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Divergent Coulomb screening in two dimensional electron gas driven by microwaves

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Two dimensional electron gas in perpendicular magnetic field, driven by microwave *ac*- field is studied. The magneto-conductivity and the diffusion constant are calculated in the microscopic model. In the driven state the Einstein relation is violated. Instability of the Coulomb screening and a divergence of the effective donor electrostatic field due to the negative Ryzhii currents is predicted. This phenomenon results in the zero-resistance states observed experimentally.

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Electron systems interacting with microwaves can be driven into highly non-equilibrium state with unusual properties. For example, two dimensional electrons (2DEG) in GaAs heterostructures in weak perpendicular magnetic field show microwave induced resistance oscillations which in the limit of extreme power are capped by zero resistance states [1, 2]. Ryzhii has predicted [3] that in 2DEG with weak disorder and electron-phonon interaction, driven by microwaves an electric photo current will flow along or against the applied weak *dc*-electric field (depending on the ratio of the microwave and cyclotron frequencies ω and ω_H) in addition to the usual current. Recent works [4, 5] have clarified mechanisms of the Ryzhii current and the microwave induced resistance oscillations in disordered systems. At high microwave intensity the negative Ryzhii current overcomes the usual positive current resulting in the absolute negative conductivity. This may lead to the current domain instability and these domains could arrange themselves into a state with net zero resistance [6].

Zero resistance states may have different origins. For slow driving $\omega \ll \omega_H$ a suppressed resistance [7] in a wide range of magnetic field could be a manifestation of the 'driven' degeneracy of electron states [8]. Fast driving $\omega \gg \omega_H$ smoothes the disorder potential and the ideal state of electrons can emerge with zero resistance (not reported yet in experiments to my knowledge), similar to the induced transparency in optics. This Letter concerns with the intermediate frequencies: $\omega \sim \omega_H$, where multiple zero and high resistance states specific only for 2DEG in magnetic field appear due to the Ryzhii phenomenon.

In high mobility heterostructures a donor electrostatic potential characterized by the spacer length d is present due to Si-doped layer. In this Letter the kinetic coefficients of 2DEG – the conductivity $\sigma(E)$ and the diffusion constant $D(E)$ – are calculated for arbitrary intensity of the microwaves E in the microscopic model with the superposition of the donor disorder and a short range disorder in the region of magnetic field: $l_H \ll 2d \ll R_c$, where l_H and R_c are the magnetic length and the cyclotron radius. In this region of magnetic field the donor potentials before and after the scattering off the short range disorder are uncorrelated. The conductivity splits into the diffusion and Ryzhii parts: $\sigma(E) = \sigma_D(E) + \sigma_R(E)$, with the former being related to the diffusion constant by the Einstein relation: $\sigma_D(E) = e^2\nu(\epsilon_F)D(E)$ where $\nu(\epsilon_F)$ is the density of states on the Fermi level. The Einstein relation is violated in the driven state. The current induced by the electron relaxation due to phonon emission can be neglected [9].

If the long range disorder potential (with lengths exceeding R_c) is present in the stationary 2DEG driven by microwaves then Ryzhii currents will be locally induced even in the absence of the probe *dc*-voltage. Electrons will move away from the equilibrium density distribution, modifying the Coulomb interaction. For $k\omega_H < \omega < (k+1/2)\omega_H$, where k is an integer, the Ryzhii current is negative and the electron density $n(\mathbf{r})$ becomes more even over the 2D plane as the intensity of microwaves E is increasing. Therefore, the screening of an external potential by electrons is reduced. The relationship between the external $u_0(\mathbf{r})$ and the screened $u(\mathbf{r})$ potentials is linear: $u(\mathbf{q}) = u_0(\mathbf{q})/\varepsilon(\mathbf{q})$, where $\varepsilon(\mathbf{q})$ is the dielectric permeability of 2DEG, provided $|u(\mathbf{r})|, |u_0(\mathbf{r})| \ll \epsilon_F$. One of the central results of this Letter is that:

$$\varepsilon(\mathbf{q}) = 1 + \frac{2\pi e^2}{|\mathbf{q}|}\nu(\epsilon_F)\left(1 + \frac{\sigma_R(E)}{\sigma_D(E)}\right), \qquad (1)$$

Indeed, in the long range limit $qR_c \ll 1$ the electron transitions amount to a continuous current. The current density is the linear response with respect to two distinct sources: weak electric field and weak gradient of the density. In magnetic field, locally, the current due to ∇u has two orthogonal components: the Hall drift current along the contour of constant potential $u(\mathbf{r})$ and the longitudinal current along the local electric field ∇u. The diffusion current is dissipative unlike the drift current and there is no kinetic force to sustain the diffusion along the contour of constant potential. Therefore, the density along this contour is constant [10] and is a function of the local potential only, linear in high Landau level: $\delta n(\mathbf{r}) = -\nu u(\mathbf{r})$. Unlike ∇u, ∇n does not induce the Hall current and does not change the wave function of the drifting states. Therefore the longitudinal current reads:

$$e\mathbf{j}_l(\mathbf{r}) = \sigma(E)\nabla u(\mathbf{r}) + e^2 D(E)\nabla n(\mathbf{r}), \qquad (2)$$

In the stationary state of electron system $\mathbf{j}_l(\mathbf{r}) = 0$. The source of the electrostatic potential $u(\mathbf{r})$ (hereafter it means the electrostatic energy) is either donors or electrons:

$$u(\mathbf{q}) = u_0(\mathbf{q}) + (2\pi e^2/|\mathbf{q}|)\; n(\mathbf{q}). \qquad (3)$$

Then the dielectric permeability Eq.(1) follows from Eqs.(2), (3). Provided the Ryzhii current is negative and and at high microwave intensity we find from Eq.(1) that $\varepsilon(\mathbf{q}) \leq 0$. Usually, zeroes of $\varepsilon(\mathbf{q})$ correspond to collective modes of electron system. In this Letter zeroes of $\varepsilon(\mathbf{q})$ is shown to generate a divergent long range electric field on the percolating level that in turn results in zero resistance states.

In the Fermi liquid the transport is described in terms of non-interacting quasiparticles. The Hamiltonian of driven electrons in a disorder potential $u(\mathbf{r})$ reads:

$$\hat{H} = \sum_j \frac{1}{2m}\left(i\hbar\nabla_j + \frac{e}{c}\mathbf{A}(\mathbf{r}_j) + \frac{e}{c}\mathbf{A}(t)\right)^2 + u(\mathbf{r}_j), \qquad (4)$$

where m is the electron mass, $\mathbf{A}(\mathbf{r}) = -Hy\mathbf{e}_x$ and $\mathbf{A}(t)$ are the vector potentials of the perpendicular magnetic field H in the Landau gauge and the microwave *ac*-field: $c\mathbf{E}(t) = -d\mathbf{A}/dt$ (at high intensities the Bose factors of microwave modes are large and $\mathbf{A}(t)$ is a classical field). We use the magnetic units: $\hbar = 1$, $e = c$, $H = 1$, $\omega_H = 1/m$. In Landau level with the large number N, there are two distinct lengths: the cyclotron radius $R_c = \sqrt{2N}l_H$ and the magnetic length $l_H = \sqrt{c\hbar/eH} = 1$.

Disorder electrostatic potential in the quantum well is created by remote ionized Si donors with the charge e and uncorrelated positions in a narrow δ-doped layer parallel to the quantum well and separated by the clean spacer of width d assumed $l_H \ll 2d \ll R_c$. Donor areal density is equal to the density of electrons n. The donor external potential $u_0(\mathbf{r})$ is Gaussian with the correlation function:

$$S_0(\mathbf{q}) = \langle u_0(\mathbf{q})u_0(-\mathbf{q})\rangle = \left(\frac{2\pi e^2}{|\mathbf{q}|}\right)^2 n\exp\left(-2|\mathbf{q}|d\right), \qquad (5)$$

in the momentum range $q \ll \sqrt{n}$. Electrons screen the external potential $u_0(\mathbf{r})$ near the 2DEG plane according to Eq.(3). In high Landau levels the linear response occurs even in the limit of non-overlapping Landau levels. The correlation function of electron density perturbed by the donors is: $\langle n(\mathbf{q})n(-\mathbf{q})\rangle = n\exp\left(-2|\mathbf{q}|d\right)$, with variation: $\langle(\delta n)^2\rangle = n/2\pi(2d)^2$. For typical densities in GaAs heterostructures the screening is complete $\varepsilon(q) \gg 1$:

$$S(\mathbf{q}) = \langle u(\mathbf{q})u(-\mathbf{q})\rangle = n\exp\left(-2|\mathbf{q}|d\right)/\nu^2(\epsilon_F). \qquad (6)$$

We expand the electron Green's functions into a series over the potential $u(\mathbf{r})$, using the diagrammatic method and average it with the disorder Gaussian probability distribution using Eq.(6). The diagrams consist of electron and impurity lines. The vertex describes an electron scattering off the potential $u(\mathbf{q})$ from the state (N,p) into the state (N',p'), where p is the degeneracy index inside Landau level. The vertex:

$$V(Np, N'p', \mathbf{q}) = V_{NN'}(\mathbf{q})U(p,\mathbf{q})\delta_{p',p+q_y}, \qquad (7)$$

is the product of the magnetic phase factor, universal for all Landau levels: $U(p,\mathbf{q}) = \exp(iq_x(p+q_y/2))$, and the reduced vertex, expressed in terms of the Laguerre polynome:

$$V_{nn'} = \left(\frac{n'!}{n!}\right)^{\pm 1/2}\left(\frac{q_y \pm iq_x}{\sqrt{2}}\right)^{|n-n'|} L^{|n-n'|}_{\min(nn')}\left(\frac{q^2}{2}\right)e^{-\frac{q^2}{4}}, \qquad (8)$$

where the sign $\pm$ corresponds to $n > n'$ and $n < n'$. In the static potential the electron propagator is either retarded or advanced. We include the two vertices into the impurity line that connects them and retain only the transitions within the same Landau level N:

$$u = \int S(\mathbf{q})\,|V_{NN}(\mathbf{q})|^2\;\frac{d^2\mathbf{q}}{(2\pi)^2}. \qquad (9)$$

A realistic model of magneto-transport in high mobility GaAs heterostructures may require an additional long range disorder with the correlation function being non-zero in the region $qR_c \ll 1$. The corresponding 'extra' long range impurity line u_0 generates no magnetic phase unlike the donor line u. Another important additional disorder is the short range potential $w(\mathbf{r})$ that induces the diffusion. It may originate in the barrier layer $Ga_{1-x}Al_xAs$. Al atoms are distributed randomly and the local energy barrier fluctuates. Accordingly $w(\mathbf{r})$ is Gaussian with the correlation function: $S_w = \langle w(\mathbf{q})w(-\mathbf{q})\rangle = 1/2\pi\nu(\epsilon_F)\tau_w$. Using Eq.(9) we find the short range impurity line: $w = \omega_H/2\pi\tau_w$, and we assume $w \ll u$.

Quantum states of an electron driven by uniform microwave *ac*-field are explicitly time depend. There exists a unitary transformation to the oscillating reference frame, where the wave function of electron become time independent whereas the disorder potential becomes time dependent [5, 8]. In this frame the driven correlation function of the short range potential reads:

$$S_w(t,t';\mathbf{q}) = S_w\, e^{i\mathbf{q}\left(\mathbf{R}(t)\, -\, \mathbf{R}(t')\right)}, \qquad (10)$$

where $\mathbf{R}(t) = (X(t), Y(t))$ is the classical elliptic trajectory of charged particle in the crossed magnetic field and the microwave electric *ac*-field $E(t) = E\cos(\omega t)$ with the frequency ω and polarized linearly along the x axis:

$$\begin{aligned} X(t) &= \frac{\mathcal{E} l_H}{\sqrt{2N}}\frac{\omega_H\cos(\omega t)}{\omega^2-\omega_H^2}, \\ Y(t) &= \frac{\mathcal{E} l_H}{\sqrt{2N}}\frac{\omega_H^2\sin(\omega t)}{\omega(\omega^2-\omega_H^2)}, \end{aligned} \qquad (11)$$

where the amplitude of the *ac*-field is expressed as the energy: $\mathcal{E} = eER_c$. For the long range disorder we neglect the time-dependent phase in Eq.(10) due to small $\mathbf{q}$. For the short range disorder this phase is essential. We take integral in Eq.(9) with respect to $\mathbf{q}$ using the driven correlation function Eqs.(10), (11) and find the driven impurity line in the limit of large N:

$$w(t,t') = wJ_0^2\left(2\mathcal{R}\left(\frac{\omega(t+t')}{2}\right)\sin\frac{\omega(t-t')}{2}\right), \qquad (12)$$

where $J_m(x)$ is the Bessel function and the relative radius of the elliptic trajectory is

$$\mathcal{R}(\phi) = \frac{\mathcal{E}\omega_H}{\omega^2-\omega_H^2}\sqrt{\cos^2\phi + \frac{\omega_H^2}{\omega^2}\sin^2\phi} \qquad (13)$$

The calculation of the conductivity of 2DEG uses the Keldysh method [12] and is similar to that of Ref.[8]. For a given realization of the long-range potential, the anisotropic longitudinal conductivity is given by the current-current diagrams with exactly one short range impurity line: $\sigma_{xx}(\Omega) \pm \sigma_{yy}(\Omega) = 2\sigma_\pm(\Omega)$, where

$$\sigma_\pm(\Omega) = \frac{2e^2N}{\pi}w\sum_m\int\frac{d\epsilon}{2\pi}\left\{\begin{array}{c}P_m\\ Q_m\end{array}\right\}\mathrm{Im}G(\epsilon)\times$$
$$\times\frac{f(\epsilon)-f(\epsilon+m\omega-\Omega)}{\Omega}\mathrm{Im}G(\epsilon+m\omega-\Omega), \quad (14)$$

where index + corresponds to P_m and index − corresponds to Q_m. The *dc*- conductivity is found in the limit $\Omega = 0$:

$$\begin{aligned} Q_m &= \int\cos(m\theta)\left[J_0(x)J_2(x)+J_1^2(x)\right]\frac{d\theta d\phi}{(2\pi)^2}, \\ P_m &= \int\cos(m\theta)\left[J_0^2(x)-J_1^2(x)\right]\frac{d\theta d\phi}{(2\pi)^2}, \end{aligned} \qquad (15)$$

where $x = 2\sin(\theta/2)\mathcal{R}(\phi)$. Next we average Eq.(14) over the long-range donor potential u. In the limit $l_H \ll 2d \ll R_c$ each Green's function in Eq.(14) is averaged separately. The *dc*-conductivity is split into the two parts: $\sigma_\pm = \sigma_\pm^D + \sigma_\pm^R$, where

$$\sigma_\pm^D = -\frac{2e^2Nw}{\pi}\times$$
$$\times\sum_m\int\frac{d\epsilon}{2\pi}\left\{\begin{array}{c}P_m\\ Q_m\end{array}\right\}\mathrm{Im}G(\epsilon+m\omega)\frac{df}{d\epsilon}\mathrm{Im}G(\epsilon) \quad (16)$$

is the diffusion part and the Ryzhii part reads:

$$\sigma_\pm^R = \frac{2e^2N}{\pi}w\sum_m\int\frac{d\epsilon}{2\pi}\left\{\begin{array}{c}P_m\\ Q_m\end{array}\right\}\frac{d}{d\epsilon}\mathrm{Im}G(\epsilon)\times$$
$$\times\left(f(\epsilon+m\omega)-f(\epsilon)\right)\mathrm{Im}G(\epsilon+m\omega) \qquad (17)$$

Finally we average Eq.(16), (17) over the 'extra' long range potential u_0. For random contour of the constant potential the local axes of the conductivity tensor rotate with respect to the *ac*-field polarization axis. Therefore, the macroscopic conductivity is given by σ_+ independent of the microwave polarization in agreement with Ref.[13]. We assume that phonon relaxation will establish the Fermi-Dirac distribution function $f(\epsilon)$. An example of our resistivity $\rho_+ = \sigma_+/\sigma_{xy}^2$ is given in Fig.1 and it agrees well with the experiments [1, 2]. Parameter u_0 determines the onset of the Shubnikov-de-Haas oscillations whereas u is the purity parameter. The conductivity Eq.(16), (17) allows for the absolute negative conductivity $\sigma_+ < 0$ at large microwave field $\mathcal{E}$.

We use the Keldysh method [12] to calculate the local diffusion constant. In the non-equilibrium state of electrons driven by microwaves with a long-range gradient of the electron density, the current density, averaged over times shorter then the density relaxation time, reads:

$$\mathbf{j}(\mathbf{r},t) = \hat{\mathbf{j}}G^{+-}(\mathbf{r}t;\mathbf{r}t). \qquad (18)$$

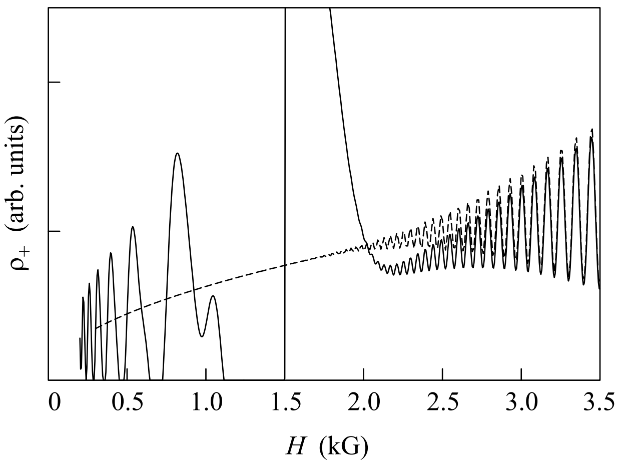

Fig.1. Positive part of the diagonal resistivity (ρ_+, Eqs. (16), (17)) with and without microwaves. x-axis is magnetic field in kGauss, $n = 3\cdot 10^{11}\,\mathrm{cm}^{-2}$, $\omega = 2\pi\cdot 57\,\mathrm{GHz}$, $E = 1.6\,\mathrm{V/cm}$. Disorder model parameters are: $u_0 = 2\,\mathrm{K}^2$, $u = \omega_H\cdot 0.03\,\mathrm{K}$, and w determines the scale of the resistivity y-axis

Diffusion is related to the short range disorder scattering, therefore, the Green's function in Eq.(18) is expanded into the diagrams with one short range impurity line. One of the Green's functions in each diagram represents a variation of the occupation number distribution $\delta f(\epsilon)$, due to the density gradient,: $\delta G^{+-} = \delta f G^A - G^R \delta f$, whereas all Green's functions to the left to δG^{+-} are retarded and all Green's functions to the right to δG^{+-} are advanced. In weakly non-equilibrium Fermi liquid $\delta f(\epsilon)$ is localized in the energy domain in the vicinity of the Fermi level. Therefore, we use ansatz:

$$\delta f(\epsilon, \mathbf{r}) = -\frac{1}{\nu(\epsilon_F)}\frac{df}{d\epsilon} n_{\mathbf{q}} \exp(i\mathbf{qr}) \qquad (19)$$

It has the quasi-classical property: $\int \delta f(\mathbf{p})\, d^2\mathbf{p}/(2\pi)^2 = n_{\mathbf{q}} \exp(i\mathbf{qr})$. In the perpendicular magnetic field we expand $\exp(i\mathbf{qr}) = 1 + iml_H^2 \mathbf{q}\times\hat{\mathbf{j}} + \ldots$ in series of small $\mathbf{q}$, neglecting the magnetic translation operator that does not contribute to the current. In the result we get a diagram shown in Fig.2. It is proportional to $\mathbf{q}n_{\mathbf{q}}$ and, therefore, gives the linear response diffusion equation:

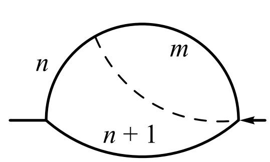

Fig.2. Diagram for the average current in the left vertex. It is a linear response to the inhomogeneous electron density with the density gradient being indicated by the arrow

$\mathbf{j} = D\nabla n$. The dashed short range impurity line in the state driven by microwaves is given by Eqs.(9), (10). The sum of all such diagrams gives the diffusion constant:

$$D_{xx} \pm D_{yy} = -\frac{4N}{\pi\nu(\epsilon_F)} w \sum_m \int \frac{d\epsilon}{2\pi} \left\{ \begin{array}{c} P_m(\omega) \\ Q_m(\omega) \end{array} \right\} \times$$
$$\times\, \mathrm{Im}G(\epsilon)\, \frac{df}{d\epsilon}\, \mathrm{Im}G(\epsilon + m\omega). \qquad (20)$$

Because $\mathrm{Im}G(\epsilon)$ is positively defined function – it is the density of states - and $df/d\epsilon < 0$, the diffusion constant is positive $D_{aa} > 0$. The diffusion constant and the diffusion conductivity Eq.(16) are related by the Einstein formula: $\sigma^D_{aa} = e^2\nu(\epsilon_F)D_{aa}$. The Einstein relation between the total conductivity and the diffusion constant is violated in the systems driven by microwaves due to the Ryzhii conductivity Eq.(17).

When the intensity of the microwaves, in magnetic fields such that $k\omega_H < \omega < (k+1/2)\omega_H$, is increasing the electron density variation, originally due to the Si donor potential, become more and more even because the negative Ryzhii current moves electrons from the places of lower electrostatic energy (higher density) into the places of higher electrostatic energy (lower density). The correlation function of the donor potential transforms from the completely screened Eq.(6) to the unscreened one Eq.(5):

$$S(q, E) = \frac{(2\pi e^2)^2\; n}{\left(q + 2\pi e^2 \nu_F[1 - |\sigma_R|/\sigma_D]\right)^2}. \qquad (21)$$

The harmonic of the potential $u(\mathbf{q})$ with $q = 0$ will diverge at the critical microwave field E_c, determined from the condition: $|\sigma^R(E_c)| = \sigma^D(E_c)$. At even higher microwave intensities $E > E_c$ the harmonics of the potential, with momenta in the shell $|\mathbf{q}_c| = \alpha(E - E_c)$, are divergent. Therefore, the local electric field will become infinite. The additional current induced by the electron relaxation due to phonons will perhaps prevent the local electric field to grow to infinity. q_c can be considered as the order parameter of the emergent random long range potential with large gradients.

In this situation the diffusion is dominated by electrons drifting in very long range $qR_c \ll 1$ random potential with very large local electric fields. The macroscopic *dc*- conductivity is inversely proportional to the magnitude of the local electric field in the vicinity of the percolating contour. Indeed, the local electric field lifts the degeneracy of the Landau level and impart a velocity and a linear dispersion to the electron: $\epsilon(\mathbf{p}) = l_H^2 \nabla u \times \mathbf{p}$. Therefore the density of states is inversely proportional to the electric field: $\nu(\epsilon_F) \sim 1/|\nabla u|$. The scattering rate is proportional to the correlation function of the short range potential: $1/\tau = 2\pi\nu(\epsilon_F)w$. In the magnetic field $D \sim 1/\tau$ due to the dominant Lorentz force. Therefore, the conductivity as well as the resistivity is almost zero and the zero-resistance state emerges.

The Hall conductivity in 2DEG with the constant electron density n and in the presence of the long range potential is determined by the drift of those electrons that are close to the percolating level in the direction perpendicular to the applied Hall electric field. It is given by $\sigma_{xy} = ecn/H$. Indeed, the total Hall voltage along the cross section of the sample can be written as $eV_H = \sum_i \epsilon_i$, where i counts percolating level crossings and $\epsilon_i = l_i|\nabla u|_i$ is the potential drop across the bunch of infinite size contours of width l_i (non-zero due to the applied Hall voltage). The Hall current is $I_H = \sum_i I_i$, where $I_i = nl_i v_i^d$, the drift velocity $\mathbf{v}_i^d = c\nabla u/H$ and n does not depend on crossing i. Therefore, the Hall conductivity $\sigma_{xy} = ecn/H$ is ideal.

In conclusion the explanation of the zero-resistance states is given. It is based on the two phenomena: the divergent Coulomb screening and the inverse dependence of the conductivity/resistivity on the local long range electric field. It has been found that in the driven state the Einstein relation is violated. The magneto-conductivity and the diffusion constant in the driven state is calculated in the framework of the microscopic model and they agree well with the experiments.

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9. The ratio of the 'relaxation' current due to phonons and the 'excitation' Ryzhii current is small in high Landau level N: $I_{rel}/I_R \sim v_F/2Ns$, where s is the sound velocity.
10. Indeed, consider the band between the two contours of the constant potential, separated by small energy ϵ with variable local width $l(x) = \epsilon/|\vec{\nabla}u|$, where x, y are the local coordinates in the direction along, perpendicular to the drift. The wave function of the drifting state is $|\psi(x,y)|^2 = \phi^2(y)/Tv_d$ [11], where $\int \phi^2(y)dy = 1$, T and $v_d(x) = c|\vec{\nabla}u|/eH$ are the period and the local velocity of the drifting trajectory. The density is $n(x) = \sum_i \int_0^l |\psi_i(x,y)|^2 dy/l(x)$, where i counts the electron states inside the band. The number of these states N_l is large and constant along the band. Therefore, the density $n = eHN_l/c\epsilon T$ is constant along the contour of constant potential.
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