Final stage of 3D Langmuir collapse

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The final stage of the evolution of a 3D Langmuir cavity is studied. The final dimensions of the cavity are found to be significant in comparison with those in the 2D case. The rf field is found to be at a high level. These results are evidence that nonlinear Landau damping plays an important role in the dissipation of wave energy.

1. Of fundamental importance to the derivation of a systematic theory of the Langmuir turbulence of plasmas is a study of the final stage of the collapse of a Langmuir cavity,[1] in which energy is transferred from Langmuir waves to fast electrons. The short temporal and spatial scales of the cavity make it difficult to study this

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process experimentally; the experiments carried out by Wong's group remain the only ones which have been carried out in this area.

There is accordingly a particularly urgent need for a numerical simulation of the final stage of the collapse. A difficulty which arises here is that in the final stage of the collapse the average hydrodynamic equations cannot be used to describe the collapse (see Ref. 3 and the literature cited in Ref. 1 regarding the numerical simulation of these equations). It becomes necessary to solve the complete kinetic problem, i.e., to use a particle method. Furthermore, the fundamental way in which the nature of the collapse varies with the dimensionality of the space[1,4] mandates solving the 3D problem, if results completely valid from the standpoint of plasma physics are to be obtained (the 2D situation was studied in Ref. 4). The net result is that a numerical simulation of the final stage of 3D Langmuir collapse is right at the limits of today's computational capabilities.

We have managed to solve this problem by making systematic use of the physical properties predicted by the theory for a collapsing cavity and by taking every possible measure to match[5] the physical parallelism of the problem with the architecture of the powerful multiprocessor complex used for the numerical simulations: the ES-1037, ES-2706 complex of the Institute of Space Research, Academy of Sciences of the USSR.[6]

A collapsing cavity has an oblate shape with a dipole charge distribution. Let us assume that the minor axis of the cavity runs along the z axis. The electric potential then has the following symmetry properties with respect to the center of the cavity:

$$\varphi(x,y,z)=\varphi(x,-y,z)=\varphi(-x,y,z)=\varphi(-x,-y,z)=-\varphi(x,y,-z).$$

By making use of these symmetry properties, we were able to restrict the study to a quarter of the cavity, specifically, the region $0\leqslant x, y\leqslant L$, $-L/2\leqslant z\leqslant L/2$, where L was varied over the interval $32r_D\leqslant L\leqslant 64r_D$. At the boundaries of the calculation region we imposed reflection conditions $\partial\varphi/\partial n|_\Gamma=0$. The ratio of the ion and electron masses was varied from 100 to 400. The initial charge distribution was specified to be

$$\rho(x,y,z)=\rho_0\sin kz(1+\cos kx)(1+\cos ky), \qquad k=\pi/L.$$

The variation of the plasma density at the initial time was calculated from the equilibrium condition:

$$\left.\frac{\delta n}{n_0}\right|_{t=0}=-\frac{1}{16\pi n_0T_e}|\mathbf{E}|^2, \qquad \mathbf{E}=-\nabla\varphi.$$

The initial distribution of electrons was chosen to be Maxwellian, while the ions were assumed to be cold. The total number of model particles was $\sim 1.8\times10^6$. The intensity of the initial distribution of wave energy at the center of the cavity, $W_{\max}/n_0T_e$ ($W_{\max}=E^2/8\pi|_{r=0}$), was varied over the interval $0.135\leqslant W_{\max}/n_0T_e\leqslant0.485$. The average value of the rf-field energy $W/n_0T_e|_{t=0}$ over the cavity was varied over the interval $0.0242\leqslant W/n_0T_e|_{t=0}\leqslant0.080$.

2. The results of the calculations provide a clear picture of the collapse (Fig. 1).

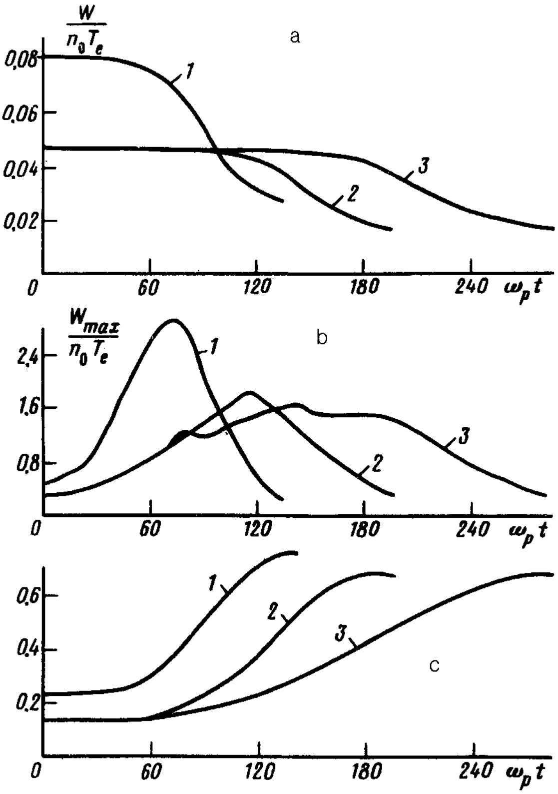

FIG. 1. Time evolution of the characteristics of a collapsing cavity. 1) $\rho_0 = 0.020$, $M/m = 100$; 2) $\rho_0 = 0.015$, $M/m = 100$; 3) $\rho_0 = 0.015$, $M/m = 400$. a—The average energy of the rf field in the cavity, $W/n_0 T_e$; b—the maximum energy of the field in the cavity, $W_{\max}/n_0 T_e$; c—spatial maximum of the density variation, $(\delta n_{\max}/n_0)-(\delta n_{\min}/n_0)$.

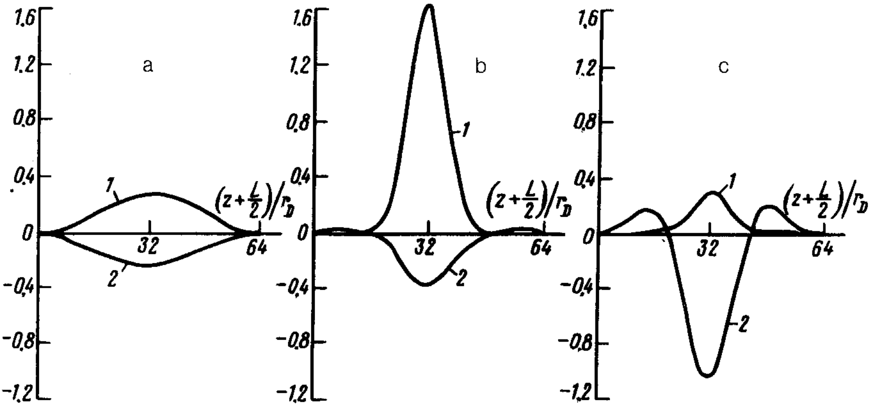

FIG. 2. Spatial profiles along the axis of the dipole in the $x = y = 0$ cross section. 1) Of $E^2/8\pi n_0 T_e$; 2) $\delta n_i/n_0$, for the version $\rho_0 = 0.015$, $M/m = 400$. a—$t = 0$; b—$t = 139.2\,\omega_p^{-1}$; c—$t = 284.0\,\omega_p^{-1}$.

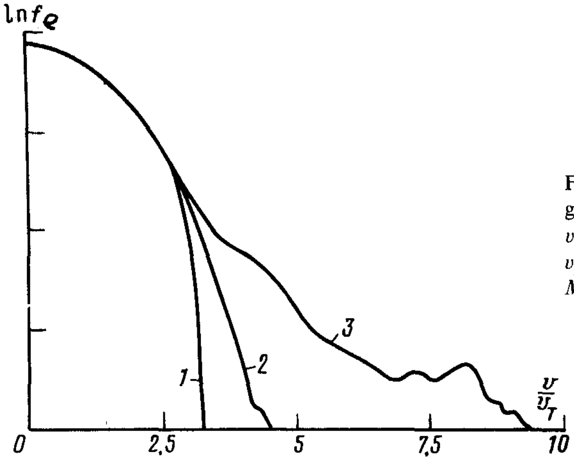

FIG. 3. Electron distribution function integrated over space and velocity. 1—v_x, v_y ($t = 0$); 2—v_y, v_z ($t = 284.0\,\omega_p^{-1}$); 3—v_x, v_y ($t = 284.0\,\omega_p^{-1}$) for the version $\rho_0 = 0.015$, $M/m = 400$.

The intensity threshold found experimentally for the process turns out to be close to that predicted theoretically.[1] In the course of the collapse, a large part (about 70%) of the energy of the rf waves is dissipated. The time scale of this dissipation is on the order of (8–9) $\times \omega_{pi}^{-1}$. According to the theoretical picture of the role played by the ion inertia,[7] most of the deepening of the density well [to $-(\delta n_i/n_0) \sim 0.7$] occurs after the rf energy reaches its maximum (Figs. 1 and 2). In these 3D calculations we do not observe the formation of a caviton just above the threshold for collapse[4]; this caviton-formation process is characteristic of the 2D case. In all versions of the calculations we see levels of the rf field and of the plasma density variation in the cavity which are substantially higher than in the corresponding 2D calculations. The increase in the wave energy at the center of the cavity precedes the deepening of the ion well (up to the time at which the maximum value is reached). The cavity remains oblate in the course of the collapse. An important result, which we see in all versions of the calculations, is that the minimum dimension of the cavity (along the axis of the dipole; Fig. 2) is fairly large [$\sim (14\text{–}16) r_D$]. In the 2D case this value is[4] $\sim 10 r_D$. This result agrees with the data of actual experiments,[2] which have previously seemed unexplainable. The explanation may be that the high levels which the rf energy reaches [up to $(W_{\max}/n_0 T_e) \sim 3$)] cause the dissipation to occur by a mechanism of Landau damping which is greatly modified by the nonlinearity and which comes to resemble a trajectory self-intersection effect. This circumstance is additionally illustrated by an analysis of the final electron velocity distribution (Fig. 3). This distribution is strongly anisotropic (the maximum acceleration occurs along the axis of the dipole). We detect the existence of electrons which have been greatly accelerated, to $v = v_{\max} = 9 v_T$ (in the 2D calculations, the maximum velocity is $v_{\max} \sim 5 v_T$). This result means, in particular, that the 3D collapse is considerably more efficient than the 2D collapse as a mechanism for producing fast electrons.

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