

# Two-body correlation functions in nuclear matter with neutron-proton condensate

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The density, spin and isospin correlation functions in nuclear matter with a neutron-proton condensate are calculated to study the possible signatures of the BEC-BCS crossover in low-density region. It is shown that criterion of the crossover (Phys. Rev. Lett. **95**, 090402 (2005)), consisting in the change of the sign of the density correlation function at low momentum transfer, fails to describe correctly the density-driven BEC-BCS transition at finite isospin asymmetry or finite temperature. As an unambiguous signature of the BEC-BCS transition, there can be used the presence (BCS regime) or absence (BEC regime) of the singularity in the momentum distribution of the quasiparticle density of states.

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*Introduction.* Neutron-proton pairing correlations play an important role in a number of contexts, including the study of medium mass  $N \approx Z$  nuclei produced at the radioactive nuclear beam facilities [1] and the process of the deuteron formation in the medium energy heavy ion collisions [2]. In the astrophysical context neutron-proton ( $np$ ) pairing correlations are relevant for the astrophysical  $r$ -process [3], responsible for the synthesis of nuclear species more massive than iron, and can play a major role in neutron star models, which permit pion or kaon condensation [4]. For not too low densities,  $np$  pairing correlations crucially depend on the overlap between neutron and proton Fermi surfaces and even small isospin asymmetry effectively destroys a condensate with  $np$  Cooper pairs due to the Pauli blocking effect [5–7]. However, under decreasing density, when neutrons and protons start to bind in deuterons and the spatial separation between deuterons, and between deuterons and extra neutrons is large, the Pauli blocking loses its efficiency in destroying a  $np$  condensate. In such a situation, despite that the isospin asymmetry may be very large, a  $np$  condensate survives and exists in the form of Bose-Einstein condensate of deuterons.

The transition from BCS superconductivity to Bose-Einstein condensation (BEC) occurs in a Fermi system, if either density is decreased or the attractive interaction between fermions is increased sufficiently. This transition was studied, first, in excitonic semiconductors [8] and then in an attractive Fermi gas [9]. Later it was realized that analogous phase transition takes place in symmetric nuclear matter, when  $np$  Cooper pairs at higher densities go over to BEC of deuterons at lower

densities [2, 5]. During this transition the chemical potential changes its sign at certain critical density (Mott transition), approaching half of the deuteron binding energy at ultra low densities. In Ref. [5] crossover from  $np$  superfluidity to BEC of deuterons was investigated in the  $T$ -matrix approach, where the pole in the  $T$ -matrix determines the critical temperature of BEC of bound states in the case of negative chemical potential and the critical temperature of appearance of  $np$  Cooper pairs in the case of positive chemical potential. The influence of isospin asymmetry on the BEC-BCS crossover in nuclear matter was studied in Ref. [10] within the BCS formalism. It has been shown that Bose-Einstein condensate is weakly affected by an additional gas of free neutrons even at very large asymmetries. The same conclusion was confirmed also in Ref. [11] on the base of the variational approach for the thermodynamic potential.

Recent upsurge of interest to the BEC-BCS crossover is caused by finding the BCS pairing in ultracold trapped quantum atom gases [12, 13]. In this study we examine the possible signatures of the BEC-BCS crossover in low-density nuclear matter. It may have interesting consequences, for example, in the far tails of the density profiles of exotic nuclei, where a deuteron condensate can exist in spite of the fact that there the density can be quite asymmetric. Besides, similar physical effects can play an important role in expanding nuclear matter, formed in heavy ion collisions, or in nuclear matter in the crust of a neutron star. The main emphasis is laid on the behavior of the density, spin and isospin correlation functions across the BEC-BCS transition region. The study is motivated by the results of Ref. [14], where

the authors state that the density correlation function of two-component ultracold fermionic gas of atoms changes sign at low momentum transfer and this represents an unambiguous signature of the BEC-BCS crossover. This statement is checked for nuclear matter taking into account additional factors: finite isospin asymmetry or finite temperature. In both cases, this criterion fails to provide a correct description of the density-driven BEC-BCS crossover and cannot serve as the universal feature of transition between two states of the system.

**Basic equations.** Superfluid states of nuclear matter are described by the normal  $f$  and anomalous  $g$  distribution functions of nucleons

$$f_{\kappa_1 \kappa_2} = \text{Tr} \varrho a_{\kappa_2}^+ a_{\kappa_1}, \quad g_{\kappa_1 \kappa_2} = \text{Tr} \varrho a_{\kappa_2} a_{\kappa_1}, \quad (1)$$

where  $\kappa \equiv (\mathbf{k}, \sigma, \tau)$ ,  $\mathbf{k}$  is momentum,  $\sigma(\tau)$  is the projection of spin (isospin) on the third axis,  $\varrho$  is the density matrix of the system. We shall study  $np$  pairing correlations in the pairing channel with total spin  $S$  and isospin  $T$  of a pair  $S = 1, T = 0$  and the projections  $S_z = T_z = 0$ . In this case the distribution functions for isospin asymmetric nuclear matter have the structure

$$\begin{aligned} f(\mathbf{k}) &= f_{00}(\mathbf{k})\sigma_0\tau_0 + f_{03}(\mathbf{k})\sigma_0\tau_3, \\ g(\mathbf{k}) &= g_{30}(\mathbf{k})\sigma_3\sigma_2\tau_2, \end{aligned} \quad (2)$$

where  $\sigma_i$  and  $\tau_k$  are the Pauli matrices in spin and isospin spaces, respectively. Using the minimum principle of the thermodynamic potential and procedure of block diagonalization [7], one can obtain expressions for the distribution functions

$$f_{00}(\mathbf{k}) = \frac{1}{2} - \frac{\xi_k}{4E_k} \left( \tanh \frac{E_k^+}{2T} + \tanh \frac{E_k^-}{2T} \right), \quad (3)$$

$$f_{03}(\mathbf{k}) = \frac{1}{4} \left( \tanh \frac{E_k^+}{2T} - \tanh \frac{E_k^-}{2T} \right), \quad (4)$$

$$g_{30}(\mathbf{k}) = -\frac{\Delta(\mathbf{k})}{4E_k} \left( \tanh \frac{E_k^+}{2T} + \tanh \frac{E_k^-}{2T} \right). \quad (5)$$

Here

$$E_k^\pm = E_k \pm \delta\mu = \sqrt{\xi_k^2 + \Delta^2(\mathbf{k})} \pm \delta\mu, \quad \xi_k = \frac{k^2}{2m} - \mu, \quad (6)$$

$\Delta$  being the energy gap in the quasiparticle excitation spectrum,  $m$  being the effective nucleon mass,  $\mu$  and  $\delta\mu$  being half of a sum and half of a difference of neutron and proton chemical potentials, respectively.

Equations, governing  $np$  pairing correlations in  $S = 1, T = 0$  pairing channel, can be obtained on the base of Green's function formalism and have the form [7, 11]

$$\Delta(\mathbf{k}) = -\frac{1}{V} \sum_{\mathbf{k}'} V(\mathbf{k}, \mathbf{k}') \frac{\Delta(\mathbf{k}')}{2E_{k'}} (1 - f(E_{k'}^+) - f(E_{k'}^-)), \quad (7)$$

$$\varrho = \frac{2}{V} \sum_{\mathbf{k}} \left( 1 - \frac{\xi_k}{E_k} [1 - f(E_k^+) - f(E_k^-)] \right) \equiv \frac{2}{V} \sum_{\mathbf{k}} n_k, \quad (8)$$

$$\alpha\varrho = \frac{2}{V} \sum_{\mathbf{k}} \left( f(E_k^-) - f(E_k^+) \right), \quad (9)$$

where  $f(E)$  is Fermi distribution function. Eq. (7) is equation for the energy gap  $\Delta$  and Eqs. (8), (9) are equations for the total density  $\varrho = \varrho_p + \varrho_n$  and neutron excess  $\delta\varrho = \varrho_n - \varrho_p \equiv \alpha\varrho$  ( $\alpha$  being the asymmetry parameter). Note that since we consider unitary superfluid state ( $\Delta\Delta^+ \propto I$ ), Eqs. (7)–(9) formally coincide with the equations for two-component isospin asymmetric superfluid with singlet spin pairing between unlike fermions. Introducing the anomalous density

$$\psi(\mathbf{k}) = \langle a_{n,k}^+ a_{p,-k}^+ \rangle = \frac{\Delta(\mathbf{k})}{2E_k} (1 - f(E_k^+) - f(E_k^-))$$

and using Eq. (8), one can represent Eq. (7) for the energy gap in the form

$$\frac{k^2}{m} \psi(\mathbf{k}) + (1 - n_k) \sum_{\mathbf{k}'} V(\mathbf{k}, \mathbf{k}') \psi(\mathbf{k}') = 2\mu\psi(\mathbf{k}). \quad (10)$$

In the limit of vanishing density,  $n_k \rightarrow 0$ , Eq. (10) goes over into the Schrödinger equation for the deuteron bound state [2, 10]. Corresponding energy eigenvalue is equal to  $2\mu$ . The change in the sign of the mean chemical potential  $\mu$  of neutrons and protons under decreasing density of nuclear matter signals the transition from the regime of large overlapping  $np$  Cooper pairs to the regime of non-overlapping bound states (deuterons).

Let us consider the two-body density correlation function

$$\mathcal{D}(\mathbf{x}, \mathbf{x}') = \text{Tr} \varrho \Delta \hat{n}(\mathbf{x}) \Delta \hat{n}(\mathbf{x}'), \quad \Delta \hat{n}(\mathbf{x}) = \hat{n}(\mathbf{x}) - \hat{n},$$

$$\begin{aligned} \hat{n}(\mathbf{x}) &\equiv \sum_{\sigma\tau} \psi_{\sigma\tau}^+(\mathbf{x}) \psi_{\sigma\tau}(\mathbf{x}) = \\ &= \frac{1}{V} \sum_{\sigma\tau \mathbf{k}\mathbf{k}'} e^{i(\mathbf{k}' - \mathbf{k})\mathbf{x}} a_{\mathbf{k}\sigma\tau}^+ a_{\mathbf{k}'\sigma\tau}, \\ \hat{n} &= \frac{1}{V} \sum_{\sigma\tau \mathbf{k}} a_{\mathbf{k}\sigma\tau}^+ a_{\mathbf{k}\sigma\tau}. \end{aligned} \quad (11)$$

Its general structure in the spatially uniform and isotropic case reads [15]

$$\mathcal{D}(\mathbf{x}, \mathbf{x}') = \varrho\delta(\mathbf{r}) + \varrho D(r), \quad \mathbf{r} = \mathbf{x} - \mathbf{x}'. \quad (12)$$

The function  $D(r)$  is called the density correlation function as well. We will be just interested in the behavior of

the function  $D(r)$ . The trace in Eq. (10) can be calculated, using definitions (1) and Wick rules. Taking into account Eqs. (2) and going to the Fourier representation

$$D(q) = \int d^3\mathbf{r} e^{i\mathbf{q}\mathbf{r}} D(r),$$

one can get

$$D(q) = I_g^{30}(q) - I_f^{00}(q) - I_f^{03}(q), \quad (13)$$

where

$$\begin{aligned} I_f^{00}(q) &= \frac{4}{\pi^3 \varrho} \int_0^\infty dr r^2 j_0(rq) \left[ \int_0^\infty dk k^2 f_{00}(k) j_0(rk) \right]^2, \\ I_f^{03}(q) &= \frac{4}{\pi^3 \varrho} \int_0^\infty dr r^2 j_0(rq) \left[ \int_0^\infty dk k^2 f_{03}(k) j_0(rk) \right]^2, \\ I_g^{30}(q) &= \frac{4}{\pi^3 \varrho} \int_0^\infty dr r^2 j_0(rq) \left[ \int_0^\infty dk k^2 g_{30}(k) j_0(rk) \right]^2. \end{aligned}$$

Here  $j_0$  is the spherical Bessel function of the first kind and zeroth order. Functions  $I_f^{00}$ ,  $I_f^{03}$  and  $I_g^{30}$  represent the normal and anomalous contributions to the density correlation function. Analogously, we can consider the two-body spin correlation function

$$S_{\mu\nu}(\mathbf{x}, \mathbf{x}') = \text{Tr} \varrho \Delta \hat{s}_\mu(\mathbf{x}) \Delta \hat{s}_\nu(\mathbf{x}'), \quad \Delta \hat{s}_\mu(\mathbf{x}) = \hat{s}_\mu(\mathbf{x}) - \hat{s}_\mu,$$

$$\begin{aligned} \hat{s}_\mu(\mathbf{x}) &\equiv \frac{1}{2} \sum_{\sigma\sigma'\tau} \psi_{\sigma\tau}^+(\mathbf{x}) (\sigma_\mu)_{\sigma\sigma'} \psi_{\sigma'\tau}(\mathbf{x}) = \\ &= \frac{1}{2V} \sum_{\sigma\sigma'\tau \mathbf{k}\mathbf{k}'} e^{i(\mathbf{k}'-\mathbf{k})\mathbf{x}} a_{\mathbf{k}\sigma\tau}^+ (\sigma_\mu)_{\sigma\sigma'} a_{\mathbf{k}'\sigma'\tau}, \quad (14) \end{aligned}$$

$$\hat{s}_\mu = \frac{1}{2V} \sum_{\sigma\sigma'\tau \mathbf{k}} a_{\mathbf{k}\sigma\tau}^+ (\sigma_\mu)_{\sigma\sigma'} a_{\mathbf{k}\sigma'\tau},$$

and the two-body isospin correlation function

$$T_{\mu\nu}(\mathbf{x}, \mathbf{x}') = \text{Tr} \varrho \Delta \hat{t}_\mu(\mathbf{x}) \Delta \hat{t}_\nu(\mathbf{x}'), \quad \Delta \hat{t}_\mu(\mathbf{x}) = \hat{t}_\mu(\mathbf{x}) - \hat{t}_\mu,$$

$$\begin{aligned} \hat{t}_\mu(\mathbf{x}) &\equiv \frac{1}{2} \sum_{\sigma\tau\tau'} \psi_{\sigma\tau}^+(\mathbf{x}) (\tau_\mu)_{\tau\tau'} \psi_{\sigma\tau'}(\mathbf{x}) = \\ &= \frac{1}{2V} \sum_{\sigma\tau\tau' \mathbf{k}\mathbf{k}'} e^{i(\mathbf{k}'-\mathbf{k})\mathbf{x}} a_{\mathbf{k}\sigma\tau}^+ (\tau_\mu)_{\tau\tau'} a_{\mathbf{k}'\sigma\tau'}, \quad (15) \end{aligned}$$

$$\hat{t}_\mu = \frac{1}{2V} \sum_{\sigma\tau\tau' \mathbf{k}} a_{\mathbf{k}\sigma\tau}^+ (\tau_\mu)_{\tau\tau'} a_{\mathbf{k}\sigma\tau'}.$$

Their general structure for isospin asymmetric nuclear matter without spin polarization is

$$S_{\mu\nu}(\mathbf{x}, \mathbf{x}') = \frac{\varrho}{4} \delta_{\mu\nu} \delta(\mathbf{r}) + \varrho S_{\mu\nu}(r), \quad (16)$$

$$T_{\mu\nu}(\mathbf{x}, \mathbf{x}') = \frac{\varrho}{4} \delta_{\mu\nu} \delta(\mathbf{r}) + \frac{\alpha\varrho}{4} i\epsilon_{\mu\nu 3} \delta(\mathbf{r}) + \varrho T_{\mu\nu}(r). \quad (17)$$

Then, calculating traces in Eqs. (14), (15), for the Fourier transforms of the spin and isospin correlation functions one can get

$$S_{\mu\nu}(q) = -\frac{1}{4} \{ \delta_{\mu\nu} (I_f^{00}(q) + I_f^{03}(q)) + (\delta_{\mu\nu} - 2\delta_{3\mu}\delta_{3\nu}) I_g^{30}(q) \}, \quad (18)$$

$$T_{\mu\nu}(q) = -\frac{1}{4} \{ \delta_{\mu\nu} (I_f^{00}(q) + I_g^{30}(q)) - (\delta_{\mu\nu} - 2\delta_{3\mu}\delta_{3\nu}) I_f^{03}(q) \}. \quad (19)$$

Note that if to put  $\nu = \mu = 3$  in Eqs. (18), (19), one gets the longitudinal spin  $S^l$  and isospin  $T^l$  correlation functions, while setting  $\mu, \nu = 1, 2$  gives the transverse spin and isospin correlation functions

$$S_{\mu\nu}^t(q) = -\frac{\delta_{\mu\nu}}{4} (I_f^{00}(q) + I_f^{03}(q) + I_g^{30}(q)) \equiv \delta_{\mu\nu} S^t(q), \quad \mu, \nu = 1, 2, \quad (20)$$

$$T_{\mu\nu}^t(q) = -\frac{\delta_{\mu\nu}}{4} (I_f^{00}(q) - I_f^{03}(q) + I_g^{30}(q)) \equiv \delta_{\mu\nu} T^t(q).$$

The following relationships between the correlation functions hold true

$$S^l(q) = \frac{D(q)}{4}, \quad S^t(q) = T^l(q). \quad (21)$$

At zero temperature and zero momentum transfer, the correlation functions satisfy the sum rule

$$\begin{aligned} S^l(q=0) &= T^l(q=0) = \\ &= -\frac{1}{2\pi^2\varrho} \int dk k^2 (f_{00}^2(k) + f_{03}^2(k) + g_{30}^2(k)) = -\frac{1}{4}, \quad (22) \end{aligned}$$

where the r.h.s. is independent of density and isospin asymmetry. Besides, the transverse isospin correlation function satisfies the relationship

$$\begin{aligned} T^t(q=0) &= -\frac{1}{2\pi^2\varrho} \int dk k^2 (f_{00}^2(k) - f_{03}^2(k) + g_{30}^2(k)) = \\ &= -\frac{1-\alpha}{4}, \quad (23) \end{aligned}$$

where the r.h.s. is independent of density.

*Correlation functions in nuclear matter with a np condensate.* Further for numerical calculations we shall use the effective zero range force, developed in Ref. [16] to reproduce the pairing gap in  $S = 1, T = 0$  pairing channel with Paris NN potential:

$$V(\mathbf{r}_1, \mathbf{r}_2) = v_0 \left\{ 1 - \eta \left( \frac{\varrho \left( \frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right)}{\varrho_0} \right)^\gamma \right\} \delta(\mathbf{r}_1 - \mathbf{r}_2) \quad (24)$$

where  $\varrho_0 = 0.16 \text{ fm}^{-3}$  is the nuclear saturation density,  $v_0 = -530 \text{ MeV} \cdot \text{fm}^3$ ,  $\eta = 0$ ,  $m = m_G$ ,  $m_G$  being the

effective mass, corresponding to the Gogny force D1S. Besides, in the gap equation (7), Eq. (24) must be supplemented with a cut-off parameter,  $\varepsilon_c = 60$  MeV.

To find the correlation functions, first, one should solve the gap equation (7) self-consistently with Eqs. (8), (9). Then the correlation functions can be determined directly from Eqs. (13), (18) and (19). The results of numerical determination of the energy gap as a function of density for different asymmetries at zero temperature are shown in Fig.1. As one can see, with increasing

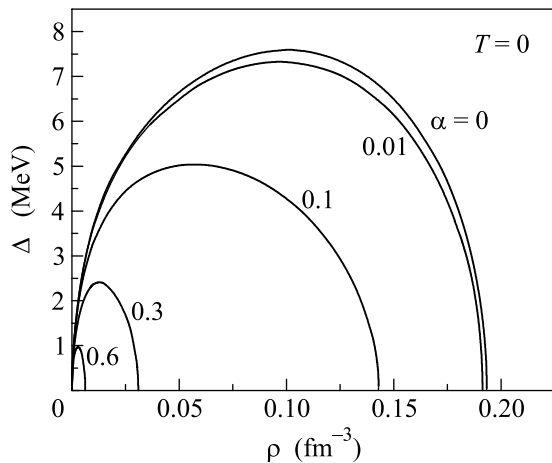


Fig.1. Energy gap as a function of density at zero temperature and different asymmetries

asymmetry the magnitude of the energy gap is decreased and the density interval, where a  $np$  condensate exists, shrinks to lower density. In reality solutions exist for any  $\alpha < 1$  (the phase curves for larger values of  $\alpha$  are not shown in Fig.1) and correspond to the formation of BEC of deuterons at very low densities of nuclear matter.

Now we consider the correlation functions  $D(q)$  and  $S^t(q)$  for symmetric nuclear matter at zero temperature, depicted in Fig.2 (at  $\alpha = 0$ ,  $T^t(q) = S^t(q)$ ). The density correlation function changes sign at low momentum transfer when the system smoothly evolves from the BEC regime to the BCS one. These two regimes are distinguished by the negative and positive values of the chemical potential  $\mu$ , respectively. In view of Eq. (21), the longitudinal spin correlation function  $S^l(q)$  changes the sign through the BEC-BCS crossover as well. The transverse spin correlation function, and, according to Eq. (21), the longitudinal and transverse isospin correlation functions change fluently between BEC and BCS limits. The behavior of the density correlation function in isospin symmetric case at zero temperature qualitatively agrees with the behavior of the density correlation function in ultracold fermionic atom gas with sin-

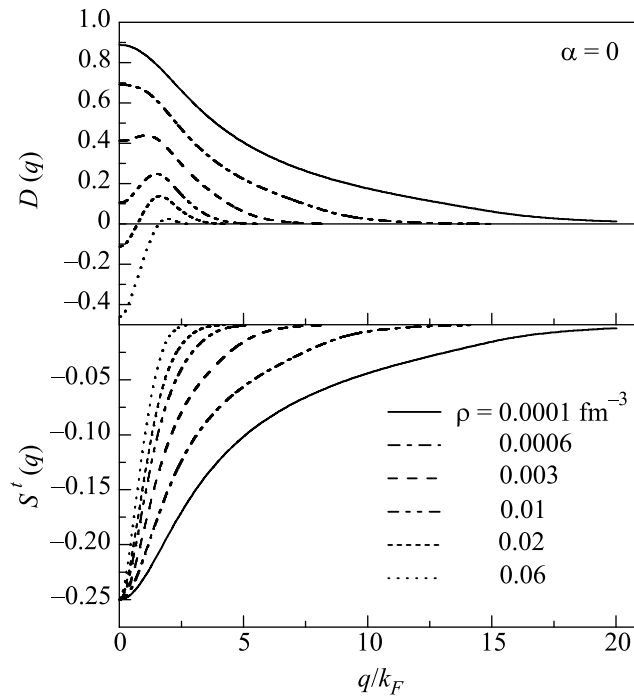


Fig.2. Density and transverse spin correlation functions as functions of momentum at zero temperature and different densities for symmetric nuclear matter

glet pairing of fermions [14]. In Ref. [14], the change in the sign of the density correlation function at low momentum transfer was considered as a signature of the BEC-BCS crossover. We would like to extend their calculations with account of finite isospin asymmetry and finite temperature.

Fig.3 shows the dependence of the density correlation function  $D(\mathbf{q} = 0)$  at zero momentum transfer as

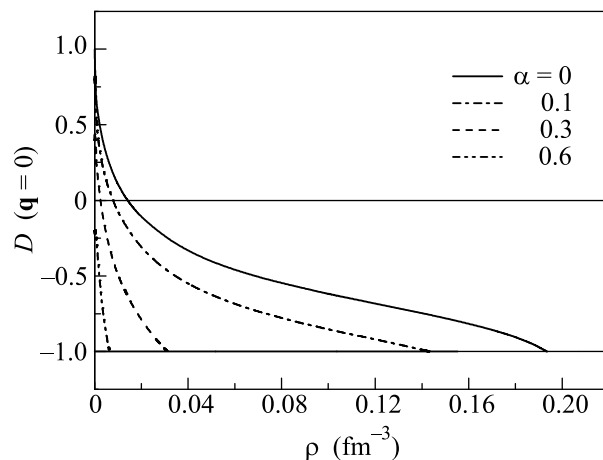


Fig.3. Density correlation function  $D(\mathbf{q} = 0)$  as a function of density at zero temperature for different isospin asymmetry parameters

a function of density for a set of various isospin asymmetry parameters and zero temperature. It is seen that with increasing the asymmetry parameter the density correlation function is decreased. For strong enough asymmetry, the function  $D(\mathbf{q} = 0)$  is always negative. In accordance with the above criterion, the density region, where the function  $D(\mathbf{q} = 0)$  has the positive or negative values, would correspond to the BEC or BCS regime, respectively. Hence, as follows from Fig.3, for strong isospin asymmetry we would have only the BCS state for all densities where a  $np$  condensate exists. Obviously, this conclusion contradicts with the behavior of the mean chemical potential  $\mu$ , being negative at very low densities for any  $\alpha < 1$ , and, hence, giving evidence to the formation of BEC of bound states [11]. Thus, at strong enough isospin asymmetry the criterion of the crossover, based on the change of the sign of the density correlation function, fails to predict the transition to the BEC of deuterons in low-density nuclear matter.

Now we consider symmetric nuclear matter at finite temperature. Fig.4 shows the dependence of the den-

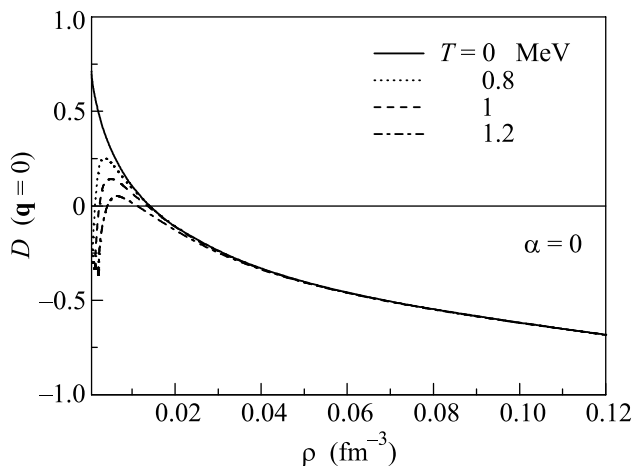


Fig.4. Density correlation function  $D(\mathbf{q} = 0)$  as a function of density at different temperatures for symmetric nuclear matter

sity correlation function  $D(\mathbf{q} = 0)$  at zero momentum transfer as a function of density for a set of various temperatures. It is seen that for not too high temperatures the density response function is non-monotone and twice changes the sign in the region of low densities. Hence, in accordance with the above criterion, we would have the density interval  $\varrho_1 < \varrho < \varrho_2$  with the BEC state, surrounded by the density regions with the BCS state. However, this conclusion contradicts with the behavior of the mean chemical potential  $\mu$  for these temperatures, being the monotone function of density and giving indication to the formation of BEC state at low densities

( $\mu < 0$ ) and the BCS state at larger densities ( $\mu > 0$ ). Thus, at finite temperature the criterion of the crossover, formulated in Ref. [14], fails to provide the correct description of the transition between two regimes.

In summary, we have calculated the density, spin and isospin correlation functions in superfluid nuclear matter with  $np$  pairing correlations, intending to find the possible signatures of the BEC-BCS crossover. It is shown that the transverse spin, and longitudinal and transverse isospin correlation functions satisfy the sum rule at zero momentum transfer and zero temperature, and change smoothly between BEC and BCS regimes. In Ref. [14], it was learned that the density correlation function in two-component ultracold fermionic atom gas with singlet pairing of fermions changes sign at low momentum transfer across the BEC-BCS transition, driven by changing the scattering length of the interaction at zero temperature. We have shown that for spin triplet pairing the longitudinal spin correlation function plays an analogous role to the density correlation function and changes the sign at low momentum transfer across the crossover in symmetric nuclear matter at zero temperature. However, while giving the satisfactory description of the density-driven BEC-BCS crossover in dilute nuclear matter at zero temperature for isospin symmetric case, this criterion fails to provide the correct description of the crossover at finite isospin asymmetry (non-equal densities of fermions of different species) or finite temperature. Hence, the criterion in Ref. [14] cannot be considered as the universal indication of the BEC-BCS transition. During the Mott transition, when the chemical potential changes sign, there is a qualitative change in the quasiparticle energy spectrum: the minimum shifts from a finite (BCS state) to zero-momentum value (BEC state) (see Eq. (6) and Ref. [17]). As such, the presence (BCS) or absence (BEC) of the singularity in the momentum distribution of the quasiparticle density of states represents the universal signature of the BEC-BCS transition. This transition may be relevant, and could give a valuable information on  $np$  pairing correlations, in low-density nuclear systems, such as tails of nuclear density distributions in exotic nuclei, produced at the radioactive nuclear beam facilities, expanding nuclear matter in heavy ion collisions, low-density nuclear matter in outer regions of neutron stars, etc.

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