Critical phenomena in the cooperative Raman scattering

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A general solution of the spatially homogeneous model for cooperative Raman scattering has been found without a "prescribed-field" approximation. The scattering process is predicted to become critical as the number of atoms in the atomic subsystem increases.

1. Cooperative Raman scattering on an extended system of two-level atoms[1-3] can be described on the basis of a one-dimensional, resonant, spatially homogeneous approximation[1] by the Hamiltonian

$$H = - \int dx \, \{ i\epsilon_\sigma^+(x) \frac{\partial}{\partial x} \epsilon_\sigma(x) + J\delta(x)\epsilon_\sigma^+(x)[\sigma^+_{\sigma\sigma'} R^- + \sigma^-_{\sigma\sigma'} R^+]\epsilon_{\sigma'}(x) \} \quad , \qquad (1)$$

where J is the constant of the two-photon interaction of light with the atom, $\epsilon_\sigma^+(x) = [E_L^+(x), E_S^+(x)]$ is the isospinor consisting of "slow"—amplitude operators of the laser field (L) and the Stokes field (S) with the commutation relations

$$[\epsilon_\sigma(x), \ \epsilon_\mu^+(y)] = \delta_{\sigma\mu}\, \delta(x-y), \qquad (2)$$

the Pauli matrices $\vec{\sigma} = (\sigma^x, \sigma^y, \sigma^z)$ and $\sigma^\pm = (1/2)(\sigma^x + i\sigma^y)$ act in the isospin space of the field, and the system of atoms is described by the spin operator $\mathbf{R} = (R^x, R^y, R^z) R^\pm = R^x \pm iR^y$ with the commutator

$$[R^i, R^j] = e^{ijk} R^k \qquad (3)$$

We will consider here the scattering of n laser photons by a system comprised of M atoms in the ground state ($R = M/2$); i.e., the initial state is described by

$$|in\rangle = (n!)^{-1/2} \int dx_1 \dots dx_n \prod_{j=1}^{n} \Phi(x_j) E_L^+(x_j) |0\rangle , \tag{4}$$

where the vacuum of the model is given by the relations

$$\epsilon_\sigma(x)|0\rangle = 0, \quad R^-|0\rangle = 0, \quad R^z|0\rangle = -\frac{M}{2}|0\rangle,$$

and $\Phi(x)$ is an arbitrary (normalized to unity) wave function of the incident L photons. The problem involves the determination of the intensity of the Stokes radiation $I_S(t-x)$ which occurs as a result of the scattering of laser light with the intensity $I_0(t-x)$.

In the class of solutions restricted by the condition[1]

$$I_S(t-x) \ll I_0 = \text{const} , \tag{5}$$

problem (1)–(4) reduces to the problem of Dicke's superradiance model[4] which allows an exact solution[5] by means of the Bethe ansatz (see the reviews by Thaker[6] and Tsvelick and Wiegmann[7]). We will find here a general solution of problem (1)–(4) [without imposing any constraints on the "given pump field" (5)] and we will analyze the critical phenomena which appear as a result of increasing the number of atoms in the system.

2. As a result of the scattering of photons by the atoms, we have the state

$$|\text{out}\rangle = \prod_{j=1}^{n} S_{j0} |in\rangle , \tag{6}$$

where $S_{j0} = \exp[iJ(\sigma_j^+ R^- + \sigma_j^- R^+)]$ is the scattering matrix of the jth particle.[8,7] The expression for the S matrix becomes simplified considerably as a result of the action on state (4),

$$S_{j0} = A + \sigma_j^- C^+ \tag{7a}$$

$$A = \cos(J\sqrt{R^- R^+}), \qquad C^+ = R^+ \frac{i\sin(J\sqrt{R^- R^+})}{\sqrt{R^- R^+}} . \tag{7b}$$

We evaluate the quantity

$$Q_n^m = \frac{(M-m)!}{m!\,M!} \langle in|(R^-)^m \hat{Q}_n (R^+)^m |in\rangle \tag{8a}$$

$$\hat{Q}_n = \Big(\prod_{j=1}^{n} S_{j0}^+\Big) R^z \Big(\prod_{j=1}^{n} S_{j0}\Big) . \tag{8b}$$

This quantity is equal, at $m=0$, to the mean value $Q(n)$ of the operator R^z in the out-state. Removing the first bracket $(A + C^-\sigma^+)$ and the last bracket $(A + \sigma^- C^+)$ in

Eq. (8), we obtain the recurrence relations

$$Q_n^m - Q_{n-1}^m = \sin^2 (J\sqrt{(m+1)(M-m)})(Q_{n-1}^{m+1} - Q_{n-1}^m) \tag{9}$$

with the initial condition $Q_0^m = -M/2 + m$. For the function $Q(n,m)$of the continuous arguments n,m, relations (9) become a Cauchy problem,

$$\frac{\partial Q}{\partial n} - \sin^2 (J\sqrt{(m+1)(M-m)})\frac{\partial Q}{\partial m} = 0 \tag{10}$$

$$Q(n,m)\Big|_{n=0} = -\frac{M}{2} + m,$$

whose solution leads to a general solution of the scattering problem (1)–(4):

$$n(t) = \int_0^{m(t)} \frac{dm'}{\sin^2[g(m')]} \tag{11a}$$

$$Q(t) = -\frac{M}{2} + m(t) \tag{11b}$$

$$I_S(t-x) = I_0(t-x)\sin^2[g(m(t-x))], \tag{11c}$$

where

$$g(m) = J\sqrt{(m+1)(M-m)}, \tag{11d}$$

and the number of laser photons $n(t)$ scattered at the time t is related to $I_0(t)$ by a straightforward relation

$$n(t) = \int_{-\infty}^{t} dt' I_0(t'). \tag{11e}$$

3. The scattering is completely determined by the maximum value $G(M) = \max[g(m)] = JM/2$ of the function $g(m)$ [Eq. (11d)] in Eq. (11a). Here we will examine the behavior of the field + atoms system as a function of M for $I_0(t) = I_0\Theta(t)$.

1) There exists a critical value $M_{\text{crit}} = 2\pi/J$ for which $G(M_{\text{crit}}) = \pi$.

2) For $M < M_{\text{crit}}$ the function $m(t)$ varies from $m = 0$ (at $t = 0$) to $m = M$ (in the limit $t \to \infty$), i.e., the system of atoms is inverted completely. The total number of Stokes photons, $N_S = \int_0^\infty dt I_S(t)$, which is always equal to the maximum value $\mu = \max[m(t)]$ of the function $m(t)$, is also equal to M. The position and value of the peak intensity of the Stokes component $I_S^{\max}$ depends on the ratio of the quantities $G(M)$ and $\pi/2$:

(a) For $G(M) < \pi/2$ the intensity maximum of the Stokes component $I_S^{\max} < I_0$ is reached when $m = M/2$. In the limiting case $G(m) \ll 1$, solution (11) becomes the solution obtained in Ref. 1. Only in this case we have $I_S^{\max} \sim M^2$.

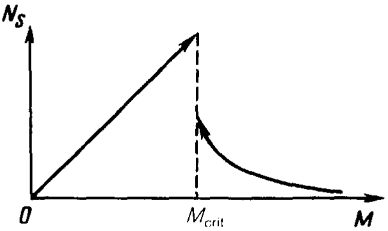

FIG. 1.

(b) For $G(M) > \pi/2$ the peak intensity of the Stokes component $I_S^{max} = I_0$ does not depend on M and is reached when $m = m_0^{(1,2)}$, where $m_0^{(1,2)}$ is determined from the condition $g(m_0^{(1,2)}) = \pi/2$.

3) In the region $M > M_{crit}$ the values of μ and m_0, determined by the conditions $g(\mu) = \pi$ and $g(m_0) = \pi/2$, are related to M by the relations

$$\mu = \frac{M}{2}\left[1 - \sqrt{1 - \left(\frac{M_{crit}}{M}\right)^2}\right] \leqslant \frac{M}{2} \tag{12a}$$

$$m_0 = \frac{M}{2}\left[1 - \sqrt{1 - \left(\frac{M_{crit}}{2M}\right)^2}\right]. \tag{12b}$$

The atomic system is not fully inverted ($\mu \leqslant M/2$) at arbitrary value of the intensity and arbitrary duration of the incident laser light. The total number N_S of Stokes photons decreases with increasing M (see Fig. 1). For $M \gg M_{crit}$ we have $N_S = M_{crit}^2/M$. The maximum intensity of the Stokes radiation is $I_S^{max} = I_0$.

For all M the shape of the Stokes radiation is that of a smooth pulse. Finally, we note that in several experiments on the cooperative Raman scattering[3] I_S^{max} attained a value of $0.7I_0$, so that the parameter G was approximately equal to the critical value of π.

It can be shown that incorporation of the anti-Stokes radiation component with the coupling constant $J_{AS} < J_S$ does not change the results materially.

[1]S. G. Rautian and B. M. Chernobrod, Zh. Eksp. Teor. Fiz. **72**, 1342 (1977) [Sov. Phys. JETP **45**, 705 (1977)].

[2]V. I. Emel'yanov and V. N. Seminogov, Zh. Eksp. Teor. Fiz. **76**, 34 (1979) [Sov. Phys. JETP **49**, 17 (1979)].

[3]A. A. Zabolotskiĭ, S. G. Rautian, V. P. Safonov, and B. M. Chernobrod, Zh. Eksp. Teor. Fiz. **86**, 1193 (1984) [Sov. Phys. JETP **59**, 696 (1984)].

[4]R. H. Dicke, Phys. Rev. **93**, 99 (1954).

[5]V. I. Rupasov and V. I. Yudson, Zh. Eksp. Teor. Fiz. **87**, 1617 (1984) [Sov. Phys. JETP **60**, 927 (1984)].

[6]H. B. Thaker, Rev. Mod. Phys. **53**, 253 (1981).

[7]A. M. Tsvelick and P. B. Wiegmann, Adv. Phys. **32**, 453 (1983).

[8]P. B. Wiegmann, J. Phys. **C14**, 1463 (1981).

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