frac{\hbar}{2\rho\omega_s(\mathbf{q})}\right)^{1/2} (e_i^s b_s^+(-\mathbf{q}) + e_i^{*s} b_s(\mathbf{q}))e^{i\mathbf{q}\boldsymbol{\rho}}, \quad (2)$$

where b_s^+ and b_s are the phonon creation and annihilation operators of polarization s, ρ is the mass density of solid, $\omega_s(\mathbf{q}) = cq$ is the phonon frequency dispersion, which we assume to be isotropic and independent of phonon polarization s.

The exciton phonon interaction in GaAs crystal can be separated into piezoelectric and deformation potential parts. Lattice deformation in a piezoelectric crystal induces a polarization density $P_i = \beta_{ijk}\partial_j u_k$ [5], where β_{ijk} is the piezoelectric tensor. This polarization interacts with the exciton dipole moment. In the limit $qd \ll 1$, the deformation potential for an exciton Θ, is a sum of the deformation potentials for an electron and a hole taken at the same spatial point. The latter represents a change of the semiconductor gap due to the local compression caused by a phonon deformation. Combining the piezoelectric and deformation parts and expanding the crystal lattice deformation in acoustic phonon modes we write the the exciton-phonon vertex in the Hamiltonian (1) as [5]

$$\Gamma_i(\mathbf{q}) = \left(\Theta q_i + ed\beta_{ijk}\frac{4\pi q_z q_j q_k}{\mathbf{q}^2}\right). \quad (3)$$

For a cubic GaAs crystal without the inversion center $\beta_{ijk} = \beta$ if all i, j, k are different and zero otherwise. In the limit of large d the piezoelectric part dominates over the deformation potential but in the experiments [1, 2] $d \approx 50A$ and both exciton-phonon interaction terms are of the same order of magnitude $\Theta \approx 4\pi ed\beta \approx 10\,\text{eV}$.

Amplitude of phonon emission is given to the lowest order of the perturbation theory by the following matrix element

$$M_{if}^s(\mathbf{q}) = < f\mathbf{q}s|H_{ph}|i0>, \quad (4)$$

between initial state of Bose gas $|i0>$ with no phonons and the final state of Bose gas $|f\mathbf{q}s>$ with just one phonon specified by the momentum $\mathbf{q}$ and the polarization s. We assume that the thermalization of the Bose gas due to particle-particle scattering is much faster than the slow cooling due to phonon emission. Thus, at any given time t the Bose gas is characterized by an effective temperature $T(t)$. This temperature defines the total gas energy $E = E(T)$. The Fermi Golden Rule gives the probability of phonon emission per unit time and one needs to multiply it by the phonon energy $\omega_s(\mathbf{q}) = cq$, to find the total energy losses

$$\frac{dE}{dt} = -\frac{2\pi}{\hbar}\sum_{f\mathbf{q}s} cq|M_{if}^s(\mathbf{q})|^2\delta(E_i - E_f - cq). \quad (5)$$

Eq.(5) has to be averaged over Gibbs distribution of the initial state with the effective temperature $T(t)$. Both the initial and the final states of the Bose gas are calculated in the interaction representation (see e.g.[6]). Particles are confined to the 2D layer and the energy losses are proportional to the area of this layer.

In the experiments [1, 2] the exciton gas is dilute $nr_0^2 \ll 1$. Popov has shown [7] that for a 2D dilute Bose gas there is a 2D Berezinski-Kosterlitz-Thouless phase transition point

$$T_c = \frac{2\pi n\hbar^2}{gm\log L}, \quad (6)$$

that separates high-T almost ideal Bose gas phase from the low-T superfluid phase. Actually, Popov theory is controlled by the large logarithm

$$L \approx -\log(nr_0^2) \approx \log\frac{E_0}{T_c}, \quad (7)$$

where n is the particle density, $E_0 = \hbar^2/r_0^2 m$, g is the particle internal degeneracy [7]. For a Bose particle $g = 2S+1$, where S is the spin of particle. It was shown in Ref.[8] that electron and hole spins flip rapidly due to the spin-orbit interaction. Thus, $g = 4$ for a GaAs exciton.

For 2D dilute non-ideal Bose gas one can distinguish three temperature regions. At high temperatures $T \gg \gg T_c \log L$, the ideal Bose gas is a good approximation. At intermediate temperatures $gT_c/L \ll T \ll T_c/\log L$, overwhelming amount of particles constitute 2D BEqC with the density

$$n_s = n\,(1 - T/T_c)\,, \quad (8)$$

whereas a small fraction of thermal particles have the bare dispersion and Bose distribution with the chemical potential $\mu \approx gT_c/L$ [7]. At low temperatures $T \ll \mu$, a

weak particle-particle interaction is crucial and the quasiparticle excitations of the Bose system acquire Bogolubov sound like dispersion. Here the transfer of momentum to impurity becomes inefficient because the quasiparticle has a vanishing scattering cross-section on a point-like impurity. In the case of excitons in GaAs crystal the intermediate-T region hardly exists at all.

At $T \gg T_b$ a phonon is emitted perpendicular to the layer. Using Eq.(5) we calculate the total energy losses here:

$$\hbar \frac{dE}{dt} = -\frac{T_b T^3}{2\pi T_{x-ph}^2} An, \tag{9}$$

where A is the total area of bilayer. It is convenient to define a characteristic exciton-phonon energy:

$$T_{x-ph} = \sqrt{\frac{\rho c^5 \hbar^3}{\Theta^2 + (4\pi d e \beta)^2/15}}. \tag{10}$$

In the case of GaAs $T_{x-ph} \approx 5\,\mathrm{K}$. Using the ideal gas equation of state: $E(T) = AnT$, we get the temperature relaxation law: $T(t) \sim 1/\sqrt{t}$.

At $T \ll T_b$ an unassisted phonon emission is forbidden. Fig.1 shows two ways of particle scattering accompanied by an acoustic phonon emission. The

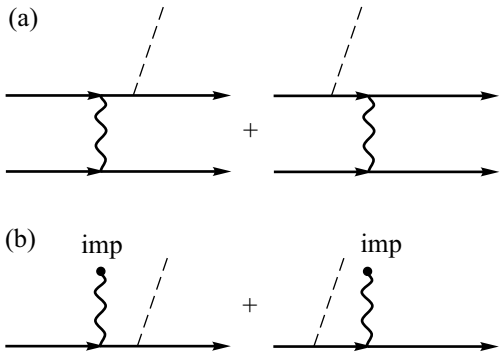

Fig.1. Amplitude of exciton-exciton scattering (a) and exciton-impurity scattering (b) accomponied by phonon emission. Full lines correspond to the propagation of exciton, dotted line correspond to the propagation of phonon. Wave line describes interaction either between excitons or exciton and impurity

left diagram shows a scattering on the second particle and the right diagram shows a scattering on impurity. First, we treat this problem in the high-T and intermediate-T regions where particles have the bare dispersion: $\epsilon = \mathbf{p}^2/2m$. 2D scattering amplitudes in both cases are isotropic and depend only on the total kinetic energy in the center of mass frame: $\mathcal{E}$ (in the impurity case $\mathcal{E} = \epsilon$) in the limit $\mathcal{E} \to 0$:

$$F(\mathcal{E}) = -\frac{2\pi\hbar^2}{m^*} \left(\log \frac{E_0}{\mathcal{E} r_0^2 m} \right)^{-1}, \tag{11}$$

where $m^* = m/2$ for particle-particle scattering and $m^* = m$ for the impurity scattering (see e.g. [7]). Both particle-particle interaction line and the impurity line on Fig.1 correspond to the scattering amplitude $F(\mathcal{E})$. The total amplitude of assisted phonon emission is universal in both cases and is given by the following matrix element:

$$M_{if}^s(\mathbf{q}) =$$

$$= C \left(F(\mathcal{E}) \tfrac{1}{cq} + F(\mathcal{E} - cq) \frac{1}{-cq} \right) e_i^s \Gamma_i(\mathbf{q}) \sqrt{\frac{q}{2\rho c}}, \tag{12}$$

where $C = 2$ for the particle-particle scattering and $C = 1$ for the impurity scattering. We neglect the phonon momentum transfer $\mathbf{q}_{||}$, to the particle because $q_{||} \ll p, p' \ll cm$, if $T \ll T_b$.

Plugging Eqs.(12), (3) into Eq.(5) and taking the integral over the final state of the Bose gas we obtain the total energy losses per unit time. In the high-T region we get:

$$\frac{dE}{dt} = -\frac{2\pi c^3}{\hbar T_{x-ph}^2} \int \left(\frac{1}{\log(E_0/\epsilon)} - \frac{1}{\log(E_0/\epsilon')} \right)^2 \times$$

$$\times K(\epsilon, T) \delta(\epsilon - \epsilon' - cq) d\epsilon d\epsilon' \frac{d^3\mathbf{q}}{(2\pi)^3}, \tag{13}$$

where $K(\epsilon, T) = 2AnN(\epsilon)(1 + 1/g)$ in the case of particle-particle scattering and $K(\epsilon, T) = An_{imp}N(\epsilon)/2$ in the case of impurity scattering. n_{imp} is the areal density of impurities and

$$N(\epsilon) = \frac{1}{\exp\left((\epsilon - \mu)/T\right) - 1} \tag{14}$$

is the Bose-Einstein occupation number. Combining the particle-particle and impurity contributions we find the total cooling rate:

$$\frac{dT}{dt} = -(4(1 + 1/g)n + n_{imp}) \frac{85}{9m} \frac{\hbar}{L^4} \frac{T^3}{T_{x-ph}^2}. \tag{15}$$

From Eqs.(9), (15) we conclude that the temperature dependence of the 2D Bose gas cooling rate is the same above and below the phonon emission threshold: $T = T_b$.

In the intermediate-T region at $g = 1$ the cooling rate is enhanced by the stimulated scattering into the BEqC final states:

$$\frac{dT}{dt} = -n_s \left(64 n_s (1 - \frac{\zeta(3)}{\zeta(2)}) + n_{imp} \frac{\zeta(3)}{\zeta(2)}\right) \times$$

$$\times \frac{2\pi\hbar^3}{m^2} \frac{\log^2 L}{L^4} \frac{T^2}{T_{x-ph}^2}. \qquad (16)$$

In the low-T region the thermodynamic equation of state reads: $E(T) = A\zeta(3)T^3/\pi s^2$, where s is the Bogolubov sound velocity: $s = \sqrt{\mu/m}$. In order to calculate the energy losses we apply the Bogolubov unitary transformation to the Hamiltonian (1):

$$H_{ph} = -\sum_{\mathbf{pq}} \frac{\mu}{2\epsilon(\mathbf{p})} \psi^+(\mathbf{p})\psi^+(-\mathbf{p}+\mathbf{q})\Gamma_i(\mathbf{q})u_i(\mathbf{q}) + \text{c.c.} \qquad (17)$$

This Hamiltonian allows emission of phonons. The cooling rate in this case is also enhanced by the condensate stimulation:

$$\frac{dT}{dt} = -\frac{\mu^2}{\hbar\pi T_{x-ph}^2} \left(1 - \frac{\zeta(4)}{\zeta(3)}\right) T^2. \qquad (18)$$

In both cases of condensate stimulation we find the temperature relaxation law: $T(t) \sim 1/t$.

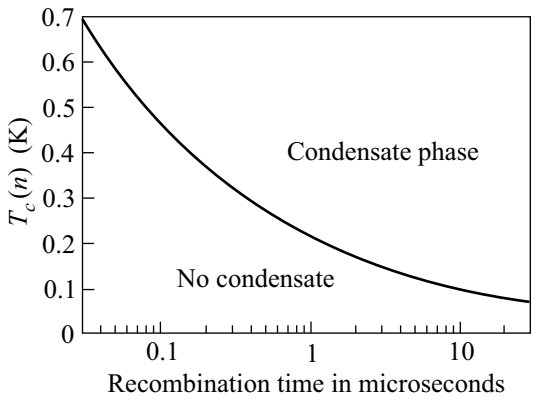

Fig.2. Kinetic phase diagram showing the posibility to reach a Bose Einstein quasi-condensate phase in the GaAs indirect exciton bilayer

For experimental realization of an exciton cooling the most relevant is Eq.(15). Integrating it and inserting $L \approx 6$ and $n \gg n_{imp}$ we find the overall cooling time t_c required to reach the phase transition point. Here one can distinguish two cases: i) cooling of exciton gas with constant density e.g. sustained by photoexcitation and ii) cooling of decaying exciton gas with $n(t) = n_0 \exp(-t/\tau_r)$, where τ_r is the exciton recombination time. We find:

$$t_c = C\hbar T_{x-ph}^2 / T_c(n)^3, \qquad (19)$$

where $T_c(n)$ is the BKT temperature as a function of the exciton density n (6), $C \approx 10$ in the case i) and $C \approx 30$ in the case ii). Note that C does not depend on exciton mass m and in the case ii) the best condition for reaching T_c occurs at $t = \tau_r/3$. Eq.(19) defines a line in the bilayer parameter space: (n, τ_r) or equivalently $(T_c(n), \tau_r)$, separating the two kinetic phases – one that can condensate and the second that remains above T_c during the exciton life time τ_r. Fig.2 shows this borderline for the case of a thin GaAs bilayer.

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