# Possibilities of a muon method for studying type II superconductors

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All the characteristics of a 2D Abrikosov vortex lattice can be found through an analysis of the Fourier spectrum of the transverse polarization of muons in type II superconductors.

The possibilities of a muon method for studying type II superconductors were first examined in Ref. 1. Numerical methods were used to find the polarization of muons in triangular and square Abrikosov lattices in fields close to $H_{c_2}$. It was shown that the type of vortex lattice can be found from the behavior $p(t)$. Although the discovery of high-temperature superconductors has sharply intensified interest in the muon method,[2–4] its possibilities have remained uncertain to a large extent, and it is

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not being used experimentally. In the present letter we show that the muon method can determine all the characteristics of a vortex structure. All the information is embodied in the Fourier spectrum of the transverse polarization, $p_\perp(\omega)$.

High-temperature superconductors, being type II superconductors, have an extremely large Ginzburg-Landau parameter $\kappa = \lambda/\xi$ ($\lambda$ is the field penetration depth, and $\xi$ is the coherence length). The interval of external fields most convenient for study is $H_{c_1} \ll H_{\rm ext} \lesssim H_{c_2}$, i.e., the case $\xi \ll a \lesssim \lambda$, where $a$ is the distance between vortices. The field outside vortices is determined by the known solution of the London equation (Ref. 5, for example):

$$H(\rho) = B \times \left(1 + \sum_{K \neq 0} \frac{e^{i\mathbf{k}\vec{\rho}}}{1 + \lambda^2 K^2}\right), \tag{1}$$

where $B = n\phi_0$ is the average field in the superconductor, $\vec{\rho}$ is a vector in the plane of the lattice ($\rho > \xi$), **K** is a reciprocal-lattice vector, $\phi_0$ is the fluxoid, and $n$ is the density of vortices.

The results of Refs. 2–4 indicate that there is no diffusion of muons. We eliminate the slow relaxation of the polarization which is observed in the normal phase and which is caused by the interaction of the muon spin with nuclear spins of the sample. The "corrected" transverse polarization of the muon is then given by

$$p_x(t) = \int \cos H(\vec{\rho})\, t \, \frac{d^2\rho}{s_0} = \int \cos \omega t \int \delta(\omega - H(\vec{\rho})) \frac{d^2\rho}{s_0}\, d\omega . \tag{2}$$

Here and below, $H \equiv \gamma_\mu H$ is the magnetic field, expressed in frequency units. The integration is over a unit cell of areas $s_0$. It can be seen from (2) that the Fourier tranform of the transverse polarization,

$$p_x(\omega) = \int \delta(\omega - H(\vec{\rho})) \frac{d^2\rho}{s_0}, \tag{3}$$

is the field probability distribution. The probability distributions of periodic fields typically exhibit Van Hove singularities.[6] In the 2D case, these singularities would consist of one minimum ($\omega_1$), one maximum ($\omega_2$), and one saddle point ($\omega_3$). Near these points, $p_x(\omega)$ is given by (Fig. 1)

$$p_x(\omega + \Delta\omega) \sim p_1; \quad p_x(\omega_2 - \Delta\omega) \sim p_2; \quad p_3(\omega_3 + \Delta\omega) \sim p_3 \ln \frac{\omega_3}{|\Delta\omega|} . \tag{4}$$

This form of $p_x(\omega)$ has nothing in common with a Gaussian form. The role played by Van Hove singularities in an analysis of $\mu sR$ spectra in type II superconductors was first pointed out in Refs. 7 and 8. In those studies, however, the spectrum was examined near $H_{c_2}$, where it is least convenient to analyze. By measuring $p_1$, $p_2$, $\omega_1$, $\omega_2$, and $\omega_3$, we can find comprehensive information on the Abrikosov structure.

We will illustrate the procedure with the example of a square lattice with $\xi \ll a \lesssim \lambda$.

It can be shown that we have

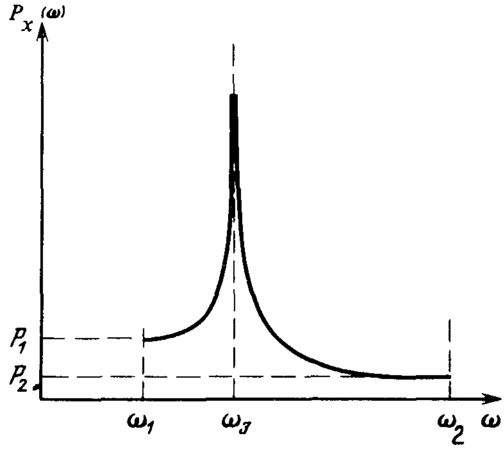


FIG. 1.

$$H(x, y) = B - \frac{\phi_0}{4\pi\lambda^2} \ln \frac{\theta_4^2(\pi\frac{x}{a})\,\theta_4^2(\pi\frac{y}{a}) - \theta_2^2(\pi\frac{x}{a})\,\theta_2^2(\pi\frac{y}{a})}{\theta_4^4} \qquad (5)$$

Here $\theta_2(x)$ and $\theta_4(x)$ are theta functions with the parameter $q = e^{-\pi}$ (Ref. 7). For the five parameters of the Fourier precession spectrum we have the values

$$\omega_1 = H(\frac{a}{2}, \frac{a}{2}) = -\frac{\phi_0}{4\pi\lambda^2} \ln 2 + B;$$

$$\omega_2 = H(\frac{\xi}{\sqrt{2}}, \frac{\xi}{\sqrt{2}}) = \frac{\phi_0}{2\pi\lambda^2} \ln \frac{a}{2K(\frac{1}{2})\xi} + B; \qquad (6)$$

$$\omega_3 = H(\frac{a}{2}, 0) = -\frac{\phi_0}{2\pi\lambda^2} \ln 2 + B;$$

Here $K(1/2)$ is the complete elliptic integral of the first kind.

Comparing $p_1$, $p_2/p_1$, and $\omega_2 - \omega_1$ independently for each pair of these quantities, we find values of $\lambda$ and $\xi/a$ which can be verified on the basis of their consistency. Knowing $\lambda$ and $\xi/a$, we can find $B$. Knowing $B$, we can find $a$. In this manner we find $\lambda$, $\xi$, and $a$ as functions of the external field and temperature and we find $H_{c_1}$ and $H_{c_2}$ in accordance with the Ginzburg-Landau theory. The type of lattice can be determined from the position of $\omega_3$.

Grain boundaries and other structural defects of the sample may disrupt the 2D nature of the Abrikosov structure. This point can be checked at a qualitative level in an experiment in a longitudinal field. If the vortex structure is three-dimensional, the longitudinal polarization will relax (generally in an oscillatory manner) to some value $\rho(\infty) = p_0 \cos^2\varphi < 1$. Here $\varphi$ is the angle between the external and internal fields, and the average is over the volume of the target.

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