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Measurements of the Dalitz Plot Parameters for $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$ Decays

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The g, h, and k Dalitz plot parameters, which are coefficients in a series expansion of the squared module of the matrix element $|M(u,v)|^2 \propto 1 + gu + hu^2 + kv^2$ (u, v are invariant variables), have been measured for $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$ decays using 35 GeV/c hadron beams at the IHEP (Protvino) accelerator. For the first time dependences of parameters and fit quality on the $\pi^0\pi^0$ mass cut were investigated. It is shown that the above mentioned expansion does not fit the experimental data near the $\pi^+\pi^-$ mass threshold and addition of the cubic terms only slightly improves the fit quality. This result points to the important role of nonanalytical terms in the matrix element that are connected with the pion rescattering. The comparison of our data with previous measurements is presented.

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1. Introduction. Experimental investigations of kaon decays play a very important role in particle physics. The results of these experiments permit to check predictions of the Standard Model and to search for new physics beyond its framework. In particular, the studies of $K \to 3\pi$ decays allow one to verify calculations based on the chiral perturbation theory.

Usually the squared module of the matrix element for $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$ decays is presented in the following form:

$$|M(u,v)|^2 \propto 1 + gu + hu^2 + kv^2, \quad (1)$$

where $u = (s_3 - s_0)/m^2_{\pi^+}$, $v = (s_2 - s_1)/m^2_{\pi^+}$, $s_i = (P_K - P_i)^2$, $s_0 = \frac{1}{3}\sum_i^3 s_i$, p_K and p_i are four-vectors of the kaon and the i-th pion, and index 3 is used for the charged pion.

The g, h, and k Dalitz plot parameters for $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$ decays measured in the three most precise experiments with K^- [1, 2] and K^+ [3] beams differ by 2 to 5 standard deviations [4]. For example, the difference of the Dalitz plot slopes g obtained in [2] and [3] is equal to 0.109 ± 0.021. As shown by our data [5] this result cannot be explained by CP violation and is most probably due to underestimation of systematic uncertainties. In this paper we present new results on the Dalitz plot parameters based on the analysis of 493k events of $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$ decays collected with the TNF-IHEP facility [5, 6].

2. Experimental Setup. Studies of charged kaon decays have been performed using 35 GeV/c positive and negative hadron beams at the IHEP accelerator. The beam intensity was monitored by four scintillation counters. Its typical value was $4 \cdot 10^6$ per 1.7 second spill. Three threshold and two differential Cherenkov counters were used to select kaons with a background of less than 1%. The products of kaon decays originating in the 58.5 m long vacuum pipe were detected by wide aperture scintillation hodoscopes and the total absorption electromagnetic calorimeter GEPARD consisting of 1968 lead-scintillator cells. The π^0 mass resolution was 12.3 MeV/c^2. The calorimeter was divided into 16 trigger elements. An anticoincidence beam counter was placed downstream of the vacuum pipe. The first level trigger T1 was formed according to the following logic formula:

$$T1 = S1 \cdot S2 \cdot S3 \cdot S4 \cdot (D1 + D2) \cdot \overline{C1} \cdot \overline{C2} \cdot \overline{C3} \cdot \overline{AC},$$

where Si, Di, Ci, and AC are logical signals from the beam, differential, threshold, and anticoincidence counters respectively. The Level 2 trigger required more than 0.8 GeV energy deposition in at least three trigger elements of the GEPARD. The details of the setup and measurement procedure can be found elsewhere [5].

3. $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$ event selection. The following criteria were used to select $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$ events [5]:

- one to three secondary tracks are reconstructed;
- the probability of the decay vertex fit is more than 5%;
- the decay vertex is inside the fiducial length of the decay pipe;

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- the number of clusters with energy above 1 GeV in the calorimeter and the number of tracks in the hodoscopes correspond to the $K^\pm \to \pi^\pm\pi^0\pi^0$ decay;
- charged pion energy exceeds 8 GeV;
- the χ^2 probability $P(\chi^2)$ of the 6C kinematic fit is more than 0.1 (all possible photon combinations are considered and the best is selected);
- event passes software Level 2 trigger.

The experimental setup was simulated using the Monte Carlo (MC) method with the GEANT 3.21 code. The setup geometry was described in detail and the data obtained in the experiment were taken into account. Among these data there were calibration coefficients for each channel of the calorimeter, the dependence of the hodoscope efficiency on the particle coordinates and correlations between kaon's spatial and angular coordinates and its momentum.

The final data sample includes 493 K completely reconstructed $K^\pm \to \pi^\pm\pi^0\pi^0$ events. The MC statistics is about four times higher. The background level estimated from the MC simulations is less than 0.25% and is mainly due to $K^\pm \to \pi^\pm\pi^0$ decays. It is shown in our paper [5] that the event distributions in the Dalitz plots for K^+ and K^- decays are identical. Taking this into account we used combined statistics for $K^\pm \to \pi^\pm\pi^0\pi^0$ decays to estimate the Dalitz plot parameters.

4. Results. Due to the finite setup resolution on the u and v variables [5] the "measured" u', v' values can differ from the true u, v for both experimental and MC events. To take this into account the Dalitz plot parameters were estimated by minimizing the following functional form:

$$\chi^2(g,h,k) = \sum_i^{Nbin} \frac{(n_i - C\cdot m_i)^2}{\sigma_i^2},$$

where n_i is the number of events in the i-th experimental Dalitz plot bin, $m_i \equiv m_i(g,h,k) = \sum_j w_{ij}$ ($w_{ij} = 1 + g\cdot u_j + h\cdot u_j^2 + k\cdot v_j^2$) is a sum of the weighted MC events in the i-th Dalitz plot bin, $C = \sum n_i/\sum m_i$ is a normalization factor and $\sigma_i^2 = n_i + C^2\cdot\sum_j w_{ij}^2$ takes into account limited MC statistics. The following values of the g, h, k parameters and elements of the correlation matrix were obtained:

$$\begin{matrix} g = 0.6259 \pm 0.0043 \\ h = 0.0551 \pm 0.0044 \\ k = 0.0082 \pm 0.0011 \end{matrix} \quad \begin{pmatrix} 1.00 & 0.90 & 0.41 \\ & 1.00 & 0.33 \\ & & 1.00 \end{pmatrix}. \qquad (2)$$

The errors quoted are statistical only, χ^2/ndf is $506/430 = 1.18$ and $P(\chi^2) = 0.0066$. It turned out that the low significance of the fit is primarily due to a difference between the experimental data and the MC simulations based on equation (1) in the threshold region of the $\pi^0\pi^0$ invariant mass M_0 (Fig.1). This discrepancy can

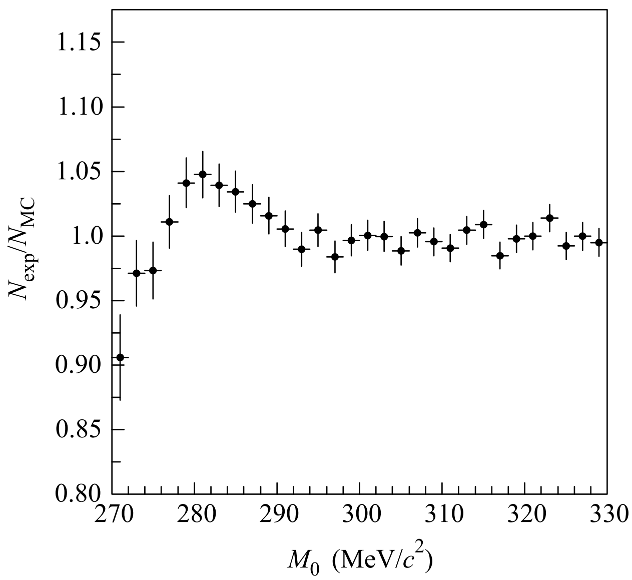

Fig.1. Ratio of experimental to MC events vs $\pi^0\pi^0$ invariant mass M_0

not be avoided by the addition of higher order terms $l\cdot u^3$ and/or $m\cdot uv^2$ in the $|M(u,v)|^2$ expansion (see Table) and might be due to nonanalitical terms in the matrix element connected to $\pi^+\pi^- \to \pi^0\pi^0$ rescattering [7]. This effect was recently considered in detail by Cabibbo [8] and Cabibbo, Isidori [9] and was observed in the experiments [10]. It plays an important role in the region of $M_0 \sim 2m_{\pi^0}$ and its contribution can be suppressed by introducing a complementary criterion of $M_0 > M_T$. It appears that the fit with $M_T \geq 290\,\text{MeV}/c^2$ results in stable values of the Dalitz plot parameters independent of the M_0 cut and a satisfactory fit significance. Introduction of the higher order terms in Eq.(1) does not change the g, h, k parameters and χ^2 value in this case. The fit with $M_T = 290\,\text{MeV}/c^2$ gave the following results:

$$\begin{matrix} g = 0.6339 \pm 0.0046 \\ h = 0.0593 \pm 0.0088 \\ k = 0.0083 \pm 0.0013 \end{matrix} \quad \begin{pmatrix} 1.00 & 0.52 & 0.43 \\ & 1.00 & 0.16 \\ & & 1.00 \end{pmatrix} \qquad (3)$$

and $\chi^2/ndf = 1.04$, $P(\chi^2) = 0.3$.

Comparison of the data in (2) and (3) shows that the Dalitz plot slope g changes by 2 standard deviations and correlations between g, h, and h, k are much

Fit results with higher-order terms in (1)

g	0.6259 ± 0.0043	0.6151 ± 0.0051	0.6284 ± 0.0048	0.6129 ± 0.0063
h	0.0551 ± 0.0044	0.0782 ± 0.0073	0.0556 ± 0.0044	0.0795 ± 0.0077
k	0.0082 ± 0.0011	0.0080 ± 0.0011	0.0070 ± 0.0015	0.0087 ± 0.0016
l	—	0.0273 ± 0.0069	—	0.0292 ± 0.0076
m	—	—	-0.0027 ± 0.0024	0.0016 ± 0.0027
χ^2	506.1	490.2	504.8	489.8
χ^2/ndf	1.18	1.14	1.17	1.14
$P(\chi^2)$	$6.6 \cdot 10^{-3}$	$2.4 \cdot 10^{-2}$	$7.4 \cdot 10^{-3}$	$2.4 \cdot 10^{-2}$

weaker if the M_T cut is applied. Fig.2 confirms the good agreement between experimental and MC data with $M_T = 290\,\text{MeV}/c^2$.

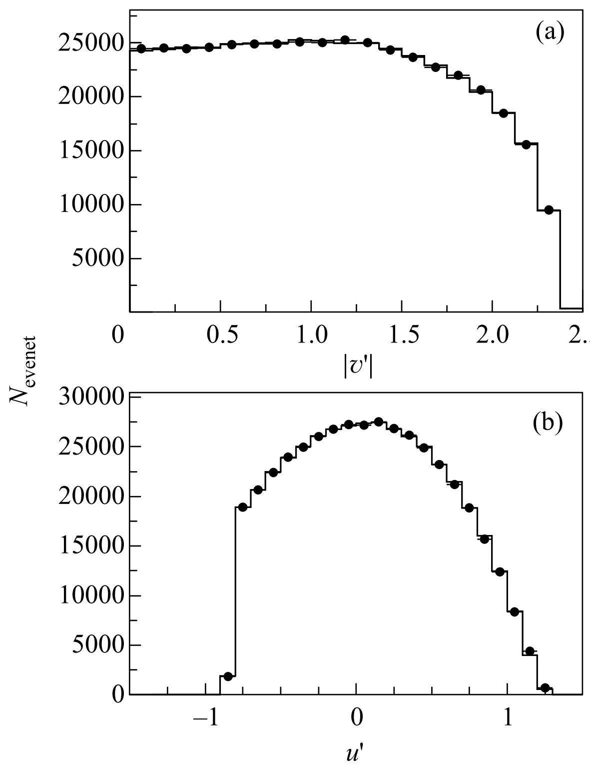

Fig.2. Event distributions projected on (a) $-|v'|$ and (b) $-v'$ axes (histogram – simulation, circles – experiment)

To estimate the systematic uncertainties of the Dalitz plot parameters we checked the stability of the results against variation of the cuts in the event selection criteria. The parameters appeared to be most sensitive to the change of the minimum gamma energies ($\Delta g = -0.0057$, $\Delta h = -0.0047$, $\Delta k = -0.0006$) and of the minimum charged pion momentum ($\Delta g = 0.0048$, $\Delta h = 0.0051$ and $\Delta k = 0.0011$). The change in the bin size by factors of 2 and 0.5 and exclusion of the bins at the Dalitz plot boundary from the fit gives $\Delta g = 0.0012$, $\Delta h = 0.0045$, and $\Delta k = 0.0004$. Uncertainties in the kaon momentum, beam profile and angular spread, as well as GEPARD calibration coefficients have no influence on the parameters. The background contribution to systematic errors turned out to be negligible. Finally, our estimations of the systematic uncertainties are the following: $\delta g = 0.0093$, $\delta h = 0.0086$, $\delta k = 0.0014$.

Fig.3 shows our results (2) together with previous data [1–4, 11–15] obtained without M_0 cut. The error bars include both systematic and statistical uncertainties. Our g and h values are in good agreement with those of Ajinenko et al [2]. We observe a deviation of the k value from zero ~ 4.5 standard deviations while Ajinenko et al reported $k = 0.001 \pm 0.002$.

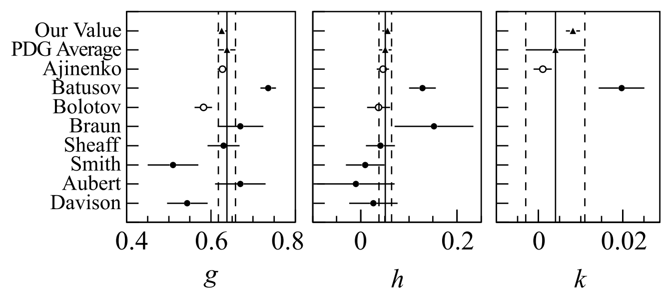

Fig.3. The Dalitz plot parameters g, h and k for the $K^+ \to \pi^+\pi^0\pi^0$ (solid circles), $K^- \to \pi^-\pi^0\pi^0$ (open circles) and $K^\pm \to \pi^\pm\pi^0\pi^0$ (triangles) decays. Vertical solid and dotted lines show the average values and their errors as calculated by the PDG [4]

5. Conclusions. The new data on the Dalitz plot parameters for $K^\pm \to \pi^\pm\pi^0\pi^0$ decays based on the analysis of $\sim 0.5M$ events collected with the TNF-IHEP facility are presented. We investigated the dependence of the Dalitz plot parameters and the fit quality of the Eq. (1) to the experimental data on the $\pi^0\pi^0$ invariant mass cut M_T. It is shown that without the cut the fit quality is rather poor while $M_0 \geq 290\,\text{MeV}/c^2$ cut results in a satisfactory fit significance. This cut changes the slope g by 2 standard deviations and weakens the correlations between g, h and h, k parameters. We consider these results to be an evidence of the important contribution of $\pi^+\pi^- \to \pi^0\pi^0$ rescattering to the matrix element of the $K^\pm \to \pi^\pm\pi^0\pi^0$ decay in the threshold region of the $\pi^0\pi^0$ invariant mass. Thus pion rescattering should be taken into account when high statistics data on $K^\pm \to \pi^\pm\pi^0\pi^0$ decays are analyzed. The obtained values of g and h parameters agree (within the errors quoted) with those published in [2] while our k value differs from zero by ~ 4.5 standard deviations in contradiction with the result of [2] which is compatible with zero.

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