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Monopole Decay in a Variable External Field

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The rate of monopole decay into a dyon and an electron in an inhomogeneous external electric field is calculated by semiclassical methods. Comparison is made to an earlier result where this quantity was calculated for a constant field. Experimental and cosmological tests are suggested.

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1. Spontaneous and Induced Decay in External Fields. Spontaneous non-perturbative processes of particle production in QFT (Schwinger processes), or vacuum decay processes have been studied since the papers of Euler–Heisenberg and Schwinger on e^+e^- generation in a constant electromagnetic field [1, 2]. False vacuum decay in a scalar field theory with a stable and a metastable vacuum states was first treated in [3]. Later this problem was given a 1-loop treatment [4, 5], with both the exponent and the preexponential factor of decay probability calculated. Spontaneous monopole production in a constant magnetic field was studied in [6].

Consideration of induced Schwinger processes was a natural extension of the scope of the problems described above [7, 8]. The term "induced" denotes the situation in which the initial state is not vacuum, but rather contains some particle(s). False vacuum decay in a scalar and spinor field theory, induced by presence of an external particle was treated semiclassically in the 1-loop approximation in [9].

On the other hand, generalization of Schwinger processes can be thought of as extending the class of fields in which the appropriate process takes place. The original Euler and Heisenberg calculation in QED was performed for a constant field. For harmonic plane waves calculations had first been done by Schwinger in [2]. One can make sure that the same expression is true for adiabatically varying fields [10]. The effective Lagrangian also was calculated exactly in an electric field, dependent on time as $E(t) \sim 1/\cosh^2(\Omega t)$ in [11]. A semiclassical treatment of a broad class of fields was given in [12]. Semiclassical methods were further developed basing upon WKB approximation [13] and the so called "worldline instanton method" [6, 14, 15]. For a comprehensive review of recent developments in Euler – Heisenberg effective actions the reader may consult Dunne's review [16].

In the present short paper a combination of the both generalizations of Schwinger processes is suggested. The possibility of an induced monopole decay into a dyon and an electron was first suggested in [17]. Later it was calculated in a constant electric field up to the leading classical exponential factor [18].

The paper is organized as follows. In section 2 the general techniques of dealing particle decay in terms of Euclidean 1-particle path integral are described. The sub-barrier trajectories and the leading exponential factor are calculated in section 3. The circumstances under which the process considered might become significant for observers, are investigated in section 4. In section 5 the results are discussed.

2. Semiclassical Approximation to Path Integral. We are going to consider 't Hooft – Polyakov monopole and dyon. Monopole mass M_m is of the order of the scale M_W/α where M_W is W-boson mass, α coupling constant. At the same time, monopole and dyon sizes are of the order of magnitude M_W^{-1}, therefore in the weak coupling limit these particles are essentially classical objects.

On the other hand, 't Hooft – Polyakov monopole does not possess a well-established local field-theoretical description. That is why it is reasonable to treat monopole and dyon in terms of 1-particle theory, evaluating Feynman path integrals semiclassically with restriction to the trajectories of classical motion in imaginary (Euclidean) time, satisfying $r \gg M_W^{-1}$, where r is the typical trajectory size. It is essential to take into account only closed trajectories' contributions because only finite classical action configurations are relevant.

To find the monopole decay probability in an inhomogeneous field one has to calculate corrections to its propagator in the presence of the field.

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The full (1-loop) Green function of a monopole in an external field is the sum of all Green functions with arbitrary number of electron-dyon loop insertions

$$G(T,0) = G^{(0)}(T,0) + G^{(1)}(T,0) + G^{(2)}(T,0) + \cdots \quad (1)$$

The free propagator $G^{(0)}(T,0)$ corresponds to the diagram without the dyon-electron loop insertion, $G^{(1)}(T,0)$ denotes Green function with one insertion of the electron-dyon loop etc., see Fig.1. Note that the

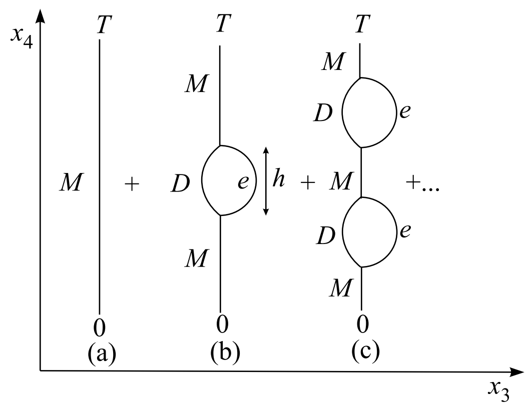

Fig.1. Full 1-loop Green function of a monopole is obtained by summing over all the insertions of electron-dyon loop into monopole's Euclidean trajectory

diagrams of Fig.1 are not Feynman graphs, but are simply classical trajectories in Ox_4x_3 plane. However, the resummation of all electron-dyon loop insertions resembles closely the analogous resummation of all one-particle irreducible vacuum polarization diagram.

Taking into account that the electric charge of the magnetic monopole is zero and using the first quantized approach, the propagator of the monopole can be written as follows

$$G^{(0)}(T,0) = \int \mathcal{D}y e^{-M_m \int_0^T \sqrt{\dot{y}^2} d\tau} \sim e^{-M_m T}\sqrt{\frac{M}{T^3}}, \quad (2)$$

where $y(t)$ is the monopole variable. The first correction comes from the Euclidean configuration labelled (b) in Fig.1.

Note that the electron and the dyon can go round the loop multiply, winding over it with some respective winding numbers m, n, so one should sum over them. However, let us limit ourselves to the term with the lowest winding numbers $m = 1$, $n = 1$. For these, the first correction due to the electron-dyon loop is given as

$$G^{(1)}(T,0) = \int \mathcal{D}y e^{-M_m \int \sqrt{\dot{y}^2} d\tau} \mathcal{D}x \mathcal{D}z e^{-S[x,z,A]}, \quad (3)$$

where action $S[x(u), z(v), A]$ is the action for the charged particles (the dyon and the electron) in the external field A_μ, $x(t)$ and $z(t)$ are respectively electron and dyon coordinates, their worldlines being parameterized by u, v

$$S[x,z,A] = m\int \sqrt{\dot{x}^2} du + ie\int A^{ext}(x)\dot{x} du + + M_d \int \sqrt{\dot{z}^2} dv - ie\int A^{ext}(z)\dot{z} dv. \quad (4)$$

Monopole coordinate is easily integrated out, because the static monopole does not interact with the electric field,

$$G^{(1)}(T,0) \sim \int e^{-M_m (T - h[x,z,A])} \mathcal{D}x \mathcal{D}z e^{-S[x,z,A]}, \quad (5)$$

here h is the vertical size of the loop (see Fig.1).

The remaining path integral has a more complicated structure. It always has zero modes, corresponding to shifts of the loop. This will contribute to the prefactor a multiplier arising due to the substitution of variables.

The other part of the prefactor will come from the non-zero modes. Among them there will be at least one negative mode, corresponding to the overall inflation of electron-dyon loop. Presence of the extra negative modes is not evident without a special investigation [18]. In general, the path integral yields

$$G^{(1)}(T,0) = \int d^4y G^{(0)}(x,y) G^{(0)}(y+\Delta y, z) K e^{-S_{cl}}, \quad (6)$$

where S_{cl} is the action of the dyon and the electron in the external field on a classical path with the proper boundary conditions (see Fig.1). The integral over d^4y is due to integrating over all loop positions, and the two free Green's functions belong to the purely monopole trajectories. The term Δy accounts for non-zero loop size and can be neglected.

How can (6) be put into a direct correspondence to the mass shift of the particle? If the full Green's function is

$$G(x,z) \approx \frac{1}{(2\pi)^4}\int \frac{e^{ik(x-z)}}{k^2 + m^2 + \delta m^2},$$

mass shift δm^2 being imaginary or real, then expanding in the powers of δm^2 one obtains the variation of Green's function

$$\delta G(x,z) = -\delta m^2 \int d^4y G^{(0)}(x,y) G^{(0)}(y+\Delta y, z). \quad (7)$$

Comparing this with (6), one makes sure that

$$\delta m^2 = K e^{-S_{cl}}.$$

3. Exponential factor. Path integral over electron and dyon coordinates will be done by steepest descent approximation, applying the world-line instanton method [14, 15] to the induced decay. To find the exponential factor one should solve classical equations of motion for dyon and electron, find closed Euclidean trajectories and minimize the action with regard to trajectory parameters. The single-pulse electric field directed along Ox_3 axis

$$E_3 = E_0 \frac{1}{\cosh^2(\omega t)} \tag{8}$$

will be considered. Equations of motion will be of the form

$$m \frac{d}{du}\frac{\dot{x}_\mu}{\sqrt{\dot{x}^2}} = -ieF_{\mu\nu}(x)\dot{x}_\nu, \quad M_d \frac{d}{dv}\frac{\dot{z}_\mu}{\sqrt{\dot{z}^2}} = ieF_{\mu\nu}(z)\dot{z}_\nu, \tag{9}$$

where m, M_d are electron and dyon masses respectively. Here dimensionless parameters $\gamma = m\omega/eE$, $\alpha = m/M_d$, $\beta = M_m/M_d$ are introduced. The solution [14] to these equations for the field

$$A_3(x_4) = -i\frac{E}{\omega}\tan(\omega x_4)$$

will be for the electron

$$x_3(u) = \frac{m}{eE}\frac{1}{\gamma\sqrt{1+\gamma^2}}\text{arcsinh}\left(\gamma\cos(2\pi u)\right) - a, \tag{10}$$

$$x_4(u) = \frac{m}{eE}\frac{1}{\gamma}\arcsin\left(\frac{\gamma}{\sqrt{1+\gamma^2}}\sin(2\pi u)\right),$$

$$u \in [-u_0, u_0],$$

where a, u_0, are some constants. The solution for dyon is analogous, with appropriate constants b, v_0, γ substituted by γ/α. If the action for these trajectories is calculated, one obtains a four-parametric expression, depending on a, b, u_0 and v_0. The action must be minimized according to these parameters, imposing boundary conditions $x_4(u_0) = z_4(v_0)$ for the trajectory to be closed. The latter can be introduced into the equations as constraints with Lagrange multipliers. This is equivalent to using a mechanical analogy (see [8]), m, M_d and M_m being forces acting in the vertex, the action minimum corresponding to equilibrium vertex configuration. Equilibrium condition is

$$\begin{aligned} m\cos\alpha_e(u_0) - M_d\cos\alpha_d(v_0) &= 0, \\ m\sin\alpha_e(u_0) + M_d\sin\alpha_d(v_0) &= M_m, \end{aligned} \tag{11}$$

$\alpha_e(u_0)$, $\alpha_d(v_0)$ are the inclination angles of corresponding trajectories in the vertex. For the electron one has $\tan\alpha_e = \sqrt{1+\gamma^2}\cot 2\pi u_0$. Using analogous relation for the dyon and (11) it can be easily found for $M_d^2 < m^2 + M_m^2$

$$2\pi u_0 = = \arctan\left[\frac{(1+\gamma^2)((M_m+m)^2-M_d^2)(M_d^2-(M_m-m)^2)}{(m^2+M_m^2-M_d^2)^2}\right]^{\frac{1}{2}},$$

$$2\pi v_0 = = \arctan\left[\frac{(1+\frac{\gamma^2}{\alpha^2})((M_m+m)^2-M_d^2)(M_d^2-(M_m-m)^2)}{(M_m^2+M_d^2-m^2)^2}\right]^{\frac{1}{2}}.$$

It follows immediately that for the boundary conditions to be properly satisfied, dyon mass should lie within the interval $M_m - m < M_d < M_m + m$. The action on the extremal trajectory is

$$S_{cl}(\gamma) = S_e(\gamma) + S_d(\gamma) - 2M_m x_4(u_0), \tag{12}$$

where, if $M_d^2 < m^2 + M_m^2$

$$S_e(\gamma) = \frac{2\pi m^2}{eE\gamma^2}\left(2u_0\sqrt{1+\gamma^2} - \frac{1}{\pi}\arctan\left(\frac{\tan 2\pi u_0}{\sqrt{1+\gamma^2}}\right)\right),$$

$$S_d(\gamma) = \frac{2\pi m^2}{eE\gamma^2}\left(2v_0\sqrt{1+\frac{\gamma^2}{\alpha^2}} - \frac{1}{\pi}\arctan\left(\frac{\tan 2\pi v_0}{\sqrt{1+\frac{\gamma^2}{\alpha^2}}}\right)\right). \tag{13}$$

For the case $M_d^2 > m^2 + M_m^2$

$$u_0 = \frac{1}{2} - \frac{1}{2\pi} \times \tag{14}$$

$$\times \arctan\left[\frac{(1+\gamma^2)((M_m+m)^2-M_d^2)(M_d^2-(M_m-m)^2)}{(m^2+M_m^2-M_d^2)^2}\right]^{\frac{1}{2}}.$$

Corresponding value of the action is

$$S_e(\gamma) = \tag{15}$$

$$= \frac{2\pi m^2}{eE\gamma^2}\left(2u_0\sqrt{1+\gamma^2} - \frac{1}{\pi}\arctan\left(\frac{\tan 2\pi u_0}{\sqrt{1+\gamma^2}}\right) - 1\right).$$

The relation Γ/Γ_0, where Γ_0 is monopole mass imaginary part (width) in a constant electric field is shown in Fig.2. One can easily see the increase of the particle width with the increase of γ. This increase becomes stronger when the difference between M_d and M_d grows.

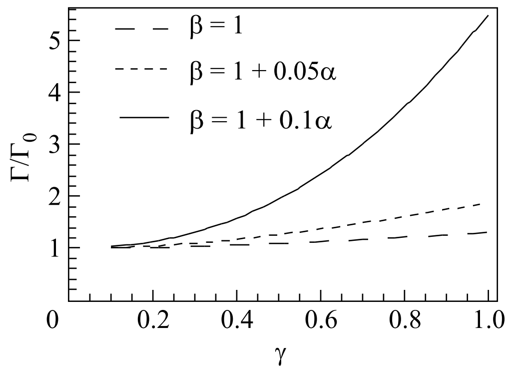

Fig.2. Γ/Γ_0 dependence on Keldysh parameter γ for the temporally inhomogeneous field

Action S tends to zero as $\gamma \to \infty$, therefore, at some point, resummation of higher winding number contributions (1) will be necessary. At sufficiently low S semiclassical approximation to the non-perturbative monopole decay becomes invalid.

As it was mentioned in section 2, one has a criterion for applicability of the first quantized approach: the loop should be large enough for dyon to be treated as a point-like object. That is, typical loop size

$$r_e = \begin{cases} \dfrac{1}{\gamma}\dfrac{m}{eE}, & \gamma \gg 1 \\ \dfrac{m}{eE}, & \gamma \ll 1 \end{cases},$$

should satisfy $r_e \gg M_W^{-1}$, that is, $\omega \ll M_W$, $E_{cr}/E \gg \frac{m}{M_W}$. Here $E_{cr} = m^2/e$ is the critical field value for electron. For imaginable processes in cosmology, field switch-on rate ω is obviously less then the enormous $100\,\mathrm{GeV} \sim 10^{26}\,\mathrm{Hz}$, thus in fact the first criterion is always satisfied. The second criterion means that $E \ll 10^8 E_{cr}$, which is valid even for most of magnetic stars and Reissner-Nordstrom black holes.

However, from general arguments it becomes clear that semiclassical approximation is wrong when $S \ll\ll 1$ [14]. Therefore, the general considerations might give us a more strict criterion of applicability of the formula (12) than monopole size considerations.

3.1. *Preexponential: Negative modes.* We will limit ourselves to qualitative considerations here. The presence of the zero mode was already discussed and used in the resummation above. It can be assumed, due to the fact that for $\gamma \to 0$ the situation is totally analogous to that described in [18]: at sufficiently small γ if $M_d^2 < M_m^2 + m^2$ there is always one negative dilatational mode, corresponding to the overall inflation of the loop. It provides a possibility for monopole mass to acquire an imaginary part. If $M_d^2 > M_m^2 + m^2$, another negative mode comes into existence (see [18]), thus making loop contribution to the monopole mass real and describing mass renormalization of a stable monopole.

3.2. *Spatially inhomogeneous field.* It has been shown [14] that in a spatially inhomogeneous field particle decay probability can be obtained by an analytic continuation of the result for the field with the same temporal inhomogeneity

$$\gamma \to i\gamma. \tag{16}$$

See also [19, 20] for a discussion of this analytic continuation. The general fact on spontaneous (induced) processes in spatially inhomogeneous fields is that the imaginary part of the effective action (particle width) decreases with the increase of γ. Physically this fact can be interpreted easily: the field characteristic size becomes too narrow, so that no pair of particles can travel a path long enough to gain energy necessary for leaving the barrier. In this paper we come to the same conclusion from (17). The explicit formula for the action reads

$$S_{cl}(\gamma) = S_e(i\gamma) + S_d(i\gamma) - 2M_m x_4(u_0, i\gamma),$$

where

$$S_e(i\gamma) = \tag{17}$$

$$= \frac{2\pi m^2}{eE\gamma^2}\left(-2u_0\sqrt{1-\gamma^2} + \frac{1}{\pi}\arctan\left(\frac{\tan 2\pi u_0}{\sqrt{1-\gamma^2}}\right)\right),$$

dyon action $S_d(i\gamma)$ given by an analogous expression.

Following the simple analytic continuation, the exponential factor for $E \sim 1/\cosh^2(\omega x_3)$ is shown. The ratio of monopole width in the spatially inhomogeneous field with parameter γ to its width in the constant field

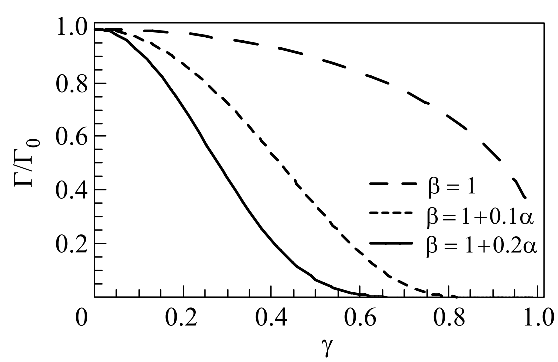

Fig.3. Γ/Γ_0 dependence on Keldysh parameter γ for the spatially inhomogeneous field

case is plotted in Fig.3. The process of monopole decay evidently becomes infinitely suppressed as $\gamma \to 1$. At higher values of inhomogeneity parameter $\gamma > 1$ decay is forbidden. This corresponds fully to the earlier results of [14] for spontaneous pair creation.

4. Monopoles in Cosmology and High-Energy Physics. The issue of magnetic monopoles, present in almost every 4-dimensional non-abelian gauge theory with a scalar Higgs field [21, 22] has long been an important problem in high-energy physics. Violation of baryon number conservation law in the presence of a magnetic monopole [23], charge quantization [24] are just a few features of monopole physics.

In particular, one should pay attention to the possible monopole decay in extremal Reissner–Nordstrom black holes, as these objects may create large electric fields (up to $2 \cdot 10^{12}$ Gs [25]). Here constant field limit $\gamma = 0$ is valid, and the value of the typical action (12) is $S \sim 10$, so that monopole decay is not infinitely suppressed by the factor of $e^{-S} \sim 10^{-4.3}$.

Let us estimate realistic values of Keldysh parameter for possible applications

$$\gamma = \frac{10^{-19} \ s}{\tau} \frac{E_{cr}}{E},$$

where τ is characteristic inhomogeneity time, E typical field value, and E_{cr} critical field value for electron ($4 \cdot 10^{13}$ Gs). The most rapid processes observed in the modern Universe have to do with pulsars and can have $\tau \sim 10^{-3}\, s$. On the other hand, typical magnetic field of a pulsar can reach the order of $10^{-1} E_{cr}$. Thus γ is extremely small in all known astrophysical processes and the stationary approximation from [18] can be used.

In terrestrial conditions, lasers with $\tau \sim$ $\sim 10^{-15} \cdots 10^{-16}$, and $E \sim 10^{-3} \cdots 10^{-5} E_{cr}$ could be within the reach of modern experimentalists. Here an interesting range of γ values $\gamma \sim 1$ is reached, where the non-stationary field limit is of some use. Typical action in this case is $S \sim 10^3$, which still suppresses monopole decay. With gamma-lasers one might hope for diminishing τ by 2–3 orders of magnitude and reach values $S \sim 1$. However, such parameters are out of reach at present time. Moreover, the semiclassical approximation breaks down at low values of S, as mentioned already.

5. Conclusion. In this short Letter the description of the induced monopole decay has been generalized to the non-stationary field case. Comparing to the stationary field configuration, it can be concluded that monopole decay is enhanced by the temporally-inhomogeneous field and suppressed by the spatially inhomogeneous field. Despite being non-perturbatively suppressed, this process might take place under some exotic conditions, e.g. in Reissner–Nordstrom black holes and in gamma or x-ray lasers.

Non-stationarity of the field becomes the key factor in the latter case, allowing the classical action on the Euclidean closed worldline to become sufficiently low and thus cease to suppress monopole decay. On the other hand, constant field approximation from [18] has been shown to remain valid we even for the most rapid processes in cosmology.

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