Pis'ma v ZhETF, vol. 82, iss. 10, pp. 710 – 713 © 2005 November 25

Parton distribution functions from the precise NNLO QCD fit

S. I. Alekhin

Institute for High Energy Physics, 142281 Protvino, Moscow reg., Russia

Submitted 6 October 2005

We report the parton distribution functions (PDFs) determined from the NNLO QCD analysis of the world inclusive DIS data with account of the precise NNLO QCD corrections to the evolution equations kernel. The value of strong coupling constant $\alpha_s^{\rm NNLO}(M_Z) = 0.1141 \pm 0.0014(\rm exp.)$, in fair agreement with one obtained using the earlier approximate NNLO kernel by van Neerven-Vogt. The intermediate bosons rates calculated in the NNLO using obtained PDFs are in agreement to the latest Run II results.

PACS: 06.20.Jr, 12.38.Bx, 13.60.Hb

Account of the higher-order QCD corrections for most of high-energy processes is important due to the value of strong coupling constant α_s is not small for a realistic kinematics. This is also true for the deep-inelastic lepton-nucleon scattering (DIS) process, which provides valuable information about structure of nucleon. However, since the next-to-next-to-leading order (NNLO) corrections were completely calculated only lately, mostly often the analysis of the DIS data was performed in the next-to-leading (NLO) approximation or, as the best, with the approximate NNLO evolution kernels derived in Ref. [1] on the basis of calculations [2, 3]. With the recently calculated exact expressions for the NNLO evolution kernels [4] one can improve available extractions of the NNLO parton distribution functions (PDFs) based on the approximate evolution kernels getting rid of the error due to kernel uncertainty. Even so consistent extraction of the NNLO PDFs from the global fits including the jet production data [5] is still unfeasible since the NNLO coefficient functions for the jet production process are not completely calculated. In this letter we describe the NNLO PDFs obtained from the updated analysis of the world data on inclusive DIS process [6], where the NNLO coefficient functions are known and full account of the NNLO corrections is possible.

We use for the analysis the charged-leptons proton/deuteron data on the DIS cross sections collected in the SLAC-CERN-HERA experiments [7] with the cuts $Q^2 > 2.5\,\rm GeV^2$, $W > 1.8\,\rm GeV$, and $x < 0.75$ imposed in order to reject the kinematical regions problematic for the perturbative QCD (pQCD) and where the nuclear corrections are particularly big. The HERA data with $Q^2 > 300\,\rm GeV^2$ were also cut off since they have marginal impact on the precision of PDFs obtained, but complicate the analysis due to account of the Z-boson contribution is required for this kinematics. The pQCD evolution input for the u-, d-, s-quarks and gluons at $Q_0^2 = 9\,\rm GeV^2$ is

$$xu_V(x,Q_0) = \frac{2}{N_u^V} x^{a_u}(1-x)^{b_u}(1+\gamma_2^u x), \tag{1}$$

$$xu_S(x,Q_0) = \frac{A_S}{N_S}\eta_u x^{a_s}(1-x)^{b_{su}}, \tag{2}$$

$$xd_V(x,Q_0) = \frac{1}{N_d^V} x^{a_d}(1-x)^{b_d}, \tag{3}$$

$$xd_S(x,Q_0) = \frac{A_S}{N^S} x^{a_s}(1-x)^{b_{sd}}, \tag{4}$$

$$xs_S(x,Q_0) = \frac{A_S}{N^S}\eta_s x^{a_s}(1-x)^{(b_{su}+b_{sd})/2}, \tag{5}$$

$$xG(x,Q_0) = A_G x^{a_G}(1-x)^{b_G}(1+\gamma_1^G\sqrt{x}+\gamma_2^G x), \tag{6}$$

where indices V and S correspond to the valence and sea distributions correspondingly. The normalization factors $N_{u,d}^V$ and A_G are calculated from other parameters using the fermion number and momentum conservation. The value of parameter N_S is defined from the condition that A_S gives total momentum carried by the sea quarks. The value of η_s is fixed at 0.42. For the PDFs parameters obtained in our fit this choice provides the value of strange sea suppression factor equal to 0.41 at $Q_0^2 = 20\,\rm GeV^2$, in agreement to the CCFR/NuTeV analysis of Ref. [8]. The b- and c-quarks contributions are accounted in the massive scheme with the $O(\alpha_s^2)$ correction of Ref. [9] included. For the lowest Q/W data used in the fit the power corrections are important and therefore we take into account the target-mass correction by Georgi-Politzer [10] and the dynamical twist-4 terms in the structure functions $F_{2,T}$ parameterized in a model-independent way as the piece-linear functions of x.

Parameters of the PDFs obtained in the NNLO fit with their errors due to statistical and systematical uncertainties in the data are given in Table 1. These PDFs are comparable to ones of Ref. [6] extracted using the approximate NNLO kernel within the errors due to the NNLO kernel uncertainty estimated in Ref. [1]. However, at small x and Q, where the NNLO corrections are enhanced, impact of the new calculations is non-negligible. With the exact NNLO corrections the QCD evolution of the gluon distribution at small x gets weaker and as a result at small x/Q the gluon distribution obtained using the precise NNLO kernel is quite different from the approximate one. In particular, while the approximate NNLO gluon distribution is negative at $Q^2 < 1.3\,\mathrm{GeV}^2$, the precise one remains positive even below $Q^2 = 1\ \mathrm{GeV}^2$ (see Fig.1). For the NLO case the positivity of gluons at small x/Q is even worse than for the approximate NNLO case due to the approximate NNLO corrections dampen the gluon evolution at small x too, therefore the account of the NNLO corrections is crucial in this respect (cf. discussion of Ref. [11]). Positivity of the PDFs is not mandatory beyond the leading order, however it allows probabilistic interpretation of the parton model and facilitates modeling of the soft processes, such as underlying events in the hadron-hadron collisions at high energies. The change of gluon distribution at small x/Q as compared to the fit with approximate NNLO evolution is rather due the change in evolution kernel than due to shift in the fitted parameters of PDFs. This is clear from comparison of the exact NNLO gluon distribution to one obtained from the approximate NNLO fit and evolved to low Q using the exact NNLO kernel (see Fig.1). In the vicinity of crossover of the gluon distribution to the negative values its relative change due to variation of the evolution kernel is quite big and therefore further fixation of the kernel at small x discussed in Ref. [12] can be substantial for the low-Q limit of PDFs.

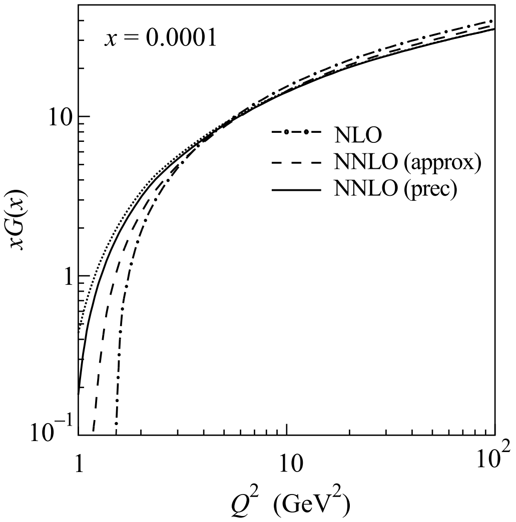

Fig.1. The gluon distributions obtained in the different variants of the analysis (solid: the fit with the exact NNLO evolution; dashes: the fit with approximate NNLO evolution; dots: the approximate NNLO gluons evolved with the exact NNLO kernel; dashed-dots: the NLO fit)

For the higher-mass kinematics at LHC numerical impact of the NNLO kernel update is not dramatic. The change in the Higgs and W/Z bosons production cross sections due to more precise definition of the NNLO PDFs is comparable to the errors coming from the PDFs uncertainties. The NNLO predictions for the longitudinal DIS structure function F_L at $x \sim 10^{-5}$ measured by the $H1$ collaboration [13] also does not change too much since they are given by the Mellin convolution of PDFs with the coefficient functions and are defined by the gluon distribution at relatively big values of x. The obtained value of the strong coupling constant

$$\alpha_s^{\mathrm{NNLO}}(M_{\mathrm{Z}}) = 0.1141 \pm 0.0014\ (\mathrm{stat} + \mathrm{syst}),$$

is in fair agreement to $\alpha_s^{\mathrm{NNLO}}(M_{\mathrm{Z}}) = 0.1143 \pm \pm 0.0014\ (\mathrm{stat} + \mathrm{syst})$ obtained in the fit of Ref. [6] with the approximate NNLO kernel and to the results of the exact NNLO analysis of the non-singlet DIS data [14].

Table 1

The PDFs parameters and χ^2/NDP obtained in the fit. The errors in parameters are obtained by propagation of the statistical and systematical errors in data

Valence quarks:	a_u	0.724 ± 0.027
	b_u	4.0194 ± 0.050
	γ_2^u	1.04 ± 0.35
	a_d	0.775 ± 0.073
	b_d	5.15 ± 0.15
Gluon:	a_G	-0.118 ± 0.021
	b_G	9.6 ± 1.2
	γ_1^G	-3.83 ± 0.51
	γ_2^G	8.4 ± 1.7
Sea quarks:	A_S	0.1586 ± 0.0089
	a_s	-0.2094 ± 0.0044
	b_{sd}	5.6 ± 1.2
	η_u	1.12 ± 0.11
	b_{su}	10.39 ± 0.88
χ^2/NDP		2534/2274

The errors in PDFs parameters given in Table 1 are calculated as a propagation of the experimental errors for the data points used in the fit. We calculate these errors using the covariance matrix method [15] taking into account statistical and systematic errors in data and correlations of the latter as well. We also take into account the theoretical errors due to possible variations of the strange suppression factor η_s and the c-quark mass m_c. For this purpose we re-calculate the error matrix for the PDFs parameters with η_s and m_c released. Since the parameters η_s and m_c are not constraint by the charged-leptons inclusive DIS data we confine their variation adding to the data sample two "measurements": $\eta_s = 0.42 \pm 0.1$ and $m_c = 1.5 \pm 0.25\,\mathrm{GeV}$ with the errors in these measurements representing our current understanding of the uncertainties in these parameters. In this approach the theoretical errors are included into the total error in PDFs and their correlations with other sources of the PDFs uncertainties are automatically taken into account. The NNLO PDFs grid for the range of $Q^2 = 0.8 \div 2 \cdot 10^8\,\mathrm{GeV}^2$ and $x = 10^{-7} \div 1$ with the total uncertainties in PDFs supplied is available directly[1)] and through the LHAPDF library[2)]. The LO and NLO PDFs grids are also supplied to provide a tool for checking sensitivity of different calculation to the QCD order of PDFs.

The NNLO inclusive rates for the intermediate boson production at the FNAL $\bar{p}p$ collider and the LHC calculated using this grid and the code of Ref. [16] with corrections of Ref. [17] are given in Table 2. The masses and widths of the W/Z bosons were set as $M_W = 80.425\,\mathrm{GeV}$, $M_Z = 91.188\,\mathrm{GeV}$, $\Gamma_W = 2.124\,\mathrm{GeV}$, $\Gamma_Z = 2.495\,\mathrm{GeV}$, squared sine of the Weinberg angle $x_W(M_Z) = 0.2312$, squared cosine of the Cabibbo angle $c_C = 0.9498$ [18]. The errors quoted in Table 2 are due to the total uncertainty in PDFs including the theoretical errors considered. The calculations are in agreement to the latest Run II results of Ref. [19] within the errors (see Fig.2). The errors in the data of Run II are bigger than one in the calculations therefore the latter can be used for better calibration of the luminosity, which gives main contribution to the measurements error.

Table 2

The NNLO inclusive rates (in nb) for the intermediate bosons production in the hadron-hadron collisions. The errors are due to the total PDFs uncertainties

	$W^{\pm}$	Z
$\bar{p}p$ (1.96 TeV)	26.11 ± 0.44	7.78 ± 0.11
pp (14 TeV)	197.0 ± 5.3	57.7 ± 1.5

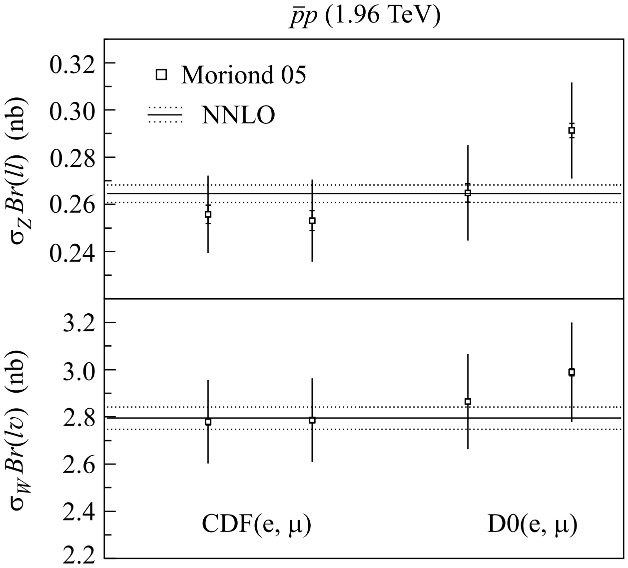

Fig.2. The NNLO calculation of the W/Z rates for Run II at Fermilab compared to the data. The dotted lines give the uncertainty in calulations due to errors in PDFs; the error bars of the data points give the total error including one due to the luminosity uncertainty. The branching ratios of the W/Z leptonic decays $Br(W \to l\nu) = 0.107$ and $Br(Z \to ll) = 0.034$ were applied

In summary, we provide update of the analysis of the world DIS inclusive data on the proton/deuteron targets with full account of the NNLO QCD corrections including the recent calculations of the exact NNLO evolution kernel. The value of α_s is in fair agreement to the earlier version of the fit based on the approximate NNLO kernels. With the exact NNLO corrections applied we observe improvement in the positivity of the gluon distributions extrapolated to small x and Q: Now we have gluons positive up to $Q = 1\,\mathrm{GeV}$, i.e. throughout kinematical region where the parton model is applicable. The NNLO W/Z-bosons rates calculated using the PDFs obtained are in agreement with the recent Run II results and can be used for better calibration of the Fermilab experiments in view of the uncertainty in the calculations due to PDFs are smaller than the experimental ones. Since these PDFs are extracted from the data for one single process they can be used for the quantitative studies of the PDFs universality that is advantage as compared to ones determined from the global fits.

I am indebted to S. Forte, S. Moch, R. Petti, and A. Vogt for stimulating discussions and S. Kulagin for development of the Mathematica package for access to the PDFs grid. The work was supported by the RFBR

[1)] http://sirius.ihep.su/~alekhin/pdfa02/

[2)] http://durpdg.dur.ac.uk/lhapdf/

grant # 03-02-17177 and by the Russian Ministry of Science grant NSh # 1695.2003.2.

1. W. L. van Neerven and A. Vogt, Phys. Lett. B **490**, 111 (2000); [arXiv:hep-ph/0007362].
2. A. Retey and J. A. Vermaseren, Nucl. Phys. B **604**, 281 (2001); [arXiv:hep-ph/0007294]; S. A. Larin, P. Nogueira, T. van Ritbergen, and J. A. M. Vermaseren, Nucl. Phys. B **492**, 338 (1997); [arXiv:hep-ph/9605317].
3. S. Catani and F. Hautmann, Nucl. Phys. B **427**, 475 (1994); [arXiv:hep-ph/9405388].
4. S. Moch, J. A. M. Vermaseren, and A. Vogt, Nucl. Phys. B **688**, 101 (2004); [arXiv:hep-ph/0403192]; A. Vogt, S. Moch, and J. A. M. Vermaseren, Nucl. Phys. B **691**, 129 (2004); [arXiv:hep-ph/0404111].
5. A. D. Martin, R. G. Roberts, W. J. Stirling, and R. S. Thorne, Phys. Lett. B **604**, 61 (2004); [arXiv:hep-ph/0410230].
6. S. Alekhin, Phys. Rev. D **68**, 014002 (2003); [arXiv:hep-ph/0211096].
7. L. W. Whitlow, E. M. Riordan, S. Dasu et al., Phys. Lett. B **282**, 475 (1992); A. C. Benvenuti et al. [BCDMS Collaboration], Phys. Lett. B **223**, 485 (1989); A. C. Benvenuti et al. [BCDMS Collaboration], Phys. Lett. B **237**, 592 (1990); M. Arneodo et al. [New Muon Collaboration], Nucl. Phys. B **483**, 3 (1997), [hep-ph/9610231]; C. Adloff et al. [H1 Collaboration], Eur. Phys. J. C **21**, 33 (2001), [arXiv:hep-ex/0012053]; S. Chekanov et al. [ZEUS Collaboration], Eur. Phys. J. C **21**, 443 (2001), [arXiv:hep-ex/0105090].
8. M. Goncharov et al. [NuTeV Collaboration], Phys. Rev. D **64**, 112006 (2001), [arXiv:hep-ex/0102049].
9. E. Laenen, S. Riemersma, J. Smith, and W. L. van Neerven, Nucl. Phys. B **392**, 229 (1993).
10. H. Georgi and H. D. Politzer, Phys. Rev. D **14**, 1829 (1976).
11. J. Huston, J. Pumplin, D. Stump, and W. K. Tung, arXiv:hep-ph/0502080.
12. G. Altarelli, R. D. Ball, and S. Forte, Nucl. Phys. B **674**, 459 (2003), [arXiv:hep-ph/0306156].
13. E. M. Lobodzinska, arXiv:hep-ph/0311180.
14. J. Blumlein, H. Bottcher, and A. Guffanti, Nucl. Phys. Proc. Suppl. **135**, 152 (2004), [arXiv:hep-ph/0407089].
15. S. I. Alekhin, arXiv:hep-ex/0005042.
16. R. Hamberg, W. L. van Neerven, and T. Matsuura, Nucl. Phys. B **359**, 343 (1991), [Erratum-ibid. B **644**, 403 (2002)].
17. R. V. Harlander and W. B. Kilgore, Phys. Rev. Lett. **88**, 201801 (2002), [arXiv:hep-ph/0201206].
18. S. Eidelman et al. [Particle Data Group], Phys. Lett. B **592**, 1 (2004).
19. A. M. Bellavance [D0 - Run II Collaboration], arXiv:hep-ex/0506025.