Pis'ma v ZhETF, vol. 82, iss. 7, pp. 435 – 439 © 2005 October 10

Charged currents, color dipoles and xF_3 at small x

R. Fiore[1]), *V. R. Zoller*[+ 1])

Dipartimento di Fisica, Universitá della Calabria, 87036 Rende Italy

Instituto Nazionale di Fisica Nucleare, Gruppo collegato di Cosenza, Italy

[+]*Institut for Theoretical and Experimental Physics, 117218 Moscow, Russia*

Submitted 18 August 2005

We develop the light-cone color dipole description of highly asymmetric diffractive interactions of left-handed and right-handed electroweak bosons. We identify the origin and estimate the strength of the left-right asymmetry effect in terms of the light-cone wave functions. We report an evaluation of the small-x neutrino-nucleon deep inelastic scattering structure functions xF_3 and $2xF_1$ and present comparison with experimental data.

PACS: 12.15.Ji, 12.38.−t

At small Bjorken x the driving term of the inclusive/diffractive excitation of charmed and (anti)strange quarks in the charged current (CC) neutrino deep inelastic scattering (DIS) is the W^+-gluon/pomeron fusion,

$$W^+g \to c\bar{s} \tag{1}$$

and

$$W^+\mathbb{P} \to c\bar{s}. \tag{2}$$

Different aspects of the CC inclusive and diffractive DIS have been discussed in [1, 2].

In the color dipole approach [3, 4] (for the review see [5]) the small-x DIS is treated in terms of the interaction of the $c\bar{s}$ color dipole of size $\mathbf{r}$ with the target proton which is described by the beam- and flavor-independent color dipole cross section $\sigma(x,r)$. Once the light-cone wave function (LCWF) of a color dipole state is specified the evaluation of observable quantities becomes a routine quantum mechanical procedure. In this communication we extend the color dipole analysis onto the CC DIS with particular emphasis on the left-right asymmetry of diffractive interactions of electroweak bosons of different helicity. We derive the relevant LCWF and evaluate the structure functions xF_3, ΔxF_3 and $2xF_1$. We focus on the vacuum exchange dominated leading $\log(1/x)$ region of $x \lesssim 0.01$.

At small x the contribution of excitation of open charm/strangeness to the absorption cross section for scalar, ($\lambda = 0$), left-handed, ($\lambda = -1$), and right-handed, ($\lambda = +1$), W-boson of virtuality Q^2, is given by the color dipole factorization formula [6, 7]

$$\sigma_\lambda(x,Q^2) = \int dz d^2\mathbf{r} \sum_{\lambda_1,\lambda_2} |\Psi_\lambda^{\lambda_1,\lambda_2}(z,\mathbf{r})|^2 \sigma(x,r). \tag{3}$$

In Eq.(3) $\Psi_\lambda^{\lambda_1,\lambda_2}(z,\mathbf{r})$ is the LCWF of the $|c\bar{s}\rangle$ state with the c quark carrying fraction z of the W^+ light-cone momentum and $\bar{s}$ with momentum fraction $1-z$. The c- and $\bar{s}$-quark helicities are $\lambda_1 = \pm 1/2$ and $\lambda_2 = \pm 1/2$, respectively. The $W^+ \to c\bar{s}$-transition vertex is specified as follows

$$gU_{cs}\bar{c}\gamma_\mu(1-\gamma_5)s,$$

where U_{cs} – is an element of the CKM-matrix and the weak charge g is related to the Fermi coupling constant G_F,

$$G_F/\sqrt{2} = g^2/m_W^2. \tag{4}$$

The polarization states of W-boson carrying the lab. frame four-momentum

$$q = (\nu, 0, 0, \sqrt{\nu^2+Q^2}) \tag{5}$$

are described by the four-vectors e_λ,

$$\begin{aligned} e_0 &= \frac{1}{Q}(\sqrt{\nu^2+Q^2}, 0, 0, \nu), \\ e_\pm &= \mp\frac{1}{\sqrt{2}}(0, 1, \pm i, 0), \end{aligned} \tag{6}$$

with unit vectors $\mathbf{e}_x$ and $\mathbf{e}_y$ in q_x- and q_y-directions, respectively. We find it convenient to use the basis of helicity spinors of Ref.[8]. Then, vector (V) and axial-vector (A) components of the LCWF

$$\Psi_\lambda^{\lambda_1,\lambda_2}(z,\mathbf{r}) = V_\lambda^{\lambda_1,\lambda_2}(z,\mathbf{r}) - A_\lambda^{\lambda_1,\lambda_2}(z,\mathbf{r}), \tag{7}$$

[1])e-mail: fiore@cs.infn.it, zoller@itep.ru

are as follows

$$V_0^{\lambda_1,\lambda_2}(z,\mathbf{r}) = \frac{\sqrt{\alpha_W N_c}}{2\pi Q}\{\delta_{\lambda_1,-\lambda_2}\left[2Q^2 z(1-z) + \right. +(m-\mu)[(1-z)m - z\mu]\left.\right] K_0(\varepsilon r) - -i\delta_{\lambda_1,\lambda_2}(2\lambda_1)e^{-i2\lambda_1\phi}(m-\mu)\varepsilon K_1(\varepsilon r)\}\,, \quad (8)$$

$$A_0^{\lambda_1,\lambda_2}(z,\mathbf{r}) = = \frac{\sqrt{\alpha_W N_c}}{2\pi Q}\{\delta_{\lambda_1,-\lambda_2}(2\lambda_1)\left[2Q^2 z(1-z) + \right. +(m+\mu)[(1-z)m + z\mu]\left.\right] K_0(\varepsilon r) + +i\delta_{\lambda_1,\lambda_2}e^{-i2\lambda_1\phi}(m+\mu)\varepsilon K_1(\varepsilon r)\}\,. \quad (9)$$

If $\lambda = \pm 1$

$$V_\lambda^{\lambda_1,\lambda_2}(z,\mathbf{r}) = -\frac{\sqrt{2\alpha_W N_c}}{2\pi}\times \times\{\delta_{\lambda_1,\lambda_2}\delta_{\lambda,2\lambda_1}[(1-z)m + z\mu]K_0(\varepsilon r) - -i(2\lambda_1)\delta_{\lambda_1,-\lambda_2}e^{i\lambda\phi}\times \times[(1-z)\delta_{\lambda,-2\lambda_1} + z\delta_{\lambda,2\lambda_1}]\,\varepsilon K_1(\varepsilon r)\}\,, \quad (10)$$

$$A_\lambda^{\lambda_1,\lambda_2}(z,\mathbf{r}) = \frac{\sqrt{2\alpha_W N_c}}{2\pi}\times \times\{\delta_{\lambda_1,\lambda_2}\delta_{\lambda,2\lambda_1}(2\lambda_1)[(1-z)m - z\mu]K_0(\varepsilon r) + +i\delta_{\lambda_1,-\lambda_2}e^{i\lambda\phi}[(1-z)\delta_{\lambda,-2\lambda_1} + z\delta_{\lambda,2\lambda_1}]\,\varepsilon K_1(\varepsilon r)\}\,, \quad (11)$$

where

$$\varepsilon^2 = z(1-z)Q^2 + (1-z)m^2 + z\mu^2 \quad (12)$$

and $K_\nu(x)$ is the modified Bessel function. We do not consider Cabibbo-suppressed transitions and

$$\alpha_W = g^2/4\pi.$$

The quark and antiquark masses are m and μ, respectively. The azimuthal angle of $\mathbf{r}$ is denoted by ϕ. To switch $W^+ \to W^-$ one should replace $m \leftrightarrow \mu$ in the equations above.

The diagonal elements of density matrix

$$\rho_{\lambda\lambda'} = \sum_{\lambda_1,\lambda_2}\Psi_\lambda^{\lambda_1,\lambda_2}\left(\Psi_{\lambda'}^{\lambda_1,\lambda_2}\right)^* \quad (13)$$

entering Eq. (3) are as follows

$$\rho_{00}(z,\mathbf{r}) = = \sum_{\lambda_1,\lambda_2}\left(\left|V_0^{\lambda_1,\lambda_2}\right|^2 + \left|A_0^{\lambda_1,\lambda_2}\right|^2\right) = \frac{2\alpha_W N_c}{(2\pi)^2 Q^2}\times \times\left\{\left[\left[2Q^2 z(1-z) + (m-\mu)[(1-z)m - z\mu]\right]^2 + \right.\right. + \left.\left[2Q^2 z(1-z) + (m+\mu)[(1-z)m + z\mu]\right]^2\right]\times \times K_0(\varepsilon r)^2 + \left.[(m-\mu)^2 + (m+\mu)^2]\varepsilon^2 K_1(\varepsilon r)^2\right\} \quad (14)$$

and for $\lambda = \lambda' = \pm 1$

$$\rho_{+1+1}(z,\mathbf{r}) = \left|\Psi_{+1}^{+1/2+1/2}\right|^2 + \left|\Psi_{+1}^{-1/2+1/2}\right|^2 = = \frac{8\alpha_W N_c}{(2\pi)^2}(1-z)^2\left[m^2 K_0(\varepsilon r)^2 + \varepsilon^2 K_1(\varepsilon r)^2\right], \quad (15)$$

$$\rho_{-1-1}(z,\mathbf{r}) = \left|\Psi_{-1}^{-1/2-1/2}\right|^2 + \left|\Psi_{-1}^{-1/2+1/2}\right|^2 = = \frac{8\alpha_W N_c}{(2\pi)^2}z^2\left[\mu^2 K_0(\varepsilon r)^2 + \varepsilon^2 K_1(\varepsilon r)^2\right]. \quad (16)$$

At $Q^2 \to 0$ the terms $\sim m^2/Q^2, \mu^2/Q^2$ in (14) remind us that W interacts with the current which is not conserved while the S-wave terms in Eqs.(15) and (16) proportional to m^2 and μ^2 remind us that this current is the parity violating $(V-A)$-current.

The density of quark-antiquark $c\bar{s}$ states in the transversely polarized W^+-boson is,

$$\rho_{TT} = \frac{1}{2}(\rho_{+1+1} + \rho_{-1-1}) = = \frac{4\alpha_W N_c}{(2\pi)^2}\left\{\left[(1-z)^2 m^2 + z^2\mu^2\right]K_0(\varepsilon r)^2 + \right. + \left.\left[(1-z)^2 + z^2\right]\varepsilon^2 K_1(\varepsilon r)^2\right\}. \quad (17)$$

One can see that our ρ_{00} and ρ_{TT} coincide with the probability densities $|\Psi_L|^2$ and $|\Psi_T|^2$ of Ref.[1] (see also [9] where z-dependence of transverse and longitudinal CC cross sections has been discussed).

The momentum partition asymmetry of both ρ_{-1-1} and ρ_{+1+1} is striking, the left-handed quark in the decay of left-handed W^+ gets the lion's share of the W^+ light-cone momentum. The nature of this phenomenon is very close to the nature of well known spin-spin correlations in the neutron β-decay, The observable which is strongly affected by this left-right asymmetry is the structure function of the neutrino-nucleon DIS named F_3. Its definition in terms of σ_R and σ_L of Eq.(3) is as follows

$$2xF_3(x,Q^2) = \frac{Q^2}{4\pi^2\alpha_W}\left[\sigma_L(x,Q^2) - \sigma_R(x,Q^2)\right]. \quad (18)$$

To estimate consequences of the left-right asymmetry for F_3 at high Q^2 such that

$$\frac{m^2}{Q^2} \ll 1,\ \frac{\mu^2}{Q^2} \ll 1 \quad (19)$$

one should take into account that the dipole cross-section $\sigma(x,r)$ in Eq.(3) is related to the un-integrated gluon structure function $\mathcal{F}(x,\kappa^2) = \partial G(x,\kappa^2)/\partial\log\kappa^2$, as follows [10]

$$\sigma(x,r) = \frac{\pi^2}{N_c} r^2 \alpha_S(r^2) \times \\ \times \int \frac{d\kappa^2 \kappa^2}{(\kappa^2+\mu_G^2)^2} \frac{4[1-J_0(\kappa r)]}{\kappa^2 r^2} \mathcal{F}(x_g,\kappa^2) \qquad (20)$$

and to the Double Leading-Log Approximation (DLLA), i.e. for small dipoles,

$$\sigma(x,r) \approx \frac{\pi^2}{N_c} r^2 \alpha_S(r^2) G(x_g, A/r^2), \qquad (21)$$

where $\mu_G = 1/R_c$ is the inverse correlation radius of perturbative gluons and $A \simeq 10$ comes from properties of the Bessel function $J_0(y)$. Because of scaling violation $G(x,Q^2)$ rises with Q^2 but the product $\alpha_S(r^2)G(x,A/r^2)$ is approximately flat in r^2. At large Q^2 the leading contribution to $\sigma_\lambda(x,Q^2)$, comes from the P-wave term, $\varepsilon^2 K_1(\varepsilon r)^2$, in Eqs.(15), (16). The asymptotic behavior of the Bessel function, $K_1(x) \simeq \simeq \exp(-x)/\sqrt{2\pi/x}$ makes the $\mathbf{r}$-integration rapidly convergent at $\varepsilon r > 1$. Integrating over $\mathbf{r}$ in Eq.(3) yields

$$\sigma_L \propto \int_0^1 dz \frac{z^2}{\varepsilon^2} \alpha_S G \sim \frac{\alpha_S G}{Q^2} \log \frac{Q^2}{\mu^2} \qquad (22)$$

and similarly

$$\sigma_R \propto \int_0^1 dz \frac{(1-z)^2}{\varepsilon^2} \alpha_S G \sim \frac{\alpha_S G}{Q^2} \log \frac{Q^2}{m^2}. \qquad (23)$$

The left-right asymmetry certainly affects also the slowly varying product $\alpha_S G$ which for the purpose of crude estimate is taken at some rescaled virtuality $\sim Q^2$ which is approximately/logarithmically the same for σ_L and σ_R. Hence,

$$\sigma_L - \sigma_R \propto \frac{\alpha_S G}{Q^2} \log \frac{m^2}{\mu^2}. \qquad (24)$$

Notice that in spite of the apparent asymmetry of z-distribution both σ_L and σ_R get equal scaling contributions from the integration domains near by the peaks $z=1$ and $z=0$, respectively. Therefore, xF_3 is free of the end-point contributions.

At $Q^2 \to 0$ and $\mu^2/m^2 \ll 1$ the cross sections σ_L and σ_R are as follows

$$\sigma_L \propto \frac{\alpha_S G}{m^2}\left(\log\frac{m^2}{\mu^2} - \frac{3}{2}\right), \ \sigma_R \propto \frac{\alpha_S G}{2m^2}. \qquad (25)$$

We evaluate $xF_3(x,Q^2)$ making use of Eqs.(3) and (20) with the differential gluon density function $\mathcal{F}(x_g,\kappa^2)$ determined in [11]. As reported in [11], the approach developed works very well in the perturbative region of high Q^2 and small $x \lesssim 0.01$. Besides, a realistic extrapolation of $\mathcal{F}(x_g,\kappa^2)$ into the soft region allows calculations at lowest Q^2 also [11]. In our calculations for $Q^2 \lesssim M^2 = 2(m^2+\mu^2)$ the gluon density $\mathcal{F}(x_g,\kappa^2)$ enters (20) at the gluon momentum fraction $x_g = x(1+M^2/Q^2)$. For large virtualities, $Q^2 \gtrsim M^2$, we put $x_g = 2x$. Direct evaluation of the proton DIS structure function $F_{2p}(x,Q^2)$ shows that this prescription corresponding to the collinear DLLA ensures a good description of experimental data on the light and heavy flavor electro-production in a wide range of the photon virtualities down to $Q^2 \sim 1\,\mathrm{GeV}^2$. The constituent quark masses are as follows $m_u = m_d = 0.2\,\mathrm{GeV}$, $m_s = 0.35\,\mathrm{GeV}$ and $m_c = 1.3\,\mathrm{GeV}$.

The xF_3 data reported by the CCFR Collaboration are presented in Fig.1. Shown is the Q^2-dependence

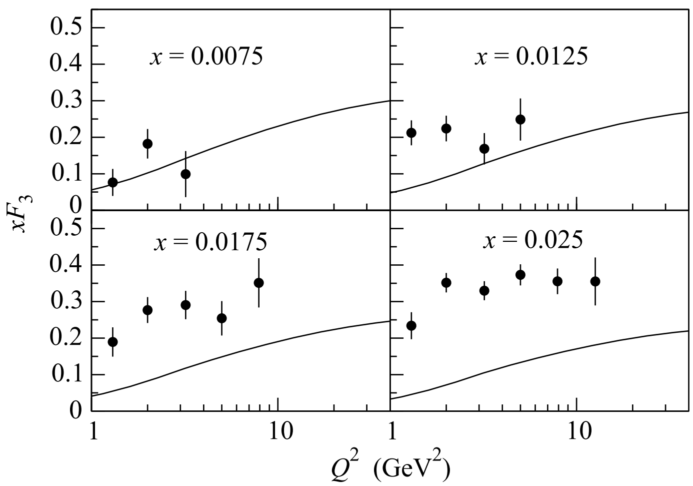

Fig.1. Data points are CCFR measurements of $xF_3(x,Q^2)$ [12]. Curves show the vacuum exchange contribution to $xF_3(x,Q^2)$

of xF_3 for several smallest values of x [12]. It should be emphasized that we focus on the vacuum exchange contribution to xF_3 corresponding to the excitation of the $c\bar{s}$ state in the process (1). Therefore, the structure function xF_3 differs from zero only due to the strong left-right asymmetry of the light-cone $|c\bar{s}\rangle$ Fock state. Shown by the solid line in Fig.1 is the pomeron exchange contribution to xF_3. The latter can be interpreted in terms of parton densities as the sea-quark component of xF_3.

Looking at Fig.1 one should bear in mind that the smallest available values of x are in fact only moderately small and there is also quite significant valence contribution to xF_3. The valence term, xV is the same for both νN and $\bar{\nu}N$ structure functions of an iso-scalar nucleon. The sea-quark term in the $xF_3^{\nu N}$ denoted by $xS(x,Q^2)$ has opposite sign for $xF_3^{\bar{\nu}N}$, the substitution $m \leftrightarrow \mu$ in Eqs.(15), (16) entails $\sigma_L \leftrightarrow \sigma_R$. Therefore,

$$xF_3^{\nu N} = xV + xS, \qquad (26)$$

and

$$xF_3^{\bar{\nu}N} = xV - xS. \quad (27)$$

One can combine the νN and $\bar{\nu}N$ structure functions to isolate the pomeron exchange term,

$$\Delta xF_3 = xF_3^{\nu N} - xF_3^{\bar{\nu}N} = 2xS. \quad (28)$$

The extraction of ΔxF_3 from CCFR $\nu_\mu Fe$ and $\bar{\nu}_\mu Fe$ differential cross section in a model-independent way has been reported in [13]. Fig.2 shows the extracted values

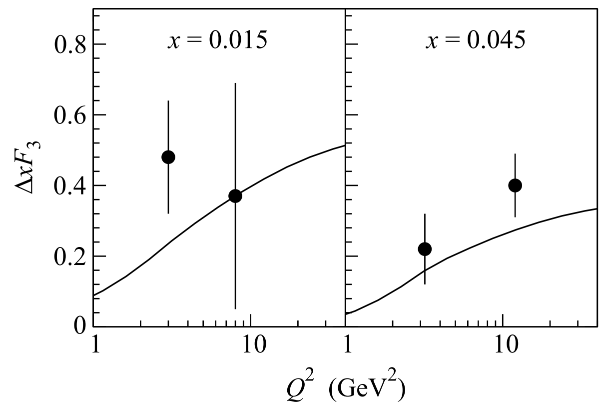

Fig.2. ΔxF_3 data as a function of Q^2 [13]. Shown by solid lines are the results of color dipole description

of ΔxF_3 as a function of Q^2 for two smallest values of x. Also shown are the results of our calculations.

After evaluating the difference of left and right cross sections let us turn to their sum and, as a consistency check, evaluate the structure function

$$2xF_1(x,Q^2) = \frac{Q^2}{4\pi^2\alpha_W}\sigma_T(x,Q^2), \quad (29)$$

where

$$\sigma_T = \frac{1}{2}\left[\sigma_L(x,Q^2) + \sigma_R(x,Q^2)\right]. \quad (30)$$

The CCFR Collaboration measurements [14] of the structure function $2xF_1$ as a function of Q^2 for three values of x are shown in Fig.3. Theory and experiment here are in qualitatively the same relations as in Fig.1. In small-x region, $x < 0.01$, dominated by the pomeron exchange our estimates are in agreement with data. For larger x the non-vacuum contributions enter the game and certain divergence shows up. This divergence will increase if we take into account the nuclear effects. Indeed, the CCFR/NuTeV structure functions $xF_3^{\nu N}$ and $xF_3^{\bar{\nu}N}$ are extracted from the νFe and $\bar{\nu}Fe$ data. The nuclear thickness factor, $T(b) = \int dzn(\sqrt{z^2+b^2})$, where

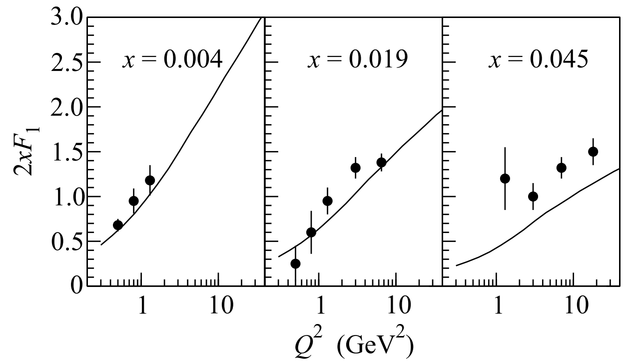

Fig.3. CCFR measurements of $2xF_1(x,Q^2)$ [14] compared with our estimates. Curves show the vacuum exchange contribution to $2xF_1(x,Q^2)$

b is the impact parameter and $n(r)$ is the nuclear matter density, $\int d^3rn(r) = A$, makes the nuclear cross section

$$\sigma_\lambda^A = A\langle\sigma_\lambda\rangle - \delta\sigma_\lambda^A, \quad (31)$$

with the nuclear shadowing term

$$\delta\sigma_\lambda^A \simeq \frac{\pi}{4}\langle\sigma_\lambda^2\rangle\int db^2T(b)^2 \quad (32)$$

very sensitive to the left-right asymmetry of the ν-nucleon cross sections. In Eqs.(31) and (32) $\langle\sigma_\lambda\rangle = \langle\Psi_\lambda|\sigma(x,r)|\Psi_\lambda\rangle$ and $\langle\sigma_\lambda^2\rangle = \langle\Psi_\lambda|\sigma(x,r)^2|\Psi_\lambda\rangle$. Hence, the nuclear shadowing correction

$$\delta xF_3 \simeq \frac{Q^2}{4\pi^2\alpha_W}\frac{\pi\langle\sigma_L^2 - \sigma_R^2\rangle}{8A}\int db^2T(b)^2, \quad (33)$$

which should be added to xF_3 extracted from the νFe data to get the "genuine" xF_3. Since $\langle\sigma_L^2\rangle \propto 1/\mu^2$ and $\langle\sigma_R^2\rangle \propto 1/m^2$, this correction is large, positive-valued and does increase xF_3 of the impulse approximation.

Summarizing, we developed the light-cone color dipole description of the left-right asymmetry effect in charged current DIS at small Bjorken x. We compared our results with experimental data and found a considerable vacuum exchange contribution to the structure functions $xF_3^{\nu N}$. This contribution is found to dominate the structure function $\Delta xF_3 = xF_3^{\nu} - xF_3^{\bar{\nu}}$ of an iso-scalar nucleon extracted from nuclear data. Theory is in reasonable agreement with data but the nuclear effects are shown can make this comparison a somewhat more complicated procedure. The color dipole analysis of nuclear effects in the CC DIS will be published elsewhere.

V.R. Z. thanks the Dipartimento di Fisica dell'Università della Calabria and the Istituto Nazionale di Fisica Nucleare - gruppo collegato di Cosenza for their warm hospitality while a part of this work was done. The work was supported in part by the Ministero Italiano dell'Istruzione, dell'Università e della Ricerca.

1. V. Barone, M. Genovese, N. N. Nikolaev et al., Phys. Lett. B **292**, 181 (1992).
2. V. Barone, U. D'Alesio, and M. Genovese, Phys. Lett. B **357**, 435 (1995); M. Bertini, M. Genovese, N. N. Nikolaev, and B. G. Zakharov, Phys. Lett. B **442**, 398 (1998); V. Barone, M. Genovese, N. N. Nikolaev et al., Phys. Lett. B **317**, 433 (1993); Ibid. B **328**, 143 (1994).
3. N. N. Nikolaev and B. G. Zakharov, Z. Phys. C **49**, 607 (1991); C **53**, 331 (1992); C **64**, 631 (1994).
4. A.H. Mueller, Nucl. Phys. B **415**, 373 (1994); A. H. Mueller and B. Patel, Nucl. Phys. B **425**, 471 (1994).
5. A. Hebecker, Phys. Rept. **331**, 1 (2000).
6. A.B. Zamolodchikov, B.Z. Kopeliovich, and L. I. Lapidus, JETP Lett. **33**, 595 (1981).
7. G. Bertsch, S. J. Brodsky, A. S. Goldhaber, and J. R. Gunion, Phys. Rev. Lett. **47**, 297 (1981).
8. G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22**, 2137 (1980); S. Brodsky, H.-Ch. Pauli, and S. Pinsky, Phys. Rept. **301**, 299 (1998).
9. V. Barone, M. Genovese, N. N. Nikolaev et al., Phys. Lett. B **328**, 143 (1994).
10. N. N. Nikolaev and B. G. Zakharov, Phys. Lett. B **332**, 184 (1994).
11. I. P. Ivanov and N. N. Nikolaev, Physics of Atomic Nucl. **64**, 753 (2001).
12. W. G. Seligman, C. G. Arroyo, L. de Barbaro et al., Phys. Rev. Lett. **79**, 1213 (1997).
13. U. K. Yang, T. Adama, A. Alton et al., Phys. Rev. Lett. **86**, 2742 (2001).
14. U. K. Yang, T. Adama, A. Alton et al., Phys. Rev. Lett. **87**, 251802 (2001).