Electrostatic pair creation and recombination in quantum plasmas

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The production of electron–positron pairs by electrostatic waves in quantum plasmas is investigated. In particular, a semi-classical governing set of equations for a self-consistent treatment of pair creation by the Schwinger mechanism in a quantum plasma is derived.

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1. Introduction. In quantum field theory, and quantum electrodynamics in particular, the vacuum is no longer trivial, but can under certain circumstances act as a medium. Effects such as elastic light-by-light scattering and photon splitting are well known examples of the influence of the nonlinear quantum vacuum [1]. The concept of pair creation is of great interest both from a principal point of view, as well as for near-future applications. In particular, the production of pairs is an intrinsically nonperturbative process, and as such poses new requirements for its theoretical treatment. Lately, there has been much interest in pair creation due to nontrivial generalizations of the constant electric field case (see e.g. [2 – 9] and references therein), as considered by Schwinger for quantum electrodynamics [10]. The next generation laser systems, such as the X-ray free electron laser, will be able to produce pairs, and this necessitates the study of collective and self-consistent pair production effects. Thus, in this spirit, the effect of pair production with circularly polarized light was considered in Refs. [11] and [12], and it was found that electromagnetic degrees of freedom were dissipated into pairs in a self-consistent manner. However, it is not unreasonable to assume that within laser-plasma and astrophysical systems, strong electrostatic fields may be built up, thus posing a new problem concerning pair creation.

In this paper we will study the effects of electrostatic pair creation in a three component plasma, consisting of ions, electrons, and positrons. The quantum properties of electrons and positrons are partially included by using a semi-classical approximation of the many-body Schrödinger system. The effect of the latter is to introduce higher order dispersion in the electron/positron momentum equations. The combined effect of pair creation and quantum plasmas can thus be studied using an effective semi-classical theory in terms of macroscopic fluid variables.

2. Theory. For fields varying slowly compared to the Compton frequency $\omega_e = m_e c^2/\hbar$, where m_e is the electron rest mass, c is the velocity of light in vacuum, and $\hbar$ is the Planck constant divided by 2π, the pair creation rate per unit phase space volume is given by $q(t, \mathbf{r}, \mathbf{p}) = q_0(t, \mathbf{r}) F(\mathbf{p})$, where [10, 13 – 15]

$$q_0 = \frac{c}{(2\pi)^3 \lambda^4} \frac{|\mathbf{E}|^2}{E_{\text{crit}}^2} \exp\left(-\pi \frac{E_{\text{crit}}}{|\mathbf{E}|}\right), \qquad (1)$$

Here $\mathbf{p}$ is the electron/positron momentum, $\mathbf{E}$ is the electric field strength, e is the magnitude of the electron charge, $E_{\text{crit}} = m_e^2 c^3/e\hbar \sim 10^{16}\,\text{V}\cdot\text{cm}^{-1}$ is the Schwinger critical field, $\lambda = \hbar/m_e c$ is the Compton wavelength, m_e is the electron rest mass, c is the speed of light in vacuum, $\hbar$ is the Planck's constant divided by 2π, and the momentum distribution function is normalized according to $\int F d^3p = 1$. Here, the small correction due to the electron/positron momentum orthogonal to the electric field is neglected. We note that the expression (1) should be used with caution, as pointed out in Ref. [6], as the electrons/positrons do not experience the local field. This can be partially remedied by averaging the field over a suitable spatial volume. Here we will at present consider the local field approximation, keeping the above in mind. Apart from the pair creation there will also be a pair recombination, or annihilation, with the rate given by $\nu_e = \sigma v_c n_p$ ($\nu_p = \sigma v_c n_e$) for the electrons (positrons), where σ is the cross section, v_c is a characteristic velocity of the recombination, and n_e (n_p) is the electron (positron) number density.

For electrostatic oscillations, the electric field is determined by Poisson's equation

$$\nabla \cdot \mathbf{E} = (-en_e + en_p + Z_i e n_i + \rho_{\text{pol}})/\epsilon_0. \qquad (2a)$$

where n_e (n_p) is the electron (positron) number density in the laboratory frame, n_i is the ion number density,

$Z_i e$ is the ion charge density and $\rho_{\rm pol}$ is the vacuum polarization charge density. Ampère's law gives

$$\partial_t \mathbf{E} = \frac{1}{\epsilon_0}\left(\frac{en_e\mathbf{p}_e}{\gamma_e m_e} - \frac{en_p\mathbf{p}_p}{\gamma_p m_p} - \frac{Z_i e n_i \mathbf{p}_i}{\gamma_i m_i} - \mathbf{j}_{\rm pol}\right), \quad (2b)$$

where the polarization current is $\mathbf{j}_{\rm pol} = \mathbf{E}q_0/|\mathbf{E}|^2$ and the vacuum polarization charge density is defined via the continuity equation $\partial_t\rho_{\rm pol} + \nabla\cdot\mathbf{j}_{\rm pol} = 0$. The particular choice of $\mathbf{j}_{\rm pol}$ gives overall energy conservation to the system where the energy of the newly created pairs is compensated by a decrease of the electrostatic energy. The fluid equations for the cold electrons, positrons, and ions are governed by [16, 12]

$$\partial_t n_e + \nabla\cdot\left(\frac{n_e\mathbf{p}_e}{\gamma_e m_e}\right) = q_0 - \sigma v_c n_e n_p, \quad (2c)$$

$$\partial_t n_p + \nabla\cdot\left(\frac{n_p\mathbf{p}_p}{\gamma_p m_p}\right) = q_0 - \sigma v_c n_e n_p, \quad (2d)$$

$$\partial_t n_i + \nabla\cdot\left(\frac{n_i\mathbf{p}_i}{\gamma_i m_i}\right) = 0, \quad (2e)$$

and

$$\left(\partial_t + \frac{\mathbf{p}_e}{\gamma_e m_e}\cdot\nabla\right)\mathbf{p}_e = -e\mathbf{E} + \frac{\hbar^2}{2m_e\gamma_e}\nabla U_{\rm Be} + + \frac{q_0}{n_e}(\mathbf{P}_e - \mathbf{p}_e) - \sigma v_c n_p(\mathbf{p}_e - \mathbf{p}_p), \quad (2f)$$

$$\left(\partial_t + \frac{\mathbf{p}_p}{\gamma_p m_p}\cdot\nabla\right)\mathbf{p}_p = e\mathbf{E} + \frac{\hbar^2}{2m_p\gamma_p}\nabla U_{\rm Bp} + + \frac{q_0}{n_p}(\mathbf{P}_p - \mathbf{p}_p) - \sigma v_c n_e(\mathbf{p}_p - \mathbf{p}_e), \quad (2g)$$

$$\left(\partial_t + \frac{\mathbf{p}_i}{\gamma_i m_i}\cdot\nabla\right)\mathbf{p}_i = Z_i e\mathbf{E}, \quad (2h)$$

valid for length scales larger than $v_F/\omega_{\rm pe}$, where v_F is the Fermi velocity and $\omega_{\rm pe} = (n_0e^2/\epsilon_0 m_e)^{1/2}$ is the electron plasma frequency for an electron density n_0. Here we have added the recombination term for the sake of generality. The index e (p) denotes the electrons (positrons), i the ions, and $\gamma_{e,p,i} = (1 + + p_{e,p,i}^2/m_{e,p,i}^2c^2)^{1/2}$ is the relativistic gamma factor. Moreover, the generalized relativistic Bohm potential is defined according to (neglecting the spin of the particles)

$$U_{{\rm B}e,p} = (n_{e,p}/\gamma_{e,p})^{-1/2}[\nabla^2(n_{e,p}/\gamma_{e,p})^{1/2} - -(1/c^2)\partial_t^2(n_{e,p}/\gamma_{e,p})^{1/2}], \quad (3)$$

obtained using a Madelung representation for the Klein-Gordon equation. In Eqs. (2f) and (2g) we have defined $\mathbf{P}_{e,p} = \int \mathbf{p}_{e,p}F\,d^3p$. Equation (2a)–(2h), together with Eq. (3), determines the semi-classical dynamics of our three component plasma where quantum mechanical correction for the electrons and positrons are taken into account.

3. One-dimensional model. We now specialize to a one-dimensional geometry. In addition to the effects discussed in the 3D model, we will assume that the electron-positron pairs are created at some distance from each other so that the energy of the particles are compensated by a decrease of the electric field energy. The Poisson equation is

$$\partial_x E = \frac{e}{\epsilon_0}(n_p - n_e + Z_i n_i), \quad (4)$$

where the ions are taken to be immobile so that $Z_i n_i = = n_0$. We suppose that the electron and positron continuity equations, respectively, are

$$\partial_t n_e + \partial_x\left(\frac{n_e p_e}{\gamma_e m_e}\right) = q_e - \sigma v_c n_e n_p, \quad (5)$$

and

$$\partial_t n_p + \partial_x\left(\frac{n_p p_p}{\gamma_p m_p}\right) = q_p - \sigma v_c n_e n_p, \quad (6)$$

and the momentum equations

$$\partial_t p_e + m_e c^2\partial_x\gamma_e = -eE \quad (7)$$

and

$$\partial_t p_p + m_p c^2\partial_x\gamma_p = eE, \quad (8)$$

where we have assumed that the cold particles are created with momenta equal to the fluid momenta of the electrons and positrons, respectively, so that the sources do not contribute to the momenta of the particles but only to the particle number densities. In this manner, we do not have heating of the particles and do not need to include pressure or take into account kinetic effects. The source terms

$$q_e = q_0 + \partial_x\left(\lambda\gamma_e q_0\frac{E_{\rm crit}}{E}\right) \quad (9)$$

and

$$q_p = q_0 - \partial_x\left(\lambda\gamma_p q_0\frac{E_{\rm crit}}{E}\right) \quad (10)$$

correspond roughly to a scenario where the creation of electrons and positrons at the position x depends on the electric field at a distance $\lambda\gamma_p E_{\rm crit}/E$ and $-\lambda\gamma_e E_{\rm crit}/E$, respectively, from x. For a stronger electric field, the distance of creation decreases, while it increases for larger γ_j. This form of the source term conserves exactly the total charge and gives an approximate energy conservation law of the form

$$\frac{d}{dt}\int_{\Omega}\left[m_e c^2(n_e\gamma_e + n_p\gamma_p - 2n_0) + \frac{\epsilon_0 E^2}{2}\right]dx = \\ = mc^2\int \frac{E_{\rm crit}}{2E} q_0 \lambda \partial_x(\gamma_e^2 - \gamma_p^2)\,dx, \tag{11}$$

where the right-hand side can be assumed to be small if $\lambda|\partial_x| \ll 1$, and where we have assumed that all fields and velocities vanish at $|x| = \infty$. The conservation law was obtained by using Ampères law

$$\partial_t E = \frac{e}{\epsilon_0}\left[\frac{n_e p_e}{\gamma_e m_e} - \frac{n_p p_p}{\gamma_p m_p} + (\gamma_e + \gamma_p)\lambda\frac{E_{\rm crit}}{E} q_0\right], \tag{12}$$

which is derived by time differentiating the Poisson equation and using the continuity equations for the particles.

4. The normalized system of equations. We now normalize the system of equations (2) by introducing the Compton wave length λ and the time scale $\tau = \hbar/m_e c^2$, thus normalizing the electric field by $E_{\rm crit}$, all momenta by $m_e c$ (with $m_p = m_e$), and number densities by our preferred density n_0. Assuming stationary ions, we then obtain the normalized and dimensionless system

$$\nabla\cdot\mathbf{E} = \omega_{\rm pe}^2(1 - n_e + n_p), \tag{13a}$$

$$\partial_t n_e + \nabla\cdot\left(\frac{n_e\mathbf{p}_e}{\gamma_e}\right) = q_0 - a n_e n_p, \tag{13b}$$

$$\partial_t n_p + \nabla\cdot\left(\frac{n_p\mathbf{p}_p}{\gamma_p}\right) = q_0 - a n_e n_p, \tag{13c}$$

and

$$\left(\partial_t + \frac{\mathbf{p}_e}{\gamma_e}\cdot\nabla\right)\mathbf{p}_e = -\mathbf{E} + \tfrac{1}{2}\nabla U_{\rm Be} + \frac{q_0}{n_e}(\mathbf{P}_e - \mathbf{p}_e) - \\ - a n_p(\mathbf{p}_e - \mathbf{p}_p), \tag{13d}$$

$$\left(\partial_t + \frac{\mathbf{p}_p}{\gamma_p}\cdot\nabla\right)\mathbf{p}_p = \mathbf{E} + \tfrac{1}{2}\nabla U_{\rm Bp} + \frac{q_0}{n_p}(\mathbf{P}_p - \mathbf{p}_p) - \\ - a n_e(\mathbf{p}_p - \mathbf{p}_e), \tag{13e}$$

where now

$$q_0 = (E^2/N_0)\exp\left(-\pi|E|^{-1}\right) \tag{14}$$

and we have used the stationary ion background density as the normalization density n_0, $N_0 = n_0 h^3/m_e^3 c^3$, the electron plasma frequency is normalized by $1/\tau$ such that $\widetilde{\omega}_{\rm pe} = (2\alpha N_0)^{1/2}/2\pi$, $\alpha = e^2/4\pi\epsilon_0\hbar c \approx 1/137$ is the fine structure constant, and we have introduced $a = N_0(\sigma/\lambda^2)(v_c/c)$.

5. Numerical analysis of the one-dimensional system. The one-dimensional system of equations (4)–(8) are normalized so that

$$\partial_x E = \widetilde{\omega}_{\rm pe}^2(1 - n_e + n_p), \tag{15a}$$

$$\partial_t n_e + \partial_x\left(\frac{n_e p_e}{\gamma_e}\right) = q_0 + \partial_x\left(\frac{\gamma_e q_0}{E}\right) - a n_e n_p, \tag{15b}$$

$$\partial_t n_p + \partial_x\left(\frac{n_p p_p}{\gamma_p}\right) = q_0 - \partial_x\left(\frac{\gamma_e q_0}{E}\right) - a n_e n_p, \tag{15c}$$

$$\left(\partial_t + \frac{p_e}{\gamma_e}\partial_x\right)p_e = -E, \tag{15d}$$

$$\left(\partial_t + \frac{p_p}{\gamma_p}\partial_x\right)p_p = E. \tag{15e}$$

and are solved numerically. The results are displayed in Figs.1–3. Initially, the electron density is perturbed lo-

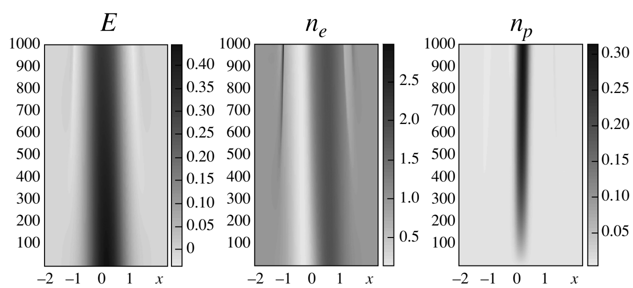

Fig.1. The electrostatic field (left panel) electron density (middle panel) and positron density (right panel) as a function of space x and time t. We used $N_0 = 0.2$ and the initial conditions $n_e = 1.01 + 2(x/L)\exp(-x^2/L^2)$ with $L = 6000$, and $n_p = 0.01$, for the electron and positron number densities, respectively

cally as $n_e = 1.01 + 2(x/L)\exp(-x^2/L^2)$ with $L = 6000$. For the positrons we take a small non-zero component $n_p = 0.01$. All particle species are initially assumed to be in rest, i.e., $p_e = p_p = 0$ at $t = 0$. We consider a dense plasma such that $N_0 = 0.2$. Initially, the maximum electric field is approximately half the Schwinger field, $E_{max} \simeq 0.5$, and we see in the right-hand panel of Fig.1 that electron-positron pairs are created so that the particle densities of the positrons are increased in a small region around $x = 0$ where the electric field amplitude has its maximum. Due to the ultra-strong electric field, the electrons are also accelerated so as to neutralize the plasma, and hence the electric field strength decreases with time. We see in Fig.2, that the pair creation rate is largest initially, when the electric field is strongest, and decreases at later times. Both the electrons and positrons are accelerated to ultra-relativistic

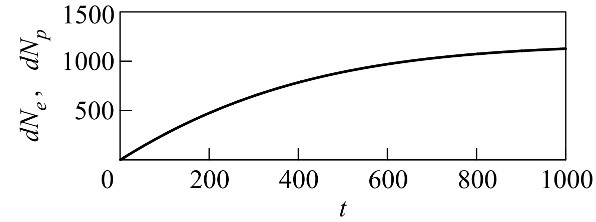

Fig.2. The total number of created electron-positron pairs, $\delta N_p = \delta N_e = N_e - N_{e,t=0}$, where $N_e = \int_\omega n_e\,dx$, as a function of time. We see a decrease in the pair creation rate, correlated with a decrease of the amplitude of the electrostatic field seen in Fig.1

speeds with gamma factors of the order 100–500, and the dynamics is essentially a balance between the kinetic energy of the particles and the potential energy stored in the electric field. We consider the energy balance in Fig.3, where we have plotted the time evolution of

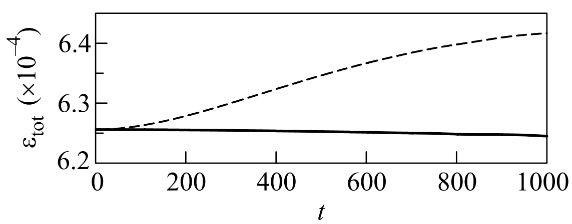

Fig.3. The total energy $\varepsilon_{\text{tot}} = \int_\Omega (n_e\gamma_e + n_p\gamma_p + E^2/2\tilde{\omega}_p^2)\,dx$ as a function of time (solid curve). The dotted curve displays the same integral for the case when the correction for the spatial displacement of the created pairs, the last terms in the right-hand sides of Eqs. (15b) and (15c), are neglected. In the latter case, the creation of electron-positron pairs lead to an increase of the total energy in the system

the sum of the total relativistic particle energies and the electrostatic energy. We see that this energy is approximately conserved (solid line) when the full expressions for the pair creation in the continuity equations (15b), (15c) are used. The last terms in the continuity equation account approximately for the model where the electron and positron is created at some spatial difference from each other so that the total energy of the newly created pair is compensated by a decrease of the electric field and hence of the electrostatic energy. If the last terms in the right-hand sides of the continuity equations are neglected, meaning that the pairs are created at the same point in space, then there is a visible increase of the total energy when the pairs are created (the dotted curve in Fig.3). However, the energy loss to the pairs is relatively small compared to the total free energy of the system that is stored initially in the electrostatic field.

6. Summary. In summary, we have presented a dynamical and self-consistent model for electron-positron pair creation by a strong electrostatic field in a dense plasma. The electrostatic field is excited self-consistently by large-amplitude electron waves, which give rise to electric fields that are comparable with Schwinger's critical field. We have derived an approximately energy-conserving one-dimensional model that takes into account the decrease of the electrostatic energy as electron-positron pairs are created. The model presented here constitutes a first step of understanding the dynamics of large-amplitude electrostatic waves that are strong enough to create electron-positron pairs. A future model should be derived from first principles of quantum electrodynamics, in order to take into account the distribution of the created electron positron pairs in momentum space and the self-consistent energy conservation of the system.

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