Pis'ma v ZhETF, vol. 83, iss. 8, pp. 376 – 379

Curved One-Dimensional Wire with Rashba Coupling as a Spin Switch

M. P. Trushin[1], *A. L. Chudnovskiy*

I. Institut für Theoretische Physik, Universität Hamburg, D-20355 Hamburg, Germany

Submitted 15 February 2006

We propose a semiconductor structure that can switch the electron spin using the interplay between the Rashba effect and geometry of the system. In detail, the structure consists of a strongly curved one-dimensional ballistic wire with intrinsic spin-orbit interactions. Here, in contrast to the previous proposals based on the straight quantum channels, the spin-switching is achievable even if the electrons are in the eigen state and spin-precession does not occur. Using parameters relevant for InAs we investigate the tunability of this effect by means of external electric and magnetic fields.

PACS: 72.25.Dc, 73.23.Ad, 73.63.Nm

In the past few years the idea to use electron spin in mesoscopic semiconductor devices has generated a lot of interest. Datta and Das [1] describe how Rashba effect [2] (with the assistance of spin-filtering contacts) can be used to modulate the current. The basic idea is that the spin precession can be controlled via Rashba spin-orbit coupling associated with the interfacial electric field present in the quantum well that contains a two-dimensional electron gas. One of the most promising materials for this purpose is the InAs semiconductors, where the tuning of the Rashba coupling by an external gate voltage was recently achieved by Grundler [3] and Matsuyama et al. [4].

Note, however, that since the implementation of spin filtering contacts into the Datta–Das device requires rather complicated design, the optical methods for generation and detection of spin-polarized currents look more preferable. In [5, 6], a non-equilibrium population of spin-up and spin-down states in quantum well structures has been experimentally established applying circularly polarized radiation. The spin polarization results in a directed motion of free carriers in the plane of a quantum well perpendicular to the direction of light propagation. Because of the spin selection rules, the spin-polarization of the current is determined by the helicity of the light and can be reversed by switching the helicity from right to left handed. Note, that the recombination of spin polarized charged carriers results in the emission of circularly polarized light. It is possible, therefore, to use the optical methods for the detection of the spin-polarization as well.

The basic element of the "conventional" spin-switch represents a straight quantum wire with spin-orbit coupling. Note, however, that if the electrons contributing to the current are in the eigen state then the spin precession does not take place here, and, therefore, the "conventional" spin-switch does not work at all (see Fig.1a). In this Letter, we propose a new type of spin-switch based on the interplay between the curvature of the electron trajectory and Rashba spin-orbit interactions. In particular, we consider a *curved* wire consisting of a semicircle with radius R attached to the infinite straight one-dimensional channels made of the same material as the curved part. Though the electrons in the input channel are in the eigen state, the spin orientation can be changed at the output of the device (see Fig.1b,c). Curved one-dimensional quantum channels in InAs [7] are expected to be used for the experimental check of the present proposal.

On the face of it, the device is similar to the one investigated by Bulgakov and Sadreev [8]. In that work, however, the authors assume *a priory* the adiabatic regime: the radius of the curvature is so large that the electrons do not feel the junction between the curved part of the wire and input/output channels. In contrast, we start from the very general solution of Schrödinger equation for the *whole* system (i.e. input channel – semicircle – output channel) and, therefore, the description of the strongly non-adiabatic regime is possible as well. Note, that flexing the quantum wire leads to the geometrically induced potential [9]. However, its effect on the electron motion is negligible in real systems [7] since the geometrical potential is much smaller than the Fermi energy.

In order to describe the spin orientation we introduce the following quantity

$$P = (j^+ - j^-)/(j^+ + j^-), \qquad (1)$$

where $j^{\pm}$ denote the probability current densities [10] with a given spin orientation, and "±" are the spin in-

[1] e-mail: mtrushin@physnet.uni-hamburg.de

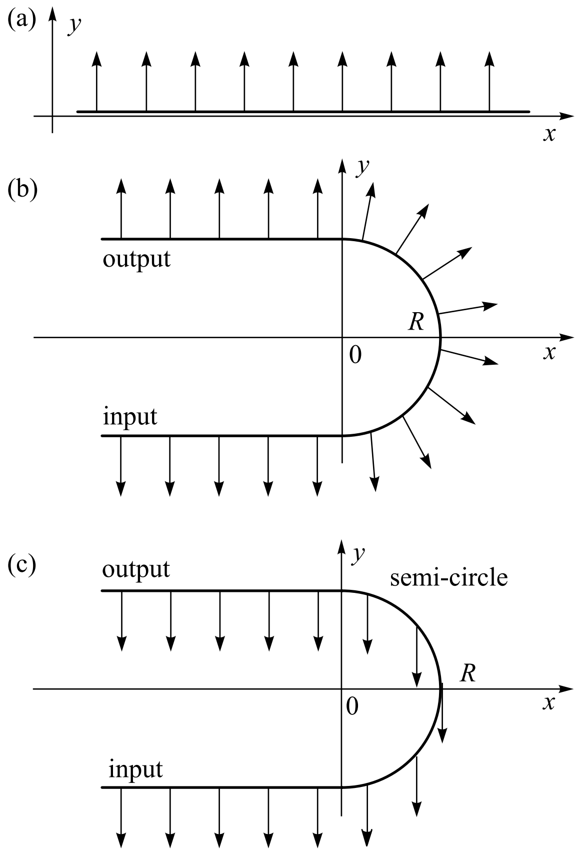

Fig.1. (a) Spin dynamics in the "conventional" spin-switch of Datta–Das type based on a straight quantum channel with Rashba coupling. The spin precession does not occur as long as the initial electrons are in the eigen state. (b) Spin dynamics in the curved quantum channel with Rashba coupling (*adiabatic* regime $\hbar^2/2\alpha m^* R \leq 1$). Here, the spin follows adiabatically the electron trajectory, and its orientation changes with respect to the laboratory coordinate system. (c) Spin dynamics in deeply *non-adiabatic* $\hbar^2/2\alpha m^* R \gg 1$ regime. Here, the electron spin is able to leave its eigen state and keep its orientation with respect to the laboratory coordinate system. The relation $\hbar^2/2\alpha m^* R$ can be tuned by the gate-voltage dependent Rashba constant α [3, 4]

dices. It seems essential to emphasize that the quantity P is controllable experimentally since the currents j^+ and j^- can be generated independently by means of absorption of circularly polarized light beams with a given helicity [5, 6]. In general, the quantity P has the meaning of projection of the probability current density spin-polarization on the spin-quantization axis. In the most important case of zero external magnetic field the spin-quantization axis is always perpendicular to the direction of the motion and lies in xy plane (see Fig.1). Note, that the spin orientation always coincides with the spin-quantization axis in the adiabatic regime, i.e. the particle does not leave its eigen state, and $P_{\text{in}} = P_{\text{out}}$. In the following we show, how one can manipulate with the output polarization P_{out} by means of external electric field (via Rashba constant).

To do that, we calculate single particle spin-split states for the system shown in Fig.1b,c. To this end, we divide the wire in three parts: input channel, semi-circle, and output channel. We use the cartesian coordinates to describe the input and output channels (the region $x < 0$ in Fig.1b,c) and the polar coordinates for the description of the curved part of the wire. The Hamiltonians describing the propagation of an electron in the input/output wires read

$$H_{\text{wire}} = \begin{pmatrix} \frac{\hbar^2}{2m^*}\hat{k}_x^2 + \varepsilon_Z & i\alpha\,\hat{k}_x \\ -i\alpha\,\hat{k}_x & \frac{\hbar^2}{2m^*}\hat{k}_x^2 - \varepsilon_Z \end{pmatrix} \tag{2}$$

whereas the propagation through the semi-circle of radius R is governed by the Hamiltonian [11]

$$H_{\text{curv}} = \begin{pmatrix} \varepsilon_0\,\hat{q}_\varphi^2 + \varepsilon_Z & \alpha\,e^{-i\varphi}\left(\hat{q}_\varphi - \frac{1}{2}\right)/R \\ \alpha\,e^{i\varphi}\left(\hat{q}_\varphi + \frac{1}{2}\right)/R & \varepsilon_0\,\hat{q}_\varphi^2 - \varepsilon_Z \end{pmatrix}. \tag{3}$$

Here

$$\hat{k}_x = -i\frac{\partial}{\partial x} - \frac{\Phi}{\Phi_0}\frac{1}{R}, \quad \hat{q}_\varphi = -i\frac{\partial}{\partial\varphi} - \frac{\Phi}{\Phi_0}$$

are the momentum and the angular momentum operators respectively, $\Phi = \pi R^2 B_z$ is a magnetic flux through the area of a ring of radius R, $\Phi_0 = 2\pi\hbar c/e$ is the flux quantum, m^* is the effective electron mass, α is the Rashba constant, $\varepsilon_0 = \hbar^2/2m^*R^2$ is the size confinement energy, $\varepsilon_Z = g^*\mu_B B_z/2$ is the Zeeman term. We adopt the vector potential $\mathbf{A}$ to be tangential to the direction of the current. Thus, in the semi-circle we choose $\mathbf{A}(x,y) = \frac{1}{2}B_z\,(x\,\mathbf{j} - y\,\mathbf{i})$, or, in cylindrical coordinates, $A_\varphi(\varphi) = \Phi/2\pi R$, whereas the vector potential in the input and output channels is determined by the continuity condition at the junction point with the curved part of the wire ($x = 0$, $y = \pm R$); hence we have $A_x = \Phi/2\pi R$.

We denote the wave functions for each part as $\Psi^{\pm}_{\text{curv}}(\varphi)$ for the semi-circle, $\Psi^{\pm}_{\text{in}}(x)$ and $\Psi^{\pm}_{\text{out}}(x)$ for the input and the output channels respectively. In order to find the wave function of the whole system, we impose boundary conditions that warrant the continuity of the wave function and its first derivative on the boundaries between the parts of the wire

$$\begin{aligned}
&\left(\Psi^+_{\text{in}} + \Psi^-_{\text{in}}\right)|_{x=0} = \left(\Psi^+_{\text{curv}} + \Psi^-_{\text{curv}}\right)|_{\varphi=-\pi/2},\\
&\left(\Psi^+_{\text{curv}} + \Psi^-_{\text{curv}}\right)|_{\varphi=\pi/2} = \left(\Psi^+_{\text{out}} + \Psi^-_{\text{out}}\right)|_{x=0},\\
&\left(\nabla\Psi^+_{\text{in}} + \nabla\Psi^-_{\text{in}}\right)|_{x=0} = \left(\nabla\Psi^+_{\text{curv}} + \nabla\Psi^-_{\text{curv}}\right)|_{\varphi=-\pi/2},\\
&\left(\nabla\Psi^+_{\text{curv}} + \nabla\Psi^-_{\text{curv}}\right)|_{\varphi=\pi/2} = \left(\nabla\Psi^+_{\text{out}} + \nabla\Psi^-_{\text{out}}\right)|_{x=0}.
\end{aligned} \tag{4}$$

Solutions of Schrödinger equations for Hamiltonians (2), (3) give us the desired spinor wave functions for the input, output and curved parts of the system. For the input channel we have

$$\Psi^{+}_{\rm in}(x)=e^{\frac{i\Phi}{\Phi_0 R}x}\begin{pmatrix} \cos\gamma^{+}\left(A_0^{+}e^{ik^{+}x}+A^{+}e^{-ik^{+}x}\right) \\ -i\sin\gamma^{+}\left(A_0^{+}e^{ik^{+}x}-A^{+}e^{-ik^{+}x}\right)\end{pmatrix}, \quad (5)$$

$$\Psi^{-}_{\rm in}(x)=e^{\frac{i\Phi}{\Phi_0 R}x}\begin{pmatrix} -i\sin\gamma^{-}\left(A_0^{-}e^{ik^{-}x}-A^{-}e^{-ik^{-}x}\right) \\ \cos\gamma^{-}\left(A_0^{-}e^{ik^{-}x}+A^{-}e^{-ik^{-}x}\right)\end{pmatrix}, \quad (6)$$

where

$$\tan\gamma^{\pm}=-\frac{\varepsilon_Z}{k^{\pm}\alpha}+\sqrt{1+\left(\frac{\varepsilon_Z}{k^{\pm}\alpha}\right)^2}. \quad (7)$$

Here, $k^{\pm}$ are the Fermi wave vectors that satisfy the dispersion relations $E_F=\hbar^2 {k^{\pm}}^2/2m^*\pm\sqrt{\alpha^2{k^{\pm}}^2+\varepsilon_Z^2}$, where E_F is the Fermi energy. In the case of zero magnetic field ($\varepsilon_Z=0$), the Fermi momenta $k^{\pm}$ take the simple form $k^{\pm}=\mp m^*\alpha/\hbar^2+k_0$, where $k_0= =\sqrt{(m^*\alpha/\hbar^2)^2+2m^*E_F/\hbar^2}$. The coefficients $A^{\pm}$ are the reflection amplitudes that have to be found by imposing the boundary conditions (4), whereas $A_0^{\pm}$ are the incident ones. For the output channel the reflection amplitudes are assumed to be zero, and the corresponding spinors read

$$\Psi^{+}_{\rm out}(x)=\begin{pmatrix} D^{+}\cos\gamma^{+}e^{i(k^{+}+\frac{\Phi}{\Phi_0 R})x} \\ iD^{+}\sin\gamma^{+}e^{i(k^{+}+\frac{\Phi}{\Phi_0 R})x}\end{pmatrix}, \quad (8)$$

$$\Psi^{-}_{\rm out}(x)=\begin{pmatrix} iD^{-}\sin\gamma^{-}e^{i(k^{-}+\frac{\Phi}{\Phi_0 R})x} \\ D^{-}\cos\gamma^{-}e^{i(k^{-}+\frac{\Phi}{\Phi_0 R})x}\end{pmatrix}. \quad (9)$$

Here $D^{\pm}$ are the transmission amplitudes. We have changed sign of $\gamma^{\pm}$ for the output wire since the electron motion changes its direction to the opposite one.

The eigenfunctions of the Hamiltonian (3) have a view

$$\Psi^{+}_{\rm curv}(\varphi)=e^{i\frac{\Phi}{\Phi_0}\varphi}\times$$
$$\begin{pmatrix} B^{+}\cos\alpha^{+}e^{i(q_R^{+}-\frac12)\varphi}+C^{+}\cos\beta^{+}e^{-i(\frac12+q_L^{+})\varphi} \\ B^{+}\sin\alpha^{+}e^{i(\frac12+q_R^{+})\varphi}-C^{+}\sin\beta^{+}e^{-i(q_L^{+}-\frac12)\varphi}\end{pmatrix}, \quad (10)$$

$$\Psi^{-}_{\rm curv}(\varphi)=e^{i\frac{\Phi}{\Phi_0}\varphi}\times$$
$$\begin{pmatrix} -B^{-}\sin\alpha^{-}e^{i(q_R^{-}-\frac12)\varphi}+C^{-}\sin\beta^{-}e^{-i(\frac12+q_L^{-})\varphi} \\ B^{-}\cos\alpha^{-}e^{i(\frac12+q_R^{-})\varphi}+C^{-}\cos\beta^{-}e^{-i(q_L^{-}-\frac12)\varphi}\end{pmatrix}, \quad (11)$$

where

$$\tan\alpha^{\pm}=\frac{\varepsilon_0 q_R^{\pm}-\varepsilon_Z}{q_R^{\pm}\alpha/R}+\sqrt{1+\left(\frac{\varepsilon_Z-\varepsilon_0 q_R^{\pm}}{q_R^{\pm}\alpha/R}\right)^2}, \quad (12)$$

$$\tan\beta^{\pm}=-\frac{\varepsilon_0 q_L^{\pm}+\varepsilon_Z}{q_L^{\pm}\alpha/R}+\sqrt{1+\left(\frac{\varepsilon_Z+\varepsilon_0 q_L^{\pm}}{q_L^{\pm}\alpha/R}\right)^2}, \quad (13)$$

and $q_R^{\pm}$ are the Fermi angular momenta for right-moving electrons that satisfy the relation

$$E_F=\frac{\varepsilon_0}{4}+\varepsilon_0 {q_R^{\pm}}^2\pm\sqrt{\left(\frac{q_R^{\pm}\alpha}{R}\right)^2+\left(q_R^{\pm}\varepsilon_0-\varepsilon_Z\right)^2}. \quad (14)$$

The relation for $q_L^{\pm}$ differs only by the sign before ε_Z. If the Zeeman effect is negligible, then the equation (14) allows the simple analytical solution with respect to $q_R^{\pm}$ ($q_L^{\pm}$)

$$q^{\pm}/R=\mp\frac{m^*\alpha}{\hbar^2}\sqrt{1+\left(\frac{\hbar^2}{2\alpha m^* R}\right)^2}+k_0, \quad (15)$$

(cf. with $k^{\pm}$). Note, that the chirality index is omitted in (15), since $q_R^{\pm}=q_L^{\pm}$ at $B_z=0$.

Imposing the boundary conditions (4) on the wave functions (5), (6), (8) – (11) we obtain a solution of the Schrödinger equation for the whole system. At this point it is pertinent to turn to the calculation of the input, reflected and transmitted current densities. Each current density is given as a sum of its two spin-polarized parts $j=j^{+}+j^{-}$, where the components $j^{\pm}$ are defined by the corresponding coefficient, i.e. $j^{\pm}_{\rm in}=|A_0^{\pm}|^2 j_0$, $j^{\pm}_{\rm refl}=-|A^{\pm}|^2 j_0$, $j^{\pm}_{\rm out}=|D^{\pm}|^2 j_0$, and

$$j_0=\frac{\hbar}{m^*}\left[k^{\pm}\pm\frac{\alpha m^*}{\hbar^2}\sin(2\gamma^{\pm})\right]. \quad (16)$$

The transmission probability is defined as $T=j_{\rm out}/j_{\rm in}$, and the reflection one as $R=j_{\rm refl}/j_{\rm in}$. Note, that the general (numerical) solution gives $T=1$ and $R=0$, which means that there is no particle backscattering.

Let us assume, that the input probability current density polarization $P_{\rm in}$ is equal to 1, i.e. $A_0^{+}=1$, $A_0^{-}=0$, and the electron spins in the input channels are aligned in y direction (Fig.1b,c). Using the general solution of Eqs. (4) we plot the dependences of $P_{\rm out}$ on the radius of curvature R (Fig.2) and Rashba constant α (Fig.2, inset). From the inset we can see immediately that the spin-swithching is achievable by means of the external electric field (via Rashba coupling). In order to explain this we solve Egs. (4) in two cases: adiabatic $\hbar^2/2m^*R\alpha\ll 1$ limit and strongly non-adiabatic $\hbar^2/(2m^*R\alpha)\gg 1$ one. The first limit is, however, not really interesting because no current density redistribution between the two spin-split modes occurs here, i.e. $|D^{+}|^2=1$ and $|D^{-}|^2=0$. Intuitively it is clear, that the curved wire does not differ too much from the straight one as long as $\hbar^2/2m^*R\alpha\ll 1$. Therefore, the

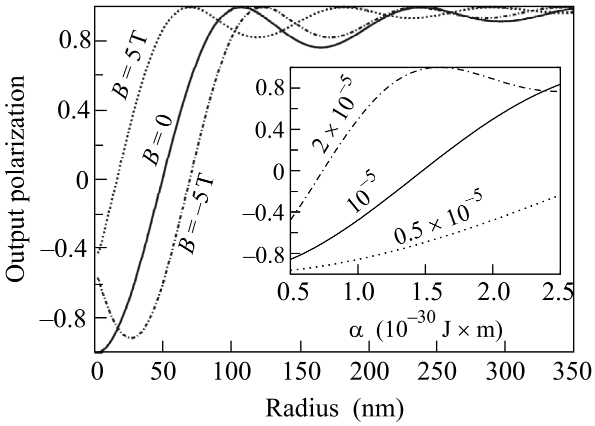

Fig.2. Output polarization P_{out} versus radius of curvature at $P_{\text{in}} = 1$ and different external magnetic fields. The parameters are $m^* = 0.033 m_e$, $E_F = 30$ meV. Inset: Output polarization versus Rashba constant α at different radius R of curvature and zero external magnetic field. Such values of α and R are achievable experimentally in InAs [3, 4, 7]

spin-polarization keeps its +100% initial value while the current flows through the system (Fig.1b).

In the opposite, strongly non-adiabatic limit, the situation changes drastically. Indeed, Eqs. (4) at $\alpha^\pm = \pi/2$, $\beta^\pm = 0$ and negligible magnetic field allow the following approximate solution: $|A^\pm|^2 = |C^\pm|^2 = 0$, $|B^\pm|^2 = \frac{1}{2}$, and $|D^\pm|^2 = \frac{1}{2} \pm \frac{1}{2}\cos\left[\pi\left(q_R^+ - q_R^-\right)\right]$. Thus, the current density redistribution occurs in the strongly non-adiabatic regime, and the polarization reads

$$P_{\text{out}} = \cos\left[\pi\left(q_R^- - q_R^+\right)\right], \qquad (17)$$

where the difference between the Fermi angular momenta has a view

$$q_R^- - q_R^+ = \frac{2m^* R\,\alpha}{\hbar^2}\sqrt{1 + \left(\frac{\hbar^2}{2\alpha\, m^* R}\right)^2}. \qquad (18)$$

Note, that if the radius of curvature is *exactly* equal to zero, then the difference (18) is equal to 1. Thus, the output spin-polarization $P_{\text{out}} = -1$, whereas the initial one was $P_{\text{in}} = +1$, i.e. spin-polarization is switched to its opposite value at $R = 0$ and $B_z = 0$. From Eq. (18) it is also clearly seen that the change of the radius is equivalent to the tuning of Rashba coupling strength. Therefore, it is possible to change output spin-polarization by tuning Rashba coupling at constant radius of curvature (Fig.2, inset).

The difference between the Fermi angular momenta q_R^- and q_R^+ depends not only on the Rashba coupling, but on the Zeeman splitting as well. Therefore, the critical values of $q_R^- - q_R^+$, when the polarization P changes the sign, are tunable by means of the external magnetic field. Unfortunately, we do not have analytical formulae for $q_{R,L}^\pm$ at non-zero magnetic fields, but one can see the effect in Fig.2. Note, that the behavior of P_{out} is sensitive to the direction of the perpendicular magnetic field for lack of chiral symmetry in the dispersion relation (14).

The major points covered by this Letter may be summarized as follows (i) strongly curved 1D wires with Rashba spin-orbit coupling can change the output spin-polarization of the electron beam up to the opposite one though the initial electrons are in the eigen state and spin-precession does not occur, (ii) the spin-switching can be governed by the external electric field (via Rashba constant, see inset in Fig.2), (iii) in addition, the spin-switch can be tuned by the magnetic field (via Zeeman effect). In our opinion, the main outcome of this Letter is that strongly curved 1D wires with Rashba spin-orbit coupling can serve in the capacity of reflectionless and high-speed spin-switchers in some special cases when the spin carriers are in the eigen states (spin-up or spin-down) and, therefore, the usage of the "conventional" spin-precession based Datta-Das scheme is not possible. This situation can take place, for example, in the experiments with the optical absorption of circularly polarized radiation [5, 6]. We believe that the interplay between Rashba spin-orbit coupling and non-zero curvature of 1D systems can find especially fruitful applications in spintronics.

The authors acknowledge financial support from DFG through Graduiertenkolleg "Physik nanostrukturierter Festkörper" (M.T.) and Sonderforschungsbereich 508 (A. C.).

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