

Optical echo in photonic crystals

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Submitted 22 December 2006

The dynamics of photonic wavepacket in the effective oscillator potential is studied. The oscillator potential is constructed on a base of one dimensional photonic crystal with a period of unit cell adiabatically varied in space. The structure has a locally equidistant discrete spectrum. This leads to an echo effect, i.e. the periodical reconstruction of the packet shape. The effect can be observed in a nonlinear response of the system. Numerical estimations for porous-silicon based structures are presented for femtosecond Ti:Sapphire laser pump.

PACS: 42.15.Eq, 42.25.Dd, 42.70.Qs

One of the strongest driving forces of the modern optics is a projection of the solid-state physics concepts on a photonic language. Except the fundamental interest, this is related to so-called photonics: a construction of optical based elements of information processing devices. Modern technology provides an opportunity to construct a photonic devices of a sunmicron and micron sizes. In particular it is worth mentioning photonic crystals [1, 2], structures with a periodic modulation of optical properties. They act on a photon the same way as a crystal lattice potential on electrons. Beauty optical analogs of several solid-state phenomena have been observed in systems, based on photonic crystals. One could mention Bloch oscillations [3, 4], optical analog of Franz-Keldysh effect [5, 6], fabrication of optical molecule [7], etc. Optical field localization in the photonic crystal based structures provides numerous nonlinear optical phenomena [8, 9].

A description of experimental results in this field commonly uses spectral(plane-wave) representation. At the same time, modern femtosecond lasers provide pulses of a small longitude. For example 100 fs impulse has a 30 μm length in vacuum and is even shorter in a material media. This is comparable to the size of a photonic crystal structure. By taking into account a finite length of the photonic wavepacket, one can predict a new range of optical effects, similar to the ones, based on electron wavepacket spatial localization. In this paper we discuss possible observation of the “optical echo” effect, i.e. periodical reconstruction of a shape of the optical pulse in a specially designed photonic crystal structure.

The simplest echo effect can be realized for a quantum particle in a harmonic oscillator potential. Let the particle being localized at the initial moment near certain spatial point x_0 . It can be characterized by the

wavefunction $\Psi_0(x)$, for example a Gaussian packet can be considered $\Psi_0(x) \propto \exp(-(x-x_0)^2/2R_0^2 + ikx)$. After the initial moment a space spreading of the pulse takes the place because of the dispersion. However, after the time equal to the period of the oscillator T , the wavepacket will reconstruct it's shape back. Indeed, all the eigenfunctions of oscillator evolve with multiple frequencies of the oscillator, so that the wavefunction of the packet $\Psi(x, t)$ repeats itself with a period T :

$$\Psi(x, t) = \sum_n C_n \phi_n(x) \exp\left(i\frac{2\pi}{T}nt\right). \quad (1)$$

One can see from (2) that the crucial circumstance here is that eigenlevels of the harmonic oscillator are equidistant in frequency domain. On the other hand energy levels of almost any pendulum are equally spaced in energy in the quasiclassical limit [10]. In this case energy split in neighboring levels is just $2\pi\hbar/T(E)$, where $T(E)$ is a period of oscillation at given energy. This means the echo effect can be observed for packets composed of locally equidistant states of the discrete spectrum of almost arbitrary potential. Echo effects can also be observed in more sophisticated cases, for example in the disordered structures [11]. The only requirement is the local equidistance of the spectrum.

Our goal is to construct an effective oscillator potential for optical pulses. This requires a modifying of dispersion relation with respect to spatial coordinate. Fabricating a structure with a refractive index being changed smoothly in a wide range is a very complicated task. However, the structures based on one-dimensional photonic crystals can be used. In order to form a structure with a discrete spectrum (i.e. localized eigenstates), we propose using a photonic crystal with a unit cell period adiabatically varied in space. Let us imagine that for the center of the structure the carrier frequency of the

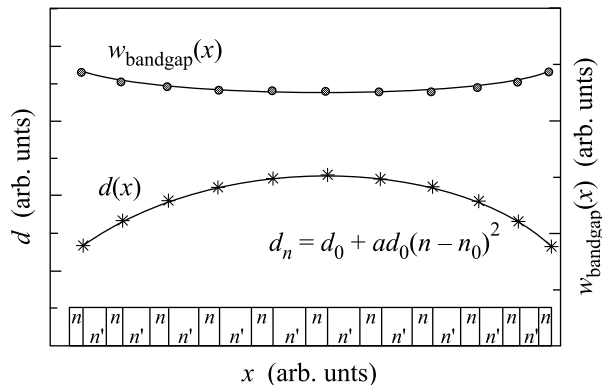


Fig.1. The variation of the period of a unit shell and the frequency of the upper band of the photonic bandgap in the photonic harmonic oscillator potential structure with respect to coordinate. The scheme of this structure is also shown

wavepacket lies just above the photonic bandgap. Consider the structure with a period decreasing to the spatial edges (Fig.1). In this case the photonic bandgap of a crystal is higher at the crystal edges, than in the geometric center. This means the wavepacket is reflected from the Bragg mirrors near the structure borders, therefore it becomes localized in the whole crystal.

The one-dimensional photonic crystals based on porous silicon are quite widespread in modern photonic technology. The procedure for their preparation is quite simple. Such photonic crystals constitute of repeating pair of layers of $n \simeq 1.5$ and $n' \simeq 2.2$ refraction indexes. The optical lengths of both layers in each repeating pair of the crystal are usually taken equal, their material dispersion can be examined as linear. We will address to consider such type of structures. The adiabatic variation of the period of such crystal is also feasible technological task. However in order to obtain sufficient number of equidistant localized levels (around 10) the whole number of pairs must be taken large enough. In our model $M = 100$ pairs of layers are considered.

In the effective local linear susceptibility approximation all layers can be characterized by two values: d_m – the length of the layer and n_m – it's refraction index. The field strength of eigenmode of each layer is given by expression:

$$\mathcal{E}_m(x) = A_m e^{-ik_m x} + B_m e^{ik_m x}, \quad k_m = \frac{w}{c} n_m, \quad (2)$$

where A_m and B_m are unknown amplitudes, which are determined by frequency of eigenmode w .

There are different ways to calculate the spectrum of the structure. One possibility would be a direct search of a solution of eigenvalue problem. The boundary conditions on the edges of the structure and on the joints

of the layers produce a set of equations for A_m, B_m for each layer. The solvability of this set requires finding a determinant of a $2M \times 2M$ matrix with some elements equal to zero. This would give an equation for w . The numerical solving of this equation produces a spectrum of whole system.

Another possible way to find a spectrum of the system is to treat it as an effective adiabatically varied periodic potential. This method is quite similar to employing Vlöke or Bloch's theorem. It fits the paradigm of applying condensed matter physics ideas on the field of optical problems. Let's zoom on pair of two layers: (d, n) and $(D - d, n')$. Their overall spatial length is D . Each eigenmode of the pair j can be presented the same way as (2):

$$\begin{aligned} \mathcal{E}_j(x) = & A_j e^{ikx\Theta(d-x)} + B_j e^{-ikx\Theta(d-x)} + \\ & + A'_j e^{ik'x\Theta(x-d)} + B'_j e^{-ik'x\Theta(x-d)}, \end{aligned} \quad (3)$$

$$k' = k \frac{n'}{n},$$

$\Theta(x)$ is the Heavyside function. The nil of the coordinates is placed at the beginning of the first layer. Similarly to Vlöke's theorem we propose that:

$$\mathcal{E}(x + D) = \mathcal{E}(x) e^{i\chi}. \quad (4)$$

Let $\Gamma = n/n'$, $k = w/c$, and $\chi_0 = kdn = k(D - d)n'$. Using the boundary conditions on the layer connections the value of Bloch's phase χ can be found [12]:

$$\cos \chi = \frac{(\Gamma + 1)^2}{4\Gamma} \cos 2\chi_0 - \frac{(\Gamma - 1)^2}{4\Gamma}. \quad (5)$$

That is equivalent to

$$\sin \frac{\chi}{2} = \frac{\Gamma + 1}{2\sqrt{\Gamma}} \sin \chi_0. \quad (6)$$

Actually, *The Vlöke theorem itself is not used*, we just use the corresponding notation. In fact, the phase χ is determined only by the parameters of local pair of layers, i.e. it is solely connected with the pair number. The expression for the value of the electric field in the pair number j can now be rewritten as:

$$\begin{aligned} \mathcal{E}(x) = \mathcal{E}_0(x) e^{i\Sigma_j}, \\ \Sigma_j = \sum_{i=0}^j \chi_i, \end{aligned} \quad (7)$$

$\mathcal{E}_0(x)$ is an expression for the spatial part of electric field in the first pair of layers. For example in odd layers [12] will appear as

$$\mathcal{E}_j(x) = (A_0(k, d_0) e^{ikx} + B_0(k, d_0) e^{-ikx}) e^{i\Sigma_j}. \quad (8)$$

The same equations can be derived for complex conjugate $\mathcal{E}_j^*(x)$. We will consider full field strength as a sum of the $\mathcal{E}_j(x)$ and $\mathcal{E}_j^*(x)$ which is in fact $2\Re\mathcal{E}_j(x)$. In this case (8) becomes

$$\mathcal{E}(x) = (A_0(k, d_0)e^{ikx} + B_0(k, d_0)e^{-ikx})e^{i\Sigma_j} + (A_0^*(k, d_0)e^{-ikx} + B_0^*(k, d_0)e^{ikx})e^{-i\Sigma_j}. \quad (9)$$

In order to get a spectrum of the eigenmodes of this system it is virtually placed in an opaque resonator. Let L be a whole length of the system. The boundary conditions on the spatial edges of the system are

$$\mathcal{E}(0) = 0, \mathcal{E}(L) = 0.$$

Overall, this gives an equation

$$\sin \Sigma_{M-1} = 0, \Sigma = \sum_{i=0}^{M-1} \chi_i. \quad (10)$$

The results of the calculation of the spectrum for ordinary one-dimensional photonic crystal and for one with a period variation are presented in Figs.2,3. The

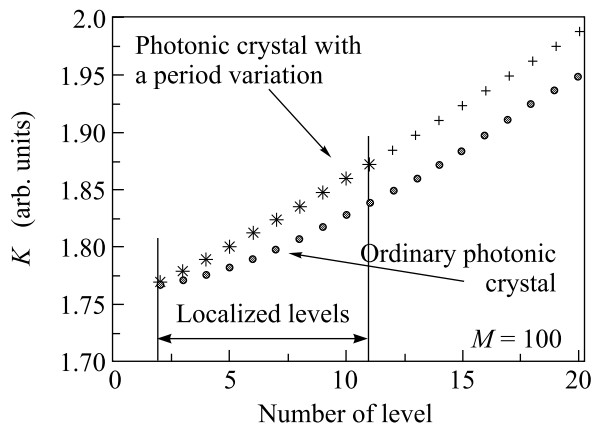


Fig.2. The part of the spectrum of ordinary photonic crystal and a photonic crystal oscillator with 10% period width variation starting from the upper edge of photonic bandgap spectrum plotted in arbitrary $K = w, c = 1$ versus the number of the level. Both structures consists of 100 pairs of layers. The important localized levels are highlighted

localized levels between the upper bands of photonic bandgaps at the crystal edges and in the center of the structure are not exactly equidistant. This means that exact reconstruction of the form of the signal won't occur and the packet will irrevocably lose it's shape after some periods. However this fact is not crucial. Let us introduce the "non-equidistance parameter" $\eta = \delta K / \Delta K$, where ΔK is an effective distance between levels and δk is an average deviation of the level interval from the

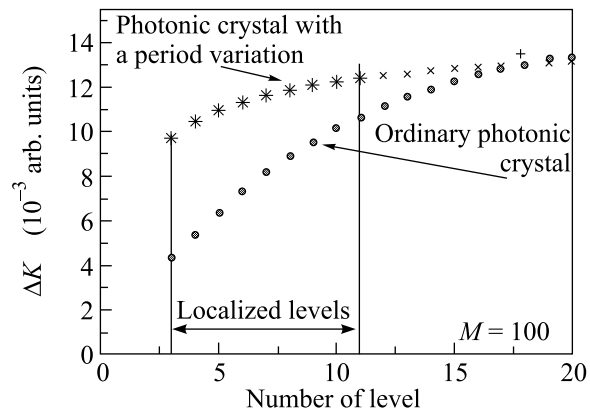
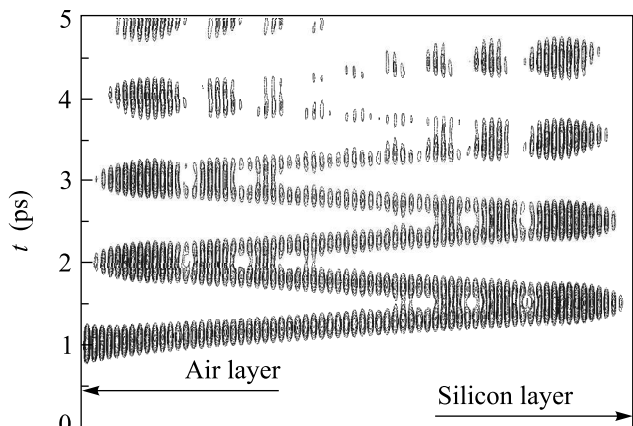


Fig.3. The difference between energy levels versus the number of the level plotted in the same region for same structures, as in Fig.1. In the frequency interval of the laser signal the photonic crystal with a period variation produces "more equidistant" energy levels than an ordinary crystal

ΔK . For an ideal oscillator $\eta = 0\%$, in case of ordinary photonic crystal $\eta = 10\%$, while in presence of adiabatic period variation $\eta = 2.5\%$. This means in a simple crystal the packet will lose it's shape approximately after 3 periods, while for the selected structure this will happen after 10 periods. This is enough for experimental purposes.

One can consider a possible experimental realization concerning distribution of a wavepacket inside the observed structure. In common optical experiments, the sample is surrounded by air and placed on a substrate. So additional wide layers of air ($n = 1$) and silicon ($n = 3.5$) are added on the bounds of the system. We take a Gaussian shaped wavepacket of a period of 130 fs and the wavelength 800 nm. These are typical values for the state of the art Ti:Sap lasers. The optical lengths of the layers on the crystal edge are $\Lambda/4 = 200$ nm. In this case the spectrum of the wavepacket lies inside the area of ~ 10 localized levels. The distribution of a signal is calculated in accordance of Eq.(2). Fig.4 shows the results of calculation. After the initial moment, most of the pulse is reflected from the structure, because its carrier frequency corresponds to the photonic bandgap of crystal edge region. However, approximately 30% part of the signal penetrates inside the oscillator due to the tunnelling and becomes localized. It starts a propagation with a periodical shape reconstruction. The localization due to the reflection of the wavepacket from the effective Bragg mirrors on the edges of the crystal leads to a periodical stops and redirections of the pulse propagation. The period of echo motion in thus created optical oscillator potential is 1 ps.



The length of the structure (20 nm, 100 pairs of layers)

Fig.4. The contour plot of the dynamics of distribution of a Gaussian wavepacket with a period of 130 fs and a 800 nm wavelength in a photonic crystal oscillator with 100 pairs of layers and a 10% period variation

Finally, let us discuss how the proposed optical echo effect can be observed in photonic crystal oscillator. One of the ways to detect the shape reconstruction is a time-resolved observation of the nonlinear response of the system. Indeed, near the stop-points of the oscillatory motion the pulse group velocity falls. Therefore, the pulse spatial length is smaller and its electromagnetic field is larger near these points, as can be observed from Fig.3. Consequently, second or third optical harmonic response of the structure should look like a consequence of sharp peaks with a sub-picosecond period. This should be seen in the autocorrelation properties of the nonlinear signal. The third harmonic seems more perspective, as the effect in higher harmonics is more pronounced. On the other hand, there maybe an experimental problem concern-

ing the absorption of the third-harmonic signal inside the Si-based structure at 800 nm pump [13, 14]. This may require usage of the infrared pump laser (Ti-sapphire + parametric oscillator system).

Authors are grateful to O.A. Aktsipetrov for his valuable comments. The work was supported by "Dynasty" foundation.

1. J. D. Joannopoulos, R. D. Meade, and J. N. Winn, *Photonic Crystals: Molding the Flow of Light*. Princeton University Press, Princeton, 1995.
2. E. Yablonovitch, *Phys. Rev. Lett.* **58**, 2059 (1987).
3. G. Lenz, *Phys. Rev. Lett.* **83**, 963 (1999).
4. C. M. de Sterke, J. E. Sipe, and L. A. Weller-Brophy, *Opt. Lett.* **16**, 1141 (1991).
5. A. P. Jauho and K. Johnsen, *Phys. Rev. Lett.* **76**, 4576 (1996).
6. H. Shen, *Phys. Rev. B* **42**, 7097 (1990).
7. I. G. Voigt-Martin, Gao Li, U. Kolb et al., *Phys. Rev. B* **59**, 6722 (1998).
8. T. V. Dolgova, A. I. Maydikovsky, M. G. Martemyanov et al., *JETP Lett.* **73**, 6 (2001).
9. T. V. Dolgova, A. I. Maidikovski, M. G. Martemyanov, et al., *JETP Lett.* **75**, 15 (2002).
10. P. V. Elutin and V. D. Krivchenkov, *Kvantovaya mekhanika s zadachami*, M.: Nauka, 1976.
11. V. N. Prigodin, B. L. Altshuler, K. B. Efetov, and S. Iida, *Phys. Rev. Lett.* **72**, 546 (1994).
12. M. B. Vinogradova, O. V. Rudenko, and A. P. Suhrkov, *Teoriya voln*, M.: Nauka, 1990.
13. E. Peterman, C. Brewer, J. Sandusky et al., *APS Meeting Abstracts*, 2003, p. 27004.
14. Y. Kanemitsu, S. Okamoto, and A. Mito, *Phys. Rev. B* **52**, 10752 (1995).