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Ultrashort Gaussian pulse-width expansion and shape deformation induced by group velocity dispersion

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Pulse-width expansion and pulse shape deformation of an ultra-short Gaussian pulse induced by both low and high order group velocity dispersion was theoretically analyzed in terms of energy conservation and coupled equations for three wave radiations. As an example, the optical parametric interaction processes in a negative uniaxial crystal $CsLiB_6O_{10}$ with 50 fs of ultra-short Gaussian pulse were simulated. The results indicate that the degree of the pulse expansion induced by low and high order group velocity dispersion is determined by both the wavelength of the incident wave and crystal length. A pulse could be expanded to 1.41 times than its initial value as a crystal length equals to the dispersion length, and further heavily expanded with decreasing wavelength and increasing crystal length. The pulse expansion induced by high order group velocity dispersion using incident wavelength 213 nm is 1.6 times than that using 532 nm in 50 fs pulse width without chirp modulation, and the symmetry deformation and the frequency pushing phenomena of ultra-short pulse shape is also found.

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1. Introduction. High repetitive laser with ultrashort pulse width, ultrashort wavelength, or near mid-IR wavelength are becoming indispensable radiation sources in many areas, particularly femtosecond (fs) ultraviolet pulsed laser is considered as a kind of effective exciting and detecting light source [1 – 3]. However, such fs laser pulse with violet or ultraviolet can not be produced directly using suitable solid state lasers, therefore, it is promising to use nonlinear optical frequency conversion technique that has become a means [4 – 6]. Recently some optical crystals have been grown and widely applied in producing ultrashort pulse with the desired laser wavelength. For example, $CsLiB_6O_{10}$ (CLBO) and $KBe_2BO_3F_2$(KBBF) [7, 8] were used to produce violet or deeply ultraviolet laser while $AgGaSe_2$ (AGSe), $AgGaS_2$ (AGS) and $ZnGeP_2$ (ZGP) [9 – 11] used to produce near-mid-far IR laser. The low and high order group velocities (GV) can be dispersed in nonlinear optical crystal during the interaction between Gaussian and quasi-Gaussian laser pulses, and its effect on the expansion of pulse width is becoming one of the hot topics [12]. In this paper, we focus on nonlinear optical crystal CLBO, and theoretical analyze and simulate the parametric interaction process, and the pulse expansion by low and high order group velocity dispersion (GVD). The correlation of the ultrashort pulse width expansion with the polarization state, the wavelengths of ultrashort pulse laser and dispersion length is simulated. The expansion of the pulse width in ultrashort pulse ordinary light induced by GVD is larger than that in extraordinary light propagating in nonlinear optical crystal. On the other hand, the pulse expansion induced by high order GVD using 213 nm in nonlinear crystal is 1.6 times than that using 532 nm in case of 50 fs pulse width without chirp modulation, and the symmetry deformation of ultra-short pulse shape have been presented.

2. Theoretical analysis and simulation. 2.1. Group velocity dispersion. When an ultrashort pulse is propagating in a nonlinear optical crystal, the propagation constant $\mathbf{k}$ at the center frequency ω_0 can be expanded as [13],

$$k(\omega) = k(\omega_0) + \left(\frac{\partial k(\omega)}{\partial \omega}\right)_{\omega_0} \delta\omega + \frac{1}{2}\left(\frac{\partial^2 k(\omega)}{\partial \omega^2}\right)_{\omega_0} \delta\omega^2 + \frac{1}{6}\left(\frac{\partial^3 k(\omega)}{\partial \omega^3}\right)_{\omega_0} \delta\omega^3 + \ldots = k(\omega_0) + \frac{1}{\beta_1}\delta\omega + \frac{1}{2\beta_2}\delta\omega^2 + \frac{1}{6\beta_3}\delta\omega^3 + \ldots \quad (1)$$

where β_1, β_2 and β_3 are GV delay coefficients, low order and high order GVD coefficient, respectively. The GV ν_g of the radiation propagating in the media is $\nu_g = 1/\beta_1$,

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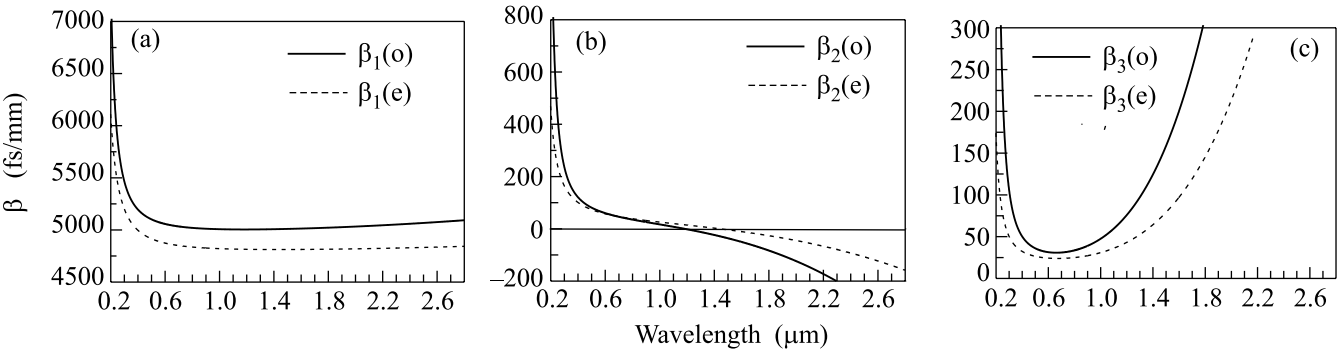

Fig.1. Group velocity dispersion coefficient for ultrashort in CLBO: (a) group velocity delay β_1, (b) low order β_2, (c) high order β_3

which is known that the propagation velocity of radiation wave ν is related to the refractive index by $\nu = c/n$, similarly we can define the GV index by $\nu_g = c/m$, then

$$m = \frac{c}{\nu_g} = n - \lambda \frac{dn}{d\lambda}, \tag{2}$$

where the GV index m_o for ordinary light is independent of the propagation direction, the index $m_e(\theta)$ for extraordinary light is a function of the propagating direction θ given by

$$m_e(\theta) = \left(\frac{\cos^2\theta}{m_e^2} + \frac{\sin^2\theta}{m_o^2} \right)^{-1/2}, \tag{3}$$

where m_o and m_e are the main GV index at $\theta = 0$ and $\theta = 90°$, respectively. Combing Eq. (2), (3) and the dispersion equation of CLBO crystal at temperature of 20 °C [13], we can obtain the dependent curves of wavelength with β_1, β_2 and β_3 respectively as shown in Fig.1. Clearly from Fig.1b, β_2 is a negative for both extraordinary light at $\lambda > 1.44\,\mu$m and ordinary light at $\lambda > 1.183\,\mu$m. This result give a substantial hint in further experimental investigations, a significant role of the symbol of the dispersion for different wavelengths cannot be neglected when considering the dispersive compensation or the effect of the chirp. On the other hand, ultrashort pulse propagates for GV, and the different frequency in the media will propagate with different velocity due to GVD effect. In the normal dispersive region of $\beta_2 > 0$, red beam is propagating faster than that of blue and the opposite situation happens in the abnormal dispersive region of $\beta_2 < 0$. If all the frequency spectrum propagate with the same speed saying $\beta_2 = 0$, the pulse width will keep constant. Any delay during the propagation process of all frequency will cause the expansion of the pulse width.

2.2. Expansion of the ultrashort Gaussian pulse.

2.2.1. Without chirp modulation. The radiation field of ultrashort Gaussian pulse at the beginning without chirp modulation can be described as

$$U(0,T) = \exp(-T^2/2T_0^2), \tag{4}$$

where T_o is half-width when pulse intensity decreases to $1/e$ of its maximum. In most cases we use the full width half maximum T_{FWHM} instead of T_o, for a Gaussian pulse, T_{FWHM} correlates with T_o by $T_{\mathrm{FWHM}} = 2(\ln 2)^{1/2} T_0$, the amplitude of pulse propagating in the media can be derived as [14]

$$U(z,T) = \left(\frac{T_0^2}{T_0^2 - i\beta_2 z} \right)^{1/2} \exp\left(-\frac{T^2}{2(T_0^2 - i\beta_2 z)} \right). \tag{5}$$

From Eq. (5), the ultrashort pulse still keeps its Gaussian form, but its pulse width increases to

$$T_i = T_0(1 + (z/L_D)^2)^{1/2}, \tag{6}$$

where

$$L_D = T_0^2/|\beta_2|, \tag{7}$$

here L_D is called as dispersive length. From Eq. (6) we know that GVD can induce the expansion of the pulse width, and the degree of the expansion is determined by the dispersive length L_D. In the media with a certain length, the pulse width is expanded largely with decreasing L_D. When Z equals to L_D, the pulse width is increased to 1.41 times of its initial value. Number simulations of $\lambda = 266$ nm pulse propagating in CLBO crystal with different crystal length at $T_o = 50$ fs are shown in Fig.2. It is evident that pulse width is gradually expanded due to GVD effect with increasing crystal length. Typically expanded pulses for o-light and e-light are displayed in Fig.3, where o-light shows a heavier expan-

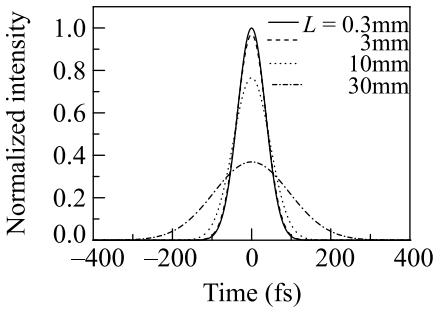

Fig.2. Pulse-width expanded in different length (in λ = = 266 nm, $T_0 = 50$ fs

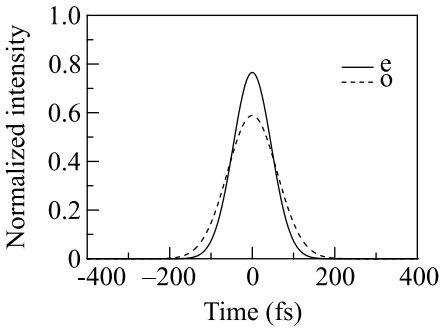

Fig.3. Pulse-width expanded on o- and e-light ($z = 10$ mm, $\lambda = 266$ nm, $T_0 = 50$ fs)

sion compared with e-light in CLBO crystal. Supposing crystal length $Z = 10$ mm, the ultrashort pulse shape with different wavelength propagating in CLBO crystal was shown in Fig.4. Clearly the expansion increases

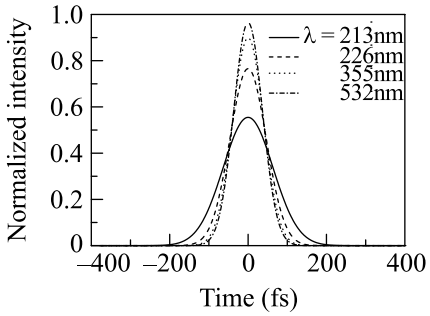

Fig.4. Pulse expanded shape on different ultrashort pulse ($z = 10$ mm, $T_0 = 50$ fs)

with decreasing wavelength. Comparing Eq. (4) with (5), we can see that chirp modulation can be induced when an incident Gaussian pulse without chirp modulation propagates in nonlinearly optical media. Eq. (5) can be changed into,

$$U(z,T) = |U(z,T)| \exp[i\phi(z,T)], \tag{8}$$

where

$$\phi(z,T) = -\frac{\mathrm{sgn}(\beta_2)(z/L_D)}{1+(z/L_D)^2}\frac{T^2}{T_0^2} + \tan^{-1}(z/L_D). \tag{9}$$

Here sgn is a symbol function. Obviously the phase $\phi(z,T)$ in Eq. (9) will change with changing time, indicating that there is a difference of $\delta\omega$ between respectively simultaneous frequency and central frequency ω_o,

$$\delta\omega = -\frac{\partial\phi}{\partial T} = \frac{2\mathrm{sgn}(\beta_2)(z/L_D)}{1+(z/L_D)^2}\frac{T}{T_0^2}. \tag{10}$$

The symbol of the tuning frequency $\delta\omega$ is determined by the symbol of GVD related parametric β_2. In a wave-front with $T < 0$, $\delta\omega$ is negative for the normal dispersive region of $\beta_2 > 0$, and becomes positive in abnormal dispersive region of $\beta_2 < 0$.

2.2.2. With chirp modulation and high order GVD. The radiation field of ultrashort Gaussian pulse at the beginning with chirp modulation can be described as

$$U(0,T) = \exp\left[-\frac{(1+iC)}{2}\frac{T^2}{T_0^2}\right]. \tag{11}$$

Here C is chirp parameter. From Eq. (8), it is indicated that the simultaneous frequency increases linearly from wave-front to wave-back of pulse when C is more than 0; while from pulse wave-back to front if C is less than 0. The symbol of the chirp parameter C can determine that of chirp modulation, and the magnitude of C can be estimated by Gaussian pulse. The spectral half width can be obtained from the Fourier transformation of Eq.(11) as $\Delta\omega = (1+C^2)^{1/2}/T_0$. When there is a chirp modulation, spectral width changes with changing chirp. Therefore by measuring $\Delta\omega$ and T_0 we can obtain the magnitude of chirp modulation from as follows,

$$U(z,T) = \left[\frac{T_0^2}{T_0^2 - i\beta_2(1+iC)}\right]^{1/2} \times$$

$$\times \exp\left[-\frac{(1+iC)T^2}{2[T_0^2 - i\beta_2 z(1+iC)]}\right]. \tag{12}$$

Now we can see from Eq. (12) that even there is chirp modulation, pulse still propagates keeping its Gaussian form. The ratio of its width T and initial pulse width T_0, which was defined as broadening factor, can be expressed as

$$\frac{T}{T_0} = \left[\left(1+\frac{C\beta_2 z}{T_0^2}\right)^2 + \left(\frac{\beta_2 z}{T_0^2}\right)^2\right]^{1/2}. \tag{13}$$

The change of pulse width is determined by both GVD parameter of β_2 and chirp parameter of C as evident

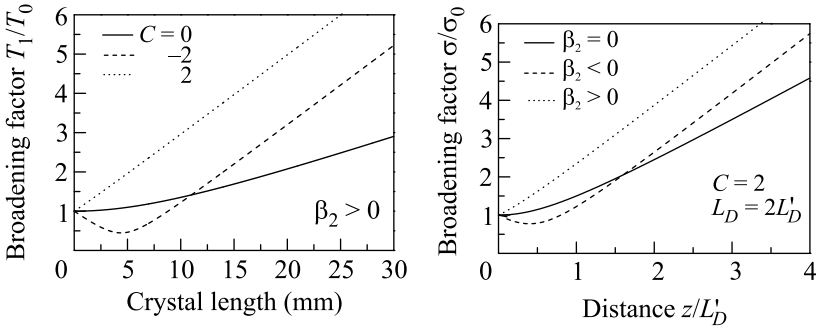

Fig.5. Expanded factor with crystall length: (a) in chirp modulation, $\lambda = 266\,\text{nm}$, (b) in high order dispersion

by Eq. (13). The Gaussian pulse width increases with increasing propagation distance if $\beta_2 C > 0$, while if $\beta_2 C < 0$, the pulse width decrease in the initial stage, and then increase with further increasing propagation distance after it reaches a minimum at

$$z_{\min} = \frac{|C|}{1+C^2} L_D,$$

the pulse width at this minimum equals to

$$T^{\min} = \frac{T_0}{(1+C^2)^{1/2}}.$$

The change of expansion factor as function of CLBO crystal length was simulated as shown in Fig.5a in $\beta > 0$ and $C = -2$, 0, and 2. It is clearly see that the pulse width is always expanded at $C = 2$. On the other hand at $C = -2$, the pulse width gradually decreases to a minimum with a $T/T_0 = 0.4472$ at $z = 4.4\,\text{mm}$, and then broadens with further increasing z. When the pulse is initially modulated by chirp and the condition of $\beta_2 C < 0$ is satisfied, because the symbol of the chirp induced by the dispersion is oppositely to that of the initial chirp, the net chirp will result in narrow pulse width until the minimum appears at these two chirps counteract each other. With further increasing propagation distance, the chirp induced by dispersion will overtake the initial chirp and become dominant, leading to the broadening of pulse width. In most cases, it is accurate enough to describe the Gaussian and quasi-Gaussian pulse width expansion only considering low order GVD. However, zero dispersion exists in nonlinear optical crystal at some dispersive wavelengths λp with low order dispersion coefficient $\beta_2 = 0$, so the effect of the high order β_3 on the pulse width expansion is dominant. Moreover, for an ultrashort pulse with a width less than 100 fs, we must consider the effect of β_3 even $\beta_2 \neq 0$. Neglecting the nonlinear interaction in the media, β_2 is zero when $\lambda = \lambda_D$, similar to $Z_{\min}$ we define the dispersion length for the high order GVD for $L'_D = T_0^3/|\beta_3|$. The pulse expansion factor can be described as [15]

$$\frac{\sigma}{\sigma_0} = \left[\left(1+\frac{C\beta_2 z}{T_0^2}\right)^2 + \left(\frac{\beta_2 z}{T_0^2}\right)^2 + (1+C^2)\left(\frac{\beta_3 z}{2T_0^3}\right)\right]^{1/2}, \tag{14}$$

where $\sigma_0 = T_0/2$, and T_0 is initial width. From Eq. (14) we know that both β_2 and β_3 affect the pulse expansion during the optical parametric interaction processes although they have different chirp modulation parameter C dependence. The distribution of β_2 to pulse expansion depends on the symbol of $\beta_2 C$, while the distribution of β_3 to the pulse expansion is independent of the symbols of β_3 and C. Supposing the pulse is modulated by the chirp parameter $C = 2$, Fig.5b presents the change of Gaussian expansion factor induced by high order GVD as a function of z/L'_D. The effect of high order GVD on the propagation of ultrashort Gaussian pulse in initially without chirp modulation was shown in Fig.6. It

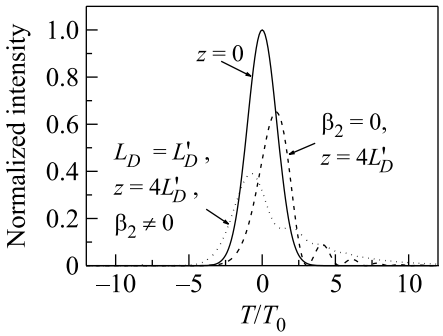

Fig.6. Symmetry deformation of Gaussian pulse width expanded in high order dispersion

can be seen that the symmetry of the Gaussian pulse is destroyed due to the effect of high order GVD at $\beta_2 = 0$, and side band vibrations appear with gradually decreasing intensity to zero. In contrast, such side band vibrations weaken obviously if $\beta_2 \neq 0$.

3. Conclusions. Although ultrashort pulse laser with different wavelength has been realized using nonlinear optical parametric effect so far, further experiments are still necessary to obtain ultrashort pulse laser with high quality beam. The pulse-width expanded and shape deformation has been changed for the ultrashort pulse of a high intensity, high repetition rate and interaction of parametric process in nonlinear medium. The expansion of ultrashort Gaussian pulse width induced by GVD in negative uniaxial crystal $CsLiB_6O_{10}$ has been theoretical analyzed. We derived that when an ultrashort width is less than 100 fs, the expansion of the pulse width in ultrashort pulse o-light induced by GVD is larger than that in e-light. Due to GVD and delay, the expansion increases with decreasing wavelength. When crystal dimension is equal to the dispersion length, the pulse width is increased to 1.41 times of its initial value, and such expansion becomes obvious with short dispersion length. We also pointed out a significant role that high order GVD is playing. The pulse expansion factor in an ultrashort Gaussian pulse induced by high order GVD increase linearly with increasing z/L'_D, and the pulse shape symmetry of the expanded pulse is destroyed by high order GVD with an appearance of side band around this pulse, therefore we conclude that high order GVD is not negligible to get novel all solid state ultrashort laser device with high quality beam. These results will be helpful to further investigation on optical parametric device of ultrashort pulse laser in terms of negative uniaxial crystal.

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