Spectrum of supersymmetric sound

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The propagation velocity of a phonino can be calculated by a purely phenomenological method. The result is $c = P/E$. The disperson law for a phonino and the region in which it exists have been found through a solution of the Bethe-Salpeter equation in the Wess-Zumino model.

Supersymmetric field models at a nonzero temperature have been under study since the early 1980s.[1–4] The physics described by these models should have been realized in the early universe. It is presently believed that a supersymmetry occurred in some form at temperatures $T > 1$ TeV.

The action describing the system is assumed to be invariant under supertransformations. On the other hand, at a nonzero temperature the state of the system is not invariant under supertransformations. This conclusion follows from simply the circumstance that bosons and fermions have different statistics, i.e., different occupation numbers. For this reason, a nonzero temperature leads to a spontaneous breaking of supersymmetry. This breaking should be manifested in the presence of a gapless fermion branch in the long-wavelength spectrum of the system. We will call such Fermi excitations "phoninos," since they are a supersymmetry analog of ordinary phonons.

Attempts to demonstrate the existence of phoninos (goldstoninos) within the Wess-Zumino model were undertaken in Refs. 2–4. Those investigators were seeking an additional gapless pole in the fermion Green's function due to an interaction. However, the question remained open. Boyanovsky[3] and Gudmunsdottir and Salomonson,[4] for example, simply discarded some of the diagrams for the self-energy function which were on the same order as those which were retained. We will show here that the presence of a phonino branch of the spectrum is a purely symmetric fact.

Let us assume that a nonzero supercurrent density $\langle S_\mu \rangle$ has been created in a system. Fast relaxation processes then lead to the establishment of a local equilibrium with a distribution function

$$\exp\left((F - H + \bar{\xi}Q)/T\right). \tag{1}$$

Here F is the free energy, H is the Hamiltonian, Q is the supercharge, and ξ is an anticommuting Majorana spinor. From distribution function (1) we easily find the expectation value

$$\langle S_\mu \rangle = \frac{1}{2T}\langle [\bar{\xi}Q, S_\mu]_+ \rangle = -\frac{i}{T}\langle \theta_{\mu\nu} \rangle \gamma_\nu \xi . \tag{2}$$

Here $\theta_{\mu\nu}$ is the energy-momentum tensor. The last equality in (2) follows from the circumstance that S_μ and $\theta_{\mu\nu}$ belong to a common supermultiplet.

It is now a simple matter to obtain the long-wavelength dynamics of $\langle S_\mu \rangle$, by making use of supercharge conservation. From the condition $\partial_\mu \langle S_\mu \rangle = 0$, according to (2), we can find a Dirac equation for the spinor ξ. This equation describes a wave which is propagating at a velocity

$$c = P/E \ , \tag{3}$$

where P is the pressure, and E the energy density. The phonino spectrum is thus of an acoustic nature, and the phonino velocity (3) is expressed in terms of exclusively the thermodynamic characteristics of the medium.

This hydrodynamic derivation of the phonino spectrum is general in nature. However, in order to find the boundaries of the region in which a phonino exists, to calculate the damping of the phonino, and to calculate the corrections to the dispersion law $\omega = ck$, we need to appeal to some model. We have carried out some specific calculations in the Wess-Zumino model (which was also used in Refs. 1–4), assuming the coupling constant g to be small.

In calculating the thermodynamic quantities which determine the phonino velocity according to (3), we can restrict the analysis to the zeroth order in the interaction. As a result, we find

$$c = \frac{1}{3}\int_0^\infty \frac{x^4\,dx}{\sqrt{x^2+1}}(n_B + n_F) \Bigg/ \int_0^\infty x^2\sqrt{x^2+1}\,dx\,(n_B + n_F). \tag{4}$$

Here

$$n_{B,F} = \left(\exp\left(\frac{m}{T}\sqrt{x^2+1}\right) \mp 1\right)^{-1} ,$$

where m is the mass of the noninteracting particles. At $T \ll m$ we have $c = T/m$, while at $T \gg m$ we have $c = 1/3$.

Incorporating the interaction leads to the result that identically seeded dispersion laws corresponding to the propagation of A, B, and ψ fields become different. The latter effect can be evaluated from the magnitude of the difference between the "squares of masses":

$$\begin{aligned} \Delta_A &= \omega_\psi^2(\mathbf{k}=0) - \omega_A^2(\mathbf{k}=0) = \frac{14}{3} g^2 m^2 \left(\frac{2T}{\pi m}\right)^{3/2} \exp\left(-\frac{m}{T}\right), \\ \Delta_B &= \omega_\psi^2(\mathbf{k}=0) - \omega_B^2(\mathbf{k}=0) = -2g^2 m^2 \left(\frac{2T}{\pi m}\right)^{3/2} \exp\left(-\frac{m}{T}\right). \end{aligned} \tag{5}$$

The last equalities in (5) hold at $T \ll m$ and in the leading approximation in g. At $T \gg m$ we have $\Delta_A \sim \Delta_B \sim g^2 T^2$.

A phonino may be thought of as a bound state of a fermion and a boson. In order to study its spectrum, we thus need to solve the corresponding Bethe-Salpeter equation.[5] This equation is shown in Fig. 1 in the leading approximation in the coupling constant. A solid line represents a fermion Green's function, a dashed line a boson

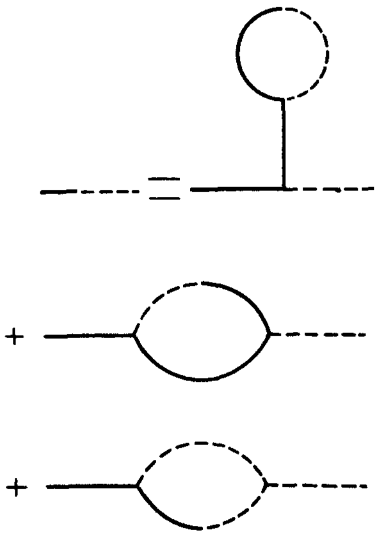

FIG. 1.

Green's function, and a mixed line a wave function of a bound fermion-boson state. The latter wave function may be interpreted as the expectation values $\langle \psi A \rangle$, $\langle \psi B \rangle$, which arise at $\langle S_\mu \rangle \neq 0$.

Since we are studying the phonino spectrum at a nonzero temperature, we should use the Keldysh diagram technique (Refs. 6 and 7; see also Refs. 8–10) for actual calculations.

The region in which a phonino exists is determined by the Landau damping mechanism,[7] which in this case means the absorption of a phonino by a boson, accompanied by its conversion into a fermion, or vice versa. This mechanism comes into play effectively at wave vectors on the order of

$$k_* = \Delta / k_T \ , \tag{6}$$

where k_T is the thermal momentum, equal to $\sqrt{mT}$ at $T \ll m$ and equal to T at $T \gg m$. At $k \gtrsim k_*$, the Landau mechanism leads to a blurring of the phonino spectrum, while at $k \ll k_*$ it causes an exponentially small contribution to the damping of the phonino.

If this damping is ignored, the equation illustrated in Fig. 1 can be used to calculate only the real part of the phonino spectrum. To determine the phonino damping, we need to consider two-loop diagrams. Studying the phonino spectrum by the method proposed here is very nearly the same as studying the spectrum of collective modes by a kinetic-equation method.[7] Here the role of the particle distribution function is played by the expectation values $\langle \psi A \rangle$ and $\langle \psi B \rangle$, and the role of the collision integral is played by the two-loop terms.

We have found solutions of the Bethe-Salpeter equation in the first two orders in k / k_*. Substituting the resulting solutions for $\langle \psi A \rangle$ and $\langle \psi B \rangle$ into the supercharge conservation law, we calculated the leading terms in the dispersion relation. We proceed now to the results.

At $T \ll m$ the real part of the phonino spectrum is given by the equation

$$\omega^2 = \frac{T^2}{m^2} k^2 \left(1 + m^2 k^2 (\Delta_A^{-1} - \Delta_B^{-1})^2\right), \tag{7}$$

where ω is the frequency, and k the wave vector, of the phonino. The damping of the phonino determined by the collision terms is given in order of magnitude by

$$\operatorname{Im} \omega \sim g^2 T k^2 / \Delta \tag{8}$$

as is always small in comparison with the frequency.

At $T \gg m$ we have $\omega \approx k/3$. The damping of the phonino depends strongly on the relation between k and $g^2 m_*$, where $m_* = m + gT$. If $k \ll g^2 m$, we have the ordinary hydrodynamic regime and $\operatorname{Im}\omega \sim k^2/m_*$. If $g^2 m_* \ll k \ll g^2 T$, the local equilibrium is greatly disrupted; this regime might be called a "zero-sound regime."[7] The damping of the phonino in this regime remains constant: $\operatorname{Im}\omega \sim g^4 m_*$.

The general features of the phonino spectrum which have been formulated here would apparently also apply to any model with a weak coupling. Accordingly, these features of the spectrum can be extended to some more interesting cases, e.g., that of a supersymmetric quark-gluon plasma.[11,12]

The results of this study will be published in more detail elsewhere.[13]

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