Pis'ma v ZhETF, vol. 83, iss. 8, pp. 420 – 424

Generation of macroscopic entangled states by means of $\chi^{(2)}$ nonlinearity without photon number resolving detection

S. A. Podoshvedov[1)]

School of Computational Sciences, Korea Institute for Advanced Study, Seoul, South Korea 130-722

Submitted 16 February 2006
Resubmitted 16 March 2006

We generalise the scheme of conditional preparation of $\chi^{(2)}$ macroscopic entangled states [S.A. Podoshvedov, JETP **129**, (2006)]. The studied system consists of the system of coupled down converters with type-I phase matching pumped simultaneously by powerful optical fields in coherent states, one auxiliary photon in superposition state of two input modes and projective measurement system. The projective measurement system involves two Hadamard gates introduced to generated output modes followed by photodetectors. Identification of macroscopic entangled states is produced by registration of one photon. No photon number resolving detection is requested for the studied scheme.

PACS: 03.65.Ud, 42.50.Dv

Since Schrödinger suggested his famous cat paradox [1], there has been great interest in generating and observing a quantum superposition of a macroscopic system. A superposition of two coherent states with a π phase difference and large amplitude of the coherent states (cat state) is considered as a realization of such a macroscopic superposition. It has been theoretically known that the cat state can be generated from a coherent state by a nonlinear interaction in a Kerr medium [2]. The same cat state passing through the Hadamard gate gives a rise to generation of coherent entangled state that may be used in quantum information processing [3 – 8]. So for the time being, there are different studies concerning use of the coherent entangled states for quantum teleportation [3], quantum computation [4], quantum nonlocality test [5], entanglement purification [6], error correction [7], and quantum metrology [8].

But despite enormous progress in theoretical study of the states with point of view of both fundamental quantum physics and possible practical applications, real experimental use of the states remains elusive entity. The Kerr nonlinearity ($\chi^{(3)}$ nonlinearity) of currently available nonlinear media is extremely small to generate required level of the superposition state of coherent states [9]. Estimations presented in [10] show, one needs an optical fiber of about 1500 km for an optical frequency of $\omega = 5 \cdot 10^{14}$ rad/s to generate a coherent superposition state with currently available Kerr nonlinearity. It is possible to make use of such long fiber in practice to use its nonlinear effect but the effects of decoherence and phase fluctuations during the propagation become very large to destroy the generated cat state. Possible methods to generate coherent entangled states are considered in [11].

Here, we present generalization of out previous work [12] to conditionally prepare two types of macroscopic entangled states in output pumping modes of the studied system without photon-number resolving detection. Our system is constructed from two down converters ($\chi^{(2)} \gg \chi^{(3)}$) with type-I phase matching and Bell state measurement scheme consisting of two Hadamard gates. To conditionally prepare the states, we use coherent state in pumping modes with one auxiliary input photon in superposition state for interaction with $\chi^{(2)}$ nonlinearity. We determine whether $\chi^{(2)}$ macroscopic entangled states are generated by postselection by means of registration of one photon in auxiliary generated modes. Given scheme does not need special detectors discriminating between one- and multi-photon number states.

We start with simplified three-mode Hamiltonian [12]

$$\widehat{H} = \frac{i\hbar r}{2}(\hat{a}_1^+ \hat{a}_2^+ \hat{a}_p - \hat{a}_p^+ \hat{a}_2 \hat{a}_1), \tag{1}$$

where all designations are the same as those used in [12]. According to [12], the output of the SPDCI with Hamiltonian (1) and input condition $|00\rangle_{12}|\alpha\rangle_p$ is given by

$$|\Psi^{(00)}\rangle = \sum_{n=0}^{\infty} |n\rangle_1 |n\rangle_2 |\Phi_n^{(00)}\rangle_p, \tag{2a}$$

with wave the following function $|\Phi_n^{(00)}\rangle_p$ in the pumping mode

$$|\Phi_n^{(00)}\rangle_p \equiv |\Phi_n^{(00)}(\alpha;\eta)\rangle_p =$$
$$= \exp(-\alpha^2/2) \sum_{l=0}^{\infty} \frac{\alpha^{l+n}}{\sqrt{(l+n)!}} f_{2(l+n),n+1}^{(00)}(\eta) |l\rangle_p, \tag{2b}$$

[1)]e-mail:sap@kias.re.kr

where $f_{2l,k}^{(00)}(\eta) \equiv f_{2l,k}^{(00)}(\eta;\tau{=}1)$, $(\{f_{2l,1}^{(00)}(\eta;0), f_{2l,2}^{(00)}(\eta;0), \ldots, f_{2l,l+1}^{(00)}(\eta;0)\} = \{1,0,\ldots,0\})$ are the wave amplitudes satisfying the system of $l+1$ differential linear equations presented in [12]. Here, $\eta = rL/2c$ with L being the crystal length is the coupling constant, $\tau = ct/L$ ($\tau \in [0;1]$) is the time scaled such that $\tau = 1$ corresponds to the crystal exit.

Consider the same Hamiltonian (1) but with other input condition $|10\rangle_{12}|\alpha\rangle_p$. Then, the same mathematical approach gives the following wave function

$$|\Psi^{(10)}\rangle = \sum_{n=0}^{\infty} |n+1\rangle_1 |n\rangle_2 |\Phi_n^{(10)}\rangle_p, \tag{3a}$$

with wave function in the pumping mode

$$|\Phi_n^{(10)}\rangle_p \equiv |\Phi_n^{(10)}(\alpha;\eta)\rangle_p =$$
$$= \exp(-\alpha^2/2) \sum_{l=0}^{\infty} \frac{\alpha^{l+n}}{\sqrt{(l+n)!}} f_{(2(l+n)+1),n+1}^{(10)}(\eta)|l\rangle_p, \tag{3b}$$

with the wave amplitudes $f_{2n+1,k}^{(10)}(\eta;\tau)$ satisfying other set of linear differential equations

$$\frac{df_{2n+1,k}^{(10)}(\eta;\tau)}{ds} = \eta(\sqrt{k(k-1)(n-k+2)} f_{2n+1,k-1}^{(10)}(\eta;\tau) -$$
$$-\sqrt{k(k+1)(n-k+1)} f_{2n+1,k+1}^{(10)}(\eta;\tau)). \tag{3c}$$

Here, $f_{2l+1,k}^{(10)}(\eta) \equiv f_{2l+1,k}^{(10)}(\eta;\tau = 1)$ $(\{f_{2l,1}^{(10)}(\eta;0), f_{2l,2}^{(10)}(\eta;0), \ldots, f_{2l,l+1}^{(10)}(\eta;0)\} = \{1,0,\ldots,0\})$ are the corresponding wave amplitudes. Similarly, the Hamiltonian (1) with input condition $|01\rangle_{12}|\alpha\rangle_p$ gives a rise the following wave function

$$|\Psi^{(01)}\rangle = \sum_{n=0}^{\infty} |n\rangle_1 |n+1\rangle_2 |\Phi_n^{(01)}\rangle_p, \tag{4}$$

where the wave functions $|\Phi_n^{(01)}\rangle_p$ are given by (3c) with substitution of the symbol (10) on (01) and $|\Phi_n^{(10)}\rangle_{p_i} = |\Phi_n^{(01)}\rangle_{p_i}$.

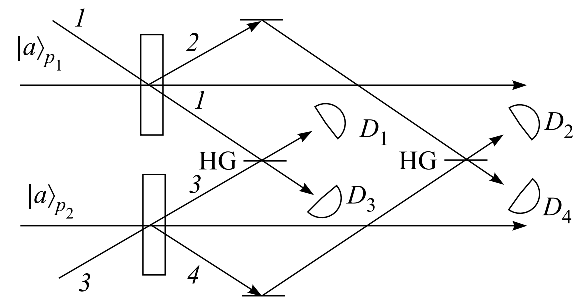
The optical scheme shown in Figure is described by the following Hamiltonian [12]

$$\widehat{H} = \frac{i\hbar r}{2}(\hat{a}_1^+ \hat{a}_2^+ \hat{a}_{p_1} - \hat{a}_{p_1}^+ \hat{a}_2 \hat{a}_1) + \hat{a}_3^+ \hat{a}_4^+ \hat{a}_{p_2} - \hat{a}_{p_2}^+ \hat{a}_4 \hat{a}_3)). \tag{5}$$

As input condition to Hamiltonian (1), we take the following state

$$|\Psi_{IN}\rangle = \frac{1}{\sqrt{2}}\{|1000\rangle + |0010\rangle\}_{1234}|\alpha\rangle_{p_1}|\alpha\rangle_{p_2}, \tag{6}$$

with real value of the quantity α of the coherent states and one photon in superposition state of two auxiliary modes 1 and 3 (subscripts 1, 2, 3, and 4 concern the corresponding input auxiliary modes, respectively). By virtue of linearity of quantum mechanics, the output of the Hamiltonian (5) with input condition (6) can be expressed as



Experimental arrangement to conditionally produce $\chi^{(2)}$ macroscopic entangled states in the output pumping modes. The system involves the system of coupled down converters with type-I phase matching simultaneously pumped by the powerful modes in coherent states with the same amplitude and one auxiliary input photon in superposition state of the modes *1* and *3*, respectively. HG is the designation for the Hadamard gate. $D_1-D_4$are the corresponding registering detectors

$$|\Psi_{OUT}\rangle = \frac{1}{\sqrt{2}}\{|\Psi_{I,II}^{(10)}\rangle + |\Psi_{I,II}^{(01)}\rangle\}, \tag{7a}$$

where

$$|\Psi_{I,II}^{(10)}\rangle = |\Psi_I^{(10)}\rangle|\Psi_{II}^{(00)}\rangle, \tag{7b}$$

$$|\Psi_{I,II}^{(01)}\rangle = |\Psi_I^{(00)}\rangle|\Psi_{II}^{(01)}\rangle. \tag{7c}$$

and the wave functions $|\Psi_I^{(10)}\rangle$ and $|\Psi_{II}^{(00)}\rangle$ with subscripts I and II concern first and second down converters, respectively.

The overall output wave function $|\Psi_{OUT}\rangle$ can be rewritten explicitly as

$$|\Psi_{OUT}\rangle = \frac{1}{\sqrt{2}}\times$$
$$\times \left\{ \begin{array}{c} |1000\rangle_{1234}|\Phi_0^{(10)}\rangle_{p_1}|\Phi_0^{(00)}\rangle_{p_2}+ \\ +|0010\rangle_{1234}|\Phi_0^{(00)}\rangle_{p_1}|\Phi_0^{(10)}\rangle_{p_2}+ \\ +|1011\rangle_{1234}|\Phi_0^{(10)}\rangle_{p_1}|\Phi_1^{(00)}\rangle_{p_2}+ \\ +|1110\rangle_{1234}|\Phi_1^{(00)}\rangle_{p_1}|\Phi_0^{(10)}\rangle_{p_2}+ \\ +|2100\rangle_{1234}|\Phi_1^{(10)}\rangle_{p_1}|\Phi_0^{(00)}\rangle_{p_2}+ \\ +|0021\rangle_{1234}|\Phi_0^{(00)}\rangle_{p_1}|\Phi_1^{(10)}\rangle_{p_2}+\ldots \end{array} \right\}, \tag{8}$$

Next step is to project the total state (8) onto one of the macroscopic states. For the purpose, we employ the Bell state measurement setup with auxiliary generated modes. According to the Figure, the first and third auxiliary modes are directed to one Hadamard gate, while the second and fourth modes are directed to other Hadamard gate. Hadamard gate produces the following transformations

$$|10\rangle_{ij} \rightarrow \frac{1}{\sqrt{2}}\{|10\rangle + |01\rangle\}_{ij}, \quad (9a)$$

$$|01\rangle_{ij} \rightarrow \frac{1}{\sqrt{2}}\{|10\rangle - |01\rangle\}_{ij}, \quad (9b)$$

$$|11\rangle_{ij} \rightarrow \frac{1}{\sqrt{2}}\{|20\rangle - |02\rangle\}_{ij}, \quad (9c)$$

$$|20\rangle_{ij} \rightarrow \frac{1}{\sqrt{2}}\{|20\rangle + \sqrt{2}|11\rangle + |02\rangle\}_{ij}, \quad (9d)$$

$$|02\rangle_{ij} \rightarrow \frac{1}{2}\{|20\rangle - \sqrt{2}|11\rangle + |02\rangle\}_{ij}. \quad (9e)$$

Then, the state $|\Psi_{OUT}\rangle$ (Eq. (8)) is transformed into $|\Psi'_{OUT}\rangle$ which has the form

$$|\Psi'_{OUT}\rangle = \frac{1}{\sqrt{2}}\times$$

$$\times\left\{\begin{array}{l} \sqrt{\frac{p_{11}^{(+)}}{2}}|1000\rangle_{1234}|\Delta_+\rangle_{p_1p_2}+ \\ +\sqrt{\frac{p_{11}^{(-)}}{2}}|0010\rangle_{1234}|\Delta_-\rangle_{p_1p_2}+ \\ +\sqrt{\frac{p_{23}^{(1)}}{2}}|2100\rangle_{1234}|\Omega_1\rangle_{p_1p_2}+ \\ +\sqrt{\frac{p_{23}^{(2)}}{2}}|2001\rangle_{1234}|\Omega_2\rangle_{p_1p_2}+ \\ +\sqrt{\frac{p_{23}^{(3)}}{2}}|0120\rangle_{1234}|\Omega_3\rangle_{p_1p_2}+ \\ +\sqrt{\frac{p_{23}^{(4)}}{2}}|0021\rangle_{1234}|\Omega_4\rangle_{p_1p_2}+ \\ +\sqrt{\frac{p_{33}^{(-)}}{2}}|1110\rangle_{1234}|\Xi_-\rangle_{p_1p_2}+ \\ +\sqrt{\frac{p_{33}^{(+)}}{2}}|1011\rangle_{1234}|\Xi_+\rangle_{p_1p_2}+\ldots \end{array}\right\}, \quad (10)$$

where we introduce the following macroscopic entangled states

$$|\Delta_\pm\rangle_{p_1p_2} = \frac{|\Phi_0^{(10)}\rangle_{p_1}|\Phi_0^{(00)}\rangle_{p_2} \pm |\Phi_0^{(00)}\rangle_{p_1}|\Phi_0^{(10)}\rangle_{p_2}}{\sqrt{p_{11}^{(\pm)}}}, \quad (11)$$

$$|\Omega_1\rangle_{p_1p_2} = \left(\begin{array}{l} |\Phi_0^{(10)}\rangle_{p_1}|\Phi_1^{(00)}\rangle_{p_2} + |\Phi_1^{(00)}\rangle_{p_1}|\Phi_0^{(10)}\rangle_{p_2}+ \\ +\frac{1}{2}(|\Phi_1^{(10)}\rangle_{p_1}|\Phi_0^{(00)}\rangle_{p_2} + |\Phi_0^{(00)}\rangle_{p_1}|\Phi_1^{(10)}\rangle_{p_2}) \end{array}\right)\Big/\sqrt{p_{23}^{(1)}}, \quad (12a)$$

$$|\Omega_2\rangle_{p_1p_2} = \left(\begin{array}{l} |\Phi_1^{(00)}\rangle_{p_1}|\Phi_0^{(10)}\rangle_{p_2} - |\Phi_0^{(10)}\rangle_{p_1}|\Phi_1^{(00)}\rangle_{p_2}+ \\ +\frac{1}{2}(|\Phi_1^{(10)}\rangle_{p_1}|\Phi_0^{(00)}\rangle_{p_2} - |\Phi_0^{(00)}\rangle_{p_1}|\Phi_1^{(10)}\rangle_{p_2}) \end{array}\right)\Big/\sqrt{p_{23}^{(2)}}, \quad (12b)$$

$$|\Omega_3\rangle_{p_1p_2} = \left(\begin{array}{l} -|\Phi_1^{(00)}\rangle_{p_1}|\Phi_0^{(10)}\rangle_{p_2} - |\Phi_0^{(10)}\rangle_{p_1}|\Phi_1^{(00)}\rangle_{p_2}+ \\ +\frac{1}{2}(|\Phi_1^{(10)}\rangle_{p_1}|\Phi_0^{(00)}\rangle_{p_2} + |\Phi_0^{(00)}\rangle_{p_1}|\Phi_1^{(10)}\rangle_{p_2}) \end{array}\right)\Big/\sqrt{p_{23}^{(3)}}, \quad (12c)$$

$$|\Omega_4\rangle_{p_1p_2} = \left(\begin{array}{l} |\Phi_0^{(10)}\rangle_{p_1}|\Phi_1^{(00)}\rangle_{p_2} - |\Phi_1^{(00)}\rangle_{p_1}|\Phi_0^{(10)}\rangle_{p_2}+ \\ +\frac{1}{2}(|\Phi_1^{(10)}\rangle_{p_1}|\Phi_0^{(00)}\rangle_{p_2} - |\Phi_0^{(00)}\rangle_{p_1}|\Phi_1^{(10)}\rangle_{p_2}) \end{array}\right)\Big/\sqrt{p_{23}^{(4)}}, \quad (12d)$$

$$|\Xi_\pm\rangle_{p_1p_2} = \frac{|\Phi_1^{(10)}\rangle_{p_1}|\Phi_0^{(00)}\rangle_{p_2} \pm |\Phi_0^{(00)}\rangle_{p_1}|\Phi_1^{(10)}\rangle_{p_2}}{\sqrt{p_{33}^{(\pm)}}}, \quad (13)$$

The normalized coefficients, in which the first subscript indicates the number of detectors that register coming photons while the second subscript indicates the total number of the detected photons, are given by

$$p_{11}^{(\pm)} = 2(\langle\Phi_0^{(00)}|\Phi_0^{(00)}\rangle\langle\Phi_0^{(10)}|\Phi_0^{(10)}\rangle \pm |\langle\Phi_0^{(00)}|\Phi_0^{(10)}\rangle|^2), \quad (14)$$

$$\begin{aligned} p_{23}^{(1)} = {} & 2\langle\Phi_0^{(10)}|\Phi_0^{(10)}\rangle\langle\Phi_1^{(00)}|\Phi_1^{(00)}\rangle+ \\ & +2\langle\Phi_0^{(10)}|\Phi_1^{(00)}\rangle\langle\Phi_1^{(00)}|\Phi_0^{(10)}\rangle+ \\ & +\sqrt{2}\langle\Phi_0^{(10)}|\Phi_1^{(10)}\rangle\langle\Phi_1^{(00)}|\Phi_0^{(00)}\rangle+ \\ & +\sqrt{2}\langle\Phi_0^{(10)}|\Phi_0^{(00)}\rangle\langle\Phi_1^{(00)}|\Phi_1^{(10)}\rangle+ \\ & +\sqrt{2}\langle\Phi_0^{(00)}|\Phi_1^{(00)}\rangle\langle\Phi_1^{(10)}|\Phi_0^{(10)}\rangle+ \\ & +\sqrt{2}\langle\Phi_0^{(00)}|\Phi_0^{(10)}\rangle\langle\Phi_1^{(10)}|\Phi_1^{(00)}\rangle+ \\ & +\langle\Phi_0^{(00)}|\Phi_0^{(00)}\rangle\langle\Phi_1^{(10)}|\Phi_1^{(10)}\rangle+ \\ & +\langle\Phi_0^{(00)}|\Phi_1^{(10)}\rangle\langle\Phi_1^{(10)}|\Phi_0^{(00)}\rangle, \end{aligned} \quad (15a)$$

$$\begin{aligned} p_{23}^{(2)} = {} & 2\langle\Phi_0^{(10)}|\Phi_0^{(10)}\rangle\langle\Phi_1^{(00)}|\Phi_1^{(00)}\rangle- \\ & -2\langle\Phi_0^{(10)}|\Phi_1^{(00)}\rangle\langle\Phi_1^{(00)}|\Phi_0^{(10)}\rangle- \\ & -\sqrt{2}\langle\Phi_0^{(10)}|\Phi_1^{(10)}\rangle\langle\Phi_1^{(00)}|\Phi_0^{(00)}\rangle+ \\ & +\sqrt{2}\langle\Phi_0^{(10)}|\Phi_0^{(00)}\rangle\langle\Phi_1^{(00)}|\Phi_1^{(10)}\rangle- \\ & +\sqrt{2}\langle\Phi_0^{(00)}|\Phi_1^{(00)}\rangle\langle\Phi_1^{(10)}|\Phi_0^{(10)}\rangle+ \\ & +\sqrt{2}\langle\Phi_0^{(00)}|\Phi_0^{(10)}\rangle\langle\Phi_1^{(10)}|\Phi_1^{(00)}\rangle+ \\ & +\langle\Phi_0^{(00)}|\Phi_0^{(00)}\rangle\langle\Phi_1^{(10)}|\Phi_1^{(10)}\rangle- \\ & -\langle\Phi_0^{(00)}|\Phi_1^{(10)}\rangle\langle\Phi_1^{(10)}|\Phi_0^{(00)}\rangle, \end{aligned} \quad (15b)$$

$$p_{23}^{(3)} = 2\langle\Phi_0^{(10)}|\Phi_0^{(10)}\rangle\langle\Phi_1^{(00)}|\Phi_1^{(00)}\rangle+$$

$$+2\langle\Phi_0^{(10)}|\Phi_1^{(00)}\rangle\langle\Phi_1^{(00)}|\Phi_0^{(10)}\rangle-$$
$$-\sqrt{2}\langle\Phi_0^{(10)}|\Phi_1^{(10)}\rangle\langle\Phi_1^{(00)}|\Phi_0^{(00)}\rangle-$$
$$-\sqrt{2}\langle\Phi_0^{(10)}|\Phi_0^{(00)}\rangle\langle\Phi_1^{(00)}|\Phi_1^{(10)}\rangle-$$
$$-\sqrt{2}\langle\Phi_0^{(00)}|\Phi_1^{(00)}\rangle\langle\Phi_1^{(10)}|\Phi_0^{(10)}\rangle-$$
$$-\sqrt{2}\langle\Phi_0^{(00)}|\Phi_0^{(10)}\rangle\langle\Phi_1^{(10)}|\Phi_1^{(00)}\rangle+$$
$$+\langle\Phi_0^{(00)}|\Phi_0^{(00)}\rangle\langle\Phi_1^{(10)}|\Phi_1^{(10)}\rangle+$$
$$+\langle\Phi_0^{(00)}|\Phi_1^{(10)}\rangle\langle\Phi_1^{(10)}|\Phi_0^{(00)}\rangle, \qquad (15c)$$

$$p_{23}^{(4)} = 2\langle\Phi_0^{(10)}|\Phi_0^{(10)}\rangle\langle\Phi_1^{(00)}|\Phi_1^{(00)}\rangle-$$
$$-2\langle\Phi_0^{(10)}|\Phi_1^{(00)}\rangle\langle\Phi_1^{(00)}|\Phi_0^{(10)}\rangle+$$
$$+\sqrt{2}\langle\Phi_0^{(10)}|\Phi_1^{(10)}\rangle\langle\Phi_1^{(00)}|\Phi_0^{(00)}\rangle-$$
$$-\sqrt{2}\langle\Phi_0^{(10)}|\Phi_0^{(00)}\rangle\langle\Phi_1^{(00)}|\Phi_1^{(10)}\rangle+$$
$$+\sqrt{2}\langle\Phi_0^{(00)}|\Phi_1^{(00)}\rangle\langle\Phi_1^{(10)}|\Phi_0^{(10)}\rangle-$$
$$-\sqrt{2}\langle\Phi_0^{(00)}|\Phi_0^{(10)}\rangle\langle\Phi_1^{(10)}|\Phi_1^{(00)}\rangle+$$
$$+\langle\Phi_0^{(00)}|\Phi_0^{(00)}\rangle\langle\Phi_1^{(10)}|\Phi_1^{(10)}\rangle-$$
$$-\langle\Phi_0^{(00)}|\Phi_1^{(10)}\rangle\langle\Phi_1^{(10)}|\Phi_0^{(00)}\rangle, \qquad (15d)$$

$$p_{33}^{(\pm)} = 2(\langle\Phi_0^{(00)}|\Phi_0^{(00)}\rangle\langle\Phi_1^{(10)}|\Phi_1^{(10)}\rangle \pm |\langle\Phi_0^{(00)}|\Phi_1^{(10)}\rangle|^2. \qquad (16)$$

As can be seen from Eq. (10), different combinations of detectors participate in identification of the macroscopic entangled states. Namely, if detector D_1 registers one photon, then the state $|\Delta_+\rangle_{p_1p_2}$ is generated with success probability $P_{11}^{(+)} = |p_{11}^{(+)}|^2/4$. If detector D_3 registers one photon, then the state $|\Delta_-\rangle_{p_1p_2}$ is generated with success probability $P_{11}^{(-)} = |p_{11}^{(-)}|^2/4$. If any pair of detectors D_1, D_2, D_3, and D_4 register three photons, one of them registers one photon and other registers two photons, then the total state $|\Psi'_{OUT}\rangle$ (Eq. (10)) is projected onto one of the states $|\Omega_i\rangle_{p_1p_2}$ $(i = 1-4)$ with corresponding success probability $P_{23}^{(i)} = |p_{23}^{(i)}|^2/8$. The outcome of the Bell state measurement resulting in threefold simultaneous detection of three photons by three detectors, (one photon per one detector), reduces the total state $|\Psi'_{OUT}\rangle$ (Eq. (11)) either onto the state $|\Xi_-\rangle_{p_1p_2}$ with success probability $P_{33}^{(-)} = |p_{33}^{(-)}|^2/8$, or the state $|\Xi_+\rangle_{p_1p_2}$ with success probability $P_{33}^{(+)} = |p_{33}^{(+)}|^2/8$, and so on. For our purpose, we are interested in events when only one photon is registered in auxiliary modes.

Let us now introduce numerical characteristics for the macroscopic states [12]. To do it, we are going to decompose the wave amplitudes $f_{2l,1}^{(00)}(\eta)$ and $f_{2l+1,1}^{(10)}(\eta)$ into asymptotic series in parameter η that always takes very small values in real experiments ($\eta \ll 1$) [12]. Up to first nonvanishing order in parameter η, we have

$$f_{2l,1}^{(00)}(\eta) = 1 - \frac{l\eta^2}{2}, \qquad (17a)$$

$$f_{2l+1,k}^{(10)}(\eta) = 1 - l\eta^2. \qquad (17b)$$

Then, the macroscopic entangled states $|\Delta_\pm\rangle_{p_1p_2}$ can be rewritten after proper renormalization taking into account asymptotic decomposition (17a) and (17b) as

$$|\Delta_\pm\rangle_{p_1p_2} \Rightarrow |\Delta_\pm(\alpha,\eta)\rangle_{p_1p_2} =$$
$$= N_\pm(\alpha,\eta)(|\alpha,\eta,10\rangle_{p_1}|\alpha,\eta,00\rangle_{p_2}\pm$$
$$\pm|\alpha,\eta,00\rangle_{p_1}|\alpha,\eta,10\rangle_{p_2}), \qquad (18a)$$

with normalization coefficient

$$N_\pm \equiv N_\pm(\alpha,\eta) = \frac{1}{\sqrt{2(1\pm|\langle\alpha,\eta,00|\alpha,\eta,10\rangle|^2)}} \qquad (18b)$$

and

$$|\alpha,\eta,ij\rangle_{p_i} = \frac{|\alpha,\eta,ij)_{p_i}}{\sqrt{(\alpha,\eta,ij|\alpha,\eta,ij)}}, \qquad (18c)$$

where $ij = 00$, 10 is the index to distinguish the modified coherent states from each other and p_i means either p_1 or p_2, respectively. Modified due to $\chi^{(2)}$ nonlinearity coherent states (non-normalized) are given by

$$|\alpha,\eta,00)_{p_i} = \exp(-\alpha^2/2)\sum_{l=0}^{\infty}\frac{\alpha^l}{\sqrt{l!}}\left(1-\frac{l\eta^2}{2}\right)|l\rangle_{p_i} \equiv$$
$$\equiv \left[1-\frac{\alpha\eta^2}{2}\left(\alpha+\frac{\partial}{\partial\alpha}\right)\right]|\alpha\rangle_{p_i}, \qquad (18d)$$

$$|\alpha,\eta,10)_{p_i} = \exp(-\alpha^2/2)\sum_{l=0}^{\infty}\frac{\alpha^l}{\sqrt{l!}}\left(1-l\eta^2\right)|l\rangle_{p_i} \equiv$$
$$\equiv \left[1-\alpha\eta^2\left(\alpha+\frac{\partial}{\partial\alpha}\right)\right]|\alpha\rangle_{p_i}. \qquad (18e)$$

Using Eqs. (18d) and (18e), one obtains the following relations for the states

$$(\alpha,\eta,00|\alpha,\eta,00) \equiv_{p_i} (\alpha,\eta,00|\alpha,\eta,00)_{p_i} =$$
$$= 1-\alpha^2\eta^2+\frac{\alpha^2\eta^4}{4}(1+\alpha^2), \qquad (19a)$$

$$(\alpha,\eta,10|\alpha,\eta,10) \equiv_{p_i} (\alpha,\eta,10|\alpha,\eta,10)_{p_i} =$$
$$= 1-2\alpha^2\eta^2+\alpha^2\eta^4(1+\alpha^2), \qquad (19b)$$

$$(\alpha,\eta,00|\alpha,\eta,10) = (\alpha,\eta,10|\alpha,\eta,00) \equiv_{p_i}$$
$$\equiv_{p_i} (\alpha,\eta,00|\alpha,\eta,10)_{p_i} =_{p_i} (\alpha,\eta,10|\alpha,\eta,00)_{p_i} =$$
$$= 1-\frac{3\alpha^2\eta^2}{2}+\frac{\alpha^2\eta^4}{2}(1+\alpha^2). \qquad (19c)$$

The probabilities for the macroscopic entangled states $|\Delta_{\pm}\rangle_{p_1p_2}$ to be observed in total state (10) in leading order in η can be estimated as

$$P_{11}^{(+)} = 1 - 3\alpha^2\eta^2, \tag{20a}$$

$$P_{11}^{(-)} \cong \frac{\alpha^2\eta^4}{16}. \tag{20b}$$

As can be seen from Eqs. (20a), (20b), the probability to generate the state $|\Delta_{+}\rangle_{p_1p_2}$ is most probable and is almost equal to one, since the condition $\alpha\eta \ll 1$ holds in practice. Following [12], it is possible to estimate the values of concurrence of the states $|\Delta_{\pm}\rangle_{p_1p_2}$. So, the concurrence of the states $|\Delta_{+}\rangle_{p_1p_2}$ is given by

$$C(|\Delta_{+}\rangle_{p_1p_2}) = 2N_{+}^2(1-a^2) = \frac{1-a^2}{1+a^2} \approx \alpha^2\eta^4/8, \tag{21}$$

while the concurrence of the state $|\Delta_{-}\rangle_{p_1p_2}$ is equal to 1 $(C(|\Delta_{+}\rangle_{p_1p_2}))$ not depending on the values of α and η [12].

In conclusion, we have proposed optical scheme consisting of system of two spontaneous parametric down converters with type-I phase matching with one auxiliary photon in superposition state of two input modes combined with Bell state measurement arrangement to conditionally produce two types of the macroscopic entangled states. Pair of the Hadamard gates is used in identification of the outcomes of the states in auxiliary modes and, as consequence, allows identifying the macroscopic entangled states in output pumping modes. The studied scheme enables to conditionally produce macroscopic entangled state $|\Delta_{+}\rangle_{p_1p_2}$ with almost unit success probability but with small amount of entanglement. The same system provides us a possibility to conditionally get macroscopic entangled state $|\Delta_{-}\rangle_{p_1p_2}$ with small success probability but with concurrence equal one. The proposed scheme requires no photon-number resolving detection, i.e. only a “YES/NO” of one of two detectors suffices.

1. E. Schrödinger, Naturwissenschaften **23**, 807, 823, 844 (1935).
2. B. Yurke and D. Stoler, Phys. Rev. Lett. **57**, 13 (1986).
3. X. Wang, Phys. Rev. A **64**, 022302 (2001); S.J. van Enk and O. Hirota, Phys. Rev. A **64**, 022313 (2001); S.J. van Enk, Phys. Rev. A **67**, 022318 (2003).
4. H. Jeong and M. S. Kim, Phys. Rev. A **65**, 042305 (2002); T. C. Ralph, A. Gilchrist, G. J. Milburn et al., Phys. Rev. A **68**, 042319 (2003).
5. D. Wilson, H. Jeong, and M. S. Kim, J. Mod. Opt. **49**, 851 (2002); H. Jeong et al., Phys. Rev. A **67**, 012106 (2003).
6. H. Jeong and M. S. Kim, Quantum Inf. Comput. **2**, 208 (2002); J. Clausen, L. Knöll, and D.-G. Welsch, Phys. Rev. A **66**, 062303 (2002).
7. P. T. Cochrane, G. J. Milburn, and W. J. Munro, Phys. Rev. A **59**, 2631 (1999); S. Glancy, H. Vasconcelos, and T. C. Ralph, quant-ph/0311093.
8. T. Ralph, Phys. Rev. A **65**, 042313 (2002); W. J. Munro et al., ibid. **66**, 023819 (2002).
9. R. W. Boyd, J. Mod. Opt. **46**, 367 (1999).
10. B. C. Sanders and G. J. Milburn, Phys. Rev. A **45**, 1919 (1992); C. C. Gerry, Phys. Rev. A **59**, 4095 (1999).
11. H. Jeong, M. S. Kim, T. C. Ralph, and B. S. Ham, Phys. Rev. A **70**, 061801(R) (2004); A. P. Lund, H. Jeong, T. C. Ralph, and M. S. Kim, Phys. Rev. A **70**, 020101(R) (2004).
12. S. A. Podoshvedov, JETP **129**, to be published.