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# Generation of two-photon KLM quantum channel

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As demonstrated by E. Knill et al. [Nature **409**, 46 (2001)] quantum teleportation and quantum logic gates with success probability close to one can be implemented using only linear optical elements, additional photons and post-selection. To do it, special quantum channels are requested to have in sight before quantum teleportation performance. Here, we propose experimental arrangement to generate two-photon KLM state different from well- known Bell states. This two-photon KLM state can be used to enhance success probability of the quantum teleportation of one-mode quantum qubit from 0.5 up to 2/3.

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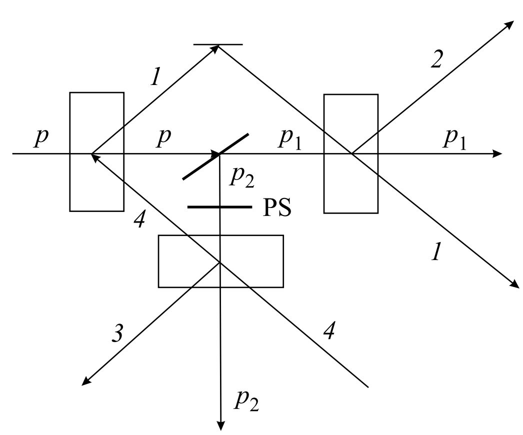
The theory of quantum computation promises to revolutionize the future of the computer technology in factoring large integers [1] and combinational searches [2]. For quantum communication purposes entangled states of the light fields are of particular interest. Such states can also be used, for example, for quantum key distribution [3] and quantum teleportation [4]. The entangled states are useful for the quantum processing, but they are hard to produce and the states tend to decohere fast. Spontaneous non-collinear parametric down converter with type-II phase matching is considered to produce true two-photon entanglement (a Bell maximally entangled state or Einstein-Podolsky-Rosen (EPR) pair) along certain directions of propagation of the generated optical beams [5]. One should mention, such EPR states give a possibility to observe, for example, process of quantum teleportation with probability of success of 50% [6]. The problem is that nonlinear interactions between individual photons are required to implement the quantum teleportation protocol that operates with 100% efficiency [7]. In order words, the inherently nonlinear Bell-state measurement must be performed for achievement of the 100% teleportation [8].

But nevertheless, it was recently recognised success probability of the quantum teleportation as well as controlled sign gates can be increased close to one by increasing the number of the ancillary optical modes, photons, and beam splitters [9]. To do it special quantum channels which we call KLM (E. Knill, R. Laflamme, G. J. Milburn) ones must be prepared before. The optical qubit interact with the KLM quantum channel by passing through a network of beam splitters and phase shifters [9]. Depending on the measurement result in ancillary modes [10], the quantum teleportation can be performed with success probability more of 50% [9]. In the paper, we propose method to generate two-photon KLM state

$$|\Psi^{(1234)}_{KLM}\rangle = \frac{1}{\sqrt{3}}\{|1100\rangle + |1001\rangle + |0011\rangle\}_{1234}, \quad (1)$$

where, henceforth, the numbers in the subscripts of the used states are related to the optical modes of the photons [11]. For example, the state $|1100\rangle_{1234}$ in Eq. (1) is a tensor product of one-photon number states where the modes and are occupied by two photons while the modes 1 and 2 have zero photons.

To generate two-photon KLM state (1), we are going to make use of the induced parametric down converter with type-I phase matching (IPDCI) with one input signal photon to the non-collinear spontaneous parametric down converter with type-I phase matching (SPDCI). For our purpose, we use experimental setup shown in Figure. First, let us consider SPDCI in detail. We are




Experimental setup to produce two-photon KLM state consisting from three coupled SPDCI. PS means phase shifter

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going to describe the SPDCI as well as IPDCI by simplified three- mode Hamiltonian [12]

$$H_1 = \frac{i\hbar r}{2}(\hat{a}_1^+\hat{a}_2^+\hat{a}_p - \hat{a}_p^+\hat{a}_2\hat{a}_1), \qquad (2)$$

where $\hat{a}_1$, $\hat{a}_2$, are the modes of down converted photons, operator $\hat{a}_p$ is the mode of the powerful beam pumping simultaneously first and second down-converted crystals through the balanced beam splitter as shown in Figure, and the coupling coefficient $r$ is related to the nonlinear second-order susceptibility tensor $\chi^{(2)}$. The simplified three-mode Hamiltonian of the non-collinear SPDCI is applicable in the case of continuous wave pumping when we neglect multi-frequency structure of the pump and use narrowband filters to choose only modes satisfying the phase matching condition. According to [12], the output function of the SPDCI with input state $|00\rangle_{12}|\alpha\rangle_p$ is given by

$$|\Psi_S^{(12)}\rangle = \sum_{n=0}^{\infty}(\alpha\beta)^n|n_1\rangle_1|n\rangle_2|\psi_{(00)}^{(n)}\rangle_p, \qquad (3a)$$

where the partial wave functions $|\psi_{(00)}^{(n)}\rangle_p$ in the pumping mode are given by

$$|\psi_{(00)}^{(n)}\rangle_p = \exp\left(-\frac{|\alpha|^2}{2}\right)\sum_{m=0}^{\infty}\frac{\alpha^m f_{n+1}^{(2(m+n))}(\beta)}{\beta^n\sqrt{(m+n)!}}|m\rangle_p, \qquad (3b)$$

where the function $f_{n+1}^{(2(m+n))}(\beta)$ is responsible for the output wave amplitude [12], the subscript (00) in the definition of the functions $|\psi_{(00)}^{(n)}\rangle_p$means the input states in the signal and idler modes were in vacuum and the superscript in the designation of the output function $|\Psi_S^{(12)}\rangle$ of the SPDCI concerns to the numbers of the generated modes. Here, the magnitude $\alpha$ is the amplitude of the coherent state input to the SPDCI and the coefficient $\beta$ is responsible for the "strength" of the SPDCI [12]. The output non-normalized wave function of the SPDCI (3b) can be significantly simplified if we make use of the $\alpha\beta \ll 1$ approximation that takes place in practice and decompose the output wave amplitudes $f_{n+1}^{(2(m+n))}(\beta)$ (Eq. (3b)) into asymptotic series in small parameter $\beta \ll 1$ to take into account only first term of the series ($f_1^{(2n)}(\beta) \sim 1$, $f_2^{(2n)}(\beta) \sim \sim \beta\sqrt{n}$, $f_3^{(2n)}(\beta) \sim \beta^2\sqrt{n(n-1)}$, ..., $f_m^{(2n)}(\beta) \approx \approx \beta^{m-1}\sqrt{n(n-1)\dots(n-m+2)}$ and so on for any $m \leq n+1$) [12]

$$|\Psi_S^{(12)}\rangle = \sum_{n=0}^{\infty}(\alpha\beta)^n|n\rangle_1|n\rangle_2|\alpha\rangle_p. \qquad (3c)$$

Now, let us consider IPDCI with one input signal photon and no photons in the idler mode, in order words, if the input conditions to Hamiltonian $H_1$ (Eq. (2)) are chosen, for example, as $|10\rangle_{12}|\alpha\rangle_p$. Following the same technique as in the case of SPDCI [12], we can write the wave function of the IPDCI as

$$|\Psi_I^{(12)}\rangle = \sum_{n=0}^{\infty}|\Psi_{(10)}^{(2n+1)}\rangle, \qquad (4a)$$

where

$$|\Psi_{(10)}^{(2n+1)}\rangle = \sum_{k=1}^{n+1} f_{k(10)}^{(2n+1)}(s;\beta)|k\rangle_1|k-1\rangle_2|n-k+1\rangle_p, \qquad (4b)$$

with the wave amplitudes $f_{k(10)}^{(2n+1)}(s;\beta)$ ($k = 1,\dots,n+ +1$) satisfying the set of linear differential equations

$$\frac{df_{k(10)}^{(2n+1)}}{ds} = \beta(\sqrt{k(k-1)(n-k+2)}f_{k-1(10)}^{(2n+1)} - -\sqrt{k(k+1)(n-k+1)}f_{k+1(10)}^{(2n+1)}. \qquad (4c)$$

The following input conditions $f_{1(10)}^{(2n)}(s=0) = = \exp(-|\alpha|^2/2)\alpha^n/\sqrt{n}!$ and $f_{k(10)}^{(2n)}(s=0) = 0$ for $k = 2,\dots,n+1$ are imposed on the Eq. (4c). Here, the symbol in the subscripts of the Eqs. (4a-e) is introduced to distinguish the states (4a-c) from the states (3a,b). The output wave function of the IPDCI $|\Psi_I^{(12)}\rangle$ (Eq. (4a)) can be rewritten as

$$|\Psi_I^{(12)}\rangle = \sum_{n=0}^{\infty}(\alpha\beta)^n|n\rangle_1|n\rangle_2|\psi_{(10}^{(n)}\rangle_p, \qquad (4d)$$

where the partial wave functions $|\psi_{(10)}^{(n)}\rangle_p$ in the pumping mode are given by

$$|\psi_{(10)}^{(n)}\rangle_p = \exp\left(-\frac{|\alpha|^2}{2}\right)\sum_{m=0}^{\infty}\frac{\alpha^m f_{n+1(10)}^{(2(m+n)+1)}(\beta)}{\beta^n\sqrt{(m+n)!}}|m\rangle_p. \qquad (4e)$$

Smallness of the parameter $\beta \ll 1$ allows for one to decompose the wave amplitudes $f_{k(10)}^{(2(m+n)+1)}(\beta)$ into asymptotic series in the small parameter $\beta$ restricting the decomposition only by first term in the case of $\beta \ll 1$. Then, we get $f_{n+1(10)}^{(2(n+m)+1)} \sim \sim \beta^n\sqrt{n+1}\sqrt{(n+m)(n+m-1)\dots(m+1)}$ and the output wave function of the IPDCI with one input signal photon in the $\alpha\beta \ll 1$ approximation is given by

$$|\Psi_I^{(12)}\rangle = \sum_{n=0}^{\infty}(\alpha\beta)^n\sqrt{n+1}|n+1\rangle_1|n\rangle_2|\alpha\rangle_p. \qquad (4f)$$

Now, we are ready to describe more detailed the experimental setup in Figure to produce the two-photon KLM state. First SPDCI produces the outcomes given by Eqs. (3a-c). We are going to make use of the $\alpha\beta \ll 1$ approximation valid in practical case to deal with only the following non-normalised wave function $|\widetilde{\Psi}_{in}\rangle = \{|00\rangle + \alpha\beta|11\rangle\}_{12}|\alpha\rangle_p$ (input to system of two coupled IPDCI in Figure) neglecting higher order in parameter $\alpha\beta \ll 1$terms in the output wave function of the SPDCI (Eq. (3c)). Next step is related to the system of two down converted crystals with identical responses pumped simultaneously by powerful pumping modes as shown in Figure. According to Figure, the pumping mode in coherent state with amplitude $\alpha$ ($|\alpha\rangle_p$) passes through the balanced beam splitter transforming to the state $|\alpha/\sqrt{2}\rangle_{p_1}|i\alpha/\sqrt{2}\rangle_{p_2}$, where the subscripts $p_1$ and $p_2$ refer to the first and second output modes of the beam splitter while the subscripts and are concerned the input modes to the beam splitter, respectively. We are going to apply phase shifter to avoid $\pi/2$ phase shift in the second pumping mode $\pi_2$. The phase-shifter is the optical element that acts on a single mode to cause a shift $\psi$ in the phase of the mode state. The underlying unitary evolution operator is $\widehat{P}_{p_2} = \exp(-\phi\hat{a}^+_{p_2}\hat{a}_{p_2})$. Then, we have the following relation $\widehat{P}_{p_2}\hat{a}^+_{p_2}\widehat{P}^+_{p_2} = \exp(-i\phi)\hat{a}^+_{p_2}$ that enables to destroy $\pi/2$ phase shift in the second pumping mode $p_2$ if we chose $\phi = \pi/2$. One of the generated modes by the SPDCI (namely mode 1) is launched into first down converted crystal while the other mode (namely mode 2 which we label as 4 in Figure) is entered to the second down converted crystal. Then finally, the input state to the two down converted crystals becomes $|\Psi_{in}\rangle = \{|0000\rangle + \alpha\beta|1001\rangle\}_{1234}|\alpha/\sqrt{2}\rangle_{p_1}|\alpha/\sqrt{2}\rangle_{p_2}$ after the beam splitter.

As consequence of such input to the down converters, the outcome of the system is divided into two groups, namely, one of the part of the state $|\Psi_{in}\rangle$ originating from the input state $|0000\rangle_{1234}|\alpha/\sqrt{2}\rangle_{p_1}|\alpha/\sqrt{2}\rangle_{p_2}$ gives a rise to the SPDCI process and other part of the input state $|\Psi_{in}\rangle$, namely, $\alpha\beta|1001\rangle_{1234}|\alpha/\sqrt{2}\rangle_{p_1}|\alpha/\sqrt{2}\rangle_{p_2}$ is responsible for the IPDCI.

We make use of the three-mode simplified Hamiltonian $H_{12}$ for the system of two down converted crystals with identical responses pumped simultaneously by powerful pump modes

$$H_{12} = H_1 + H_2 =$$
$$= \frac{i\hbar r'}{2}(\hat{a}^+_1\hat{a}^+_2\hat{a}^+_{p_1} - \hat{a}^+_{p_1}\hat{a}_2\hat{a}_1 + \hat{a}^+_3\hat{a}^+_4\hat{a}^+_{p_1} - \hat{a}^+_{p1}\hat{a}_4\hat{a}_3), \quad (5)$$

where the subscripts for quantum operators in (5) are referred to the corresponding signal, idler (modes 1–4) and pump modes (modes $p_1$ and $p_2$), the coupling constant $r'$ is related to the system of coupled down converters, and the Hamiltonians $H_1$ and $H_2$ are concerned first and second down converters in Figure, respectively. The Hamiltonian $H_{12}$ (5) gives a rise to the following wave function

$$|\Psi_{Out}\rangle = |\Psi^{(12)}_S\rangle|\Psi^{(34)}_S\rangle + \alpha\beta|\Psi^{(12)}_I\rangle|\Psi^{(34)}_I\rangle, \quad (6)$$

with the corresponding wave functions $|\Psi^{(12)}_S\rangle$ and $|\Psi^{(12)}_I\rangle$ (Eqs. (3a,b), (4d,f)) in the first and second output ports and the states $|\Psi^{(34)}_S\rangle$ and $|\Psi^{(34)}_I\rangle$ in the third and fourth output channels, respectively. That fact that the parameter $\alpha\beta$ in practical case takes a value much less of one ($\alpha\beta \ll 1$) ) gives a possibility to rewrite the output wave function only taking into account vacuum states and terms proportional to the factor $\alpha\beta$ as

$$|\Psi_{Out}\rangle = \{|0000\rangle + \alpha\beta'/\sqrt{2}(|1100\rangle + |0011\rangle) +$$
$$+ \alpha\beta|1001\rangle\}_{1234}|\alpha/\sqrt{2}\rangle_{p_1}|\alpha/\sqrt{2}\rangle_{p_2}. \quad (7)$$

Here the parameter $\beta'$ is related to the coupling constant of the system of two SPDCI pumped simultaneously through the balanced beam splitter. Finally, we get superposition of the vacuum state with two-photon KLM state (1) (Eq. (7)), if we take $\beta' = \sqrt{2}\beta$. One should mention, proposed experiment is based on use of coupled IPDCI, which is inherently random. Consequently, we can determine whether a pair of photons has been generated only by postselection produced by detectors. The same random generation of the superposition of the vacuum and Bell states occurs in majority of current experiments [5, 6], when the randomness of the generated pair is not essential. The proposed in Figure scheme can become basis for generation of other types of KLM states, for example, four-photon KLM state $|rt_2\rangle = \sum_{j=0}^{2}|\tilde{0}\rangle^j|\tilde{1}\rangle^{2-j}|\tilde{0}\rangle^{2-j}|\tilde{1}\rangle^j$ written in terms of the qubit encoding [9] by use of quantum encoding technique. The state $|rt_2\rangle$ can be used to teleport two-mode qubit with success probability of 2/3 [9].

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