Strings with boojums at their ends: topological defects of a new type in nematic liquid crystals

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Strings connecting pairs of point defects have been observed experimentally in thin layers of a nematic liquid crystal with degenerate hybrid boundary conditions for the distribution of the director field. The strings contract at a constant rate to the point of annihilation in a dissipative dynamic process.

Topological defects in distributions of vector fields are extremely important subjects of research in various branches of physics. Some clarity has now been achieved in the classification of these defects (Ref. 1, for example). On the other hand, the interactions of defects and their dynamics resulting from these interactions have received little study. This problem is attracting particular interest in connection with the possible existence of various configurations of a vector field which are similar to strings or vortices that connect point defects. Experimentally, however, no objects of a finite-string type have been observed.

Nematic liquid crystals present a favorable opportunity for studying this problem. The existence of various types of defects in these crystals has been reliably established. Studies have been made of the dynamics of individual domain walls,[1–3] disclinations,[4] and disclination pairs.[5,6] The particular boundary conditions chosen for the experiments on nematic liquid crystals which we are reporting here have made it possible to

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observe strings with point defects at their ends, to establish their structure, and to study their dynamic properties.

The basic purpose of the experiment was to create a distribution of the field of the director **n** of a three-dimensional nature in a sample of a nematic liquid crystal. For this purpose, we imposed hybrid boundary conditions on **n**: tangential (T) conditions at the lower surface of the layer of the crystal and normal (N) conditions at the upper surface. This arrangement was achieved by placing the liquid-crystal layer on an isotropic liquid surface [glycerine or α,ω-acryl-bil-(propylglycol)-2,4-toluylenedioritan (APGT)], which imposed degenerate T conditions on **n** at the lower boundary of the sample. The upper boundary of the liquid crystal was left free; since we were studying the compounds MBBA and ZhK-440, the orientation of **n** as it was nearly normal. Strings were observed in liquid-crystal films of thickness $h \approx 10$ μm.

When observed under a polarizing microscope, the liquid-crystal layer which results from spreading over the surface of the substrate exhibits so-called Schlieren textures with dark extinction bands.[2] These bands are localized in regions where the director is oriented parallel to the polarization plane of one of the Nicol prisms. The bands terminate at singular points at the T surface. These points are called "boojums" (Figs. 1 and 2).[7] The distribution of **n** near the boojums was determined from the shape of the extinction bands (Fig. 1a) and also by a decoration method[2] (Fig. 1b). It

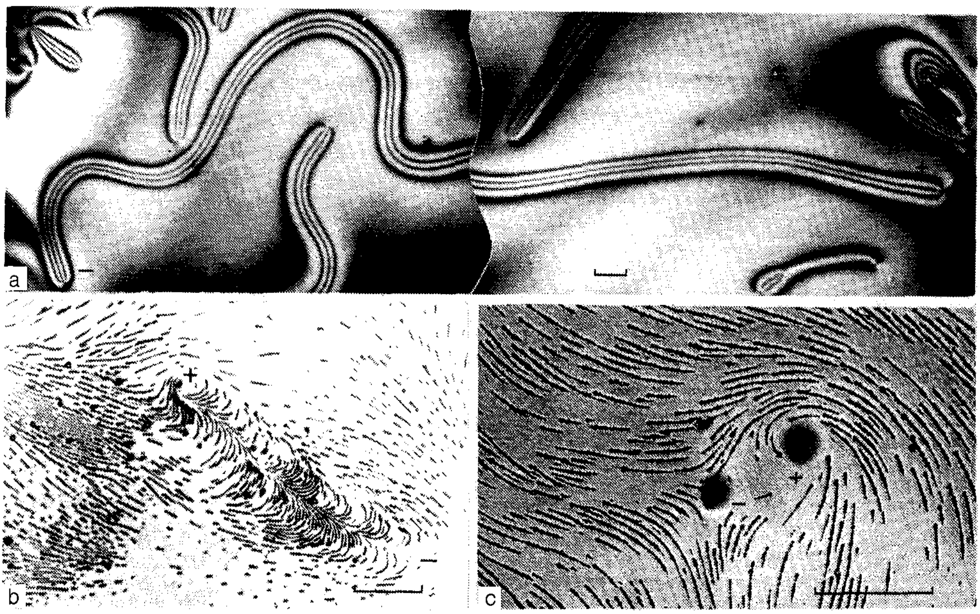

FIG. 1. a,b—Strings connecting a boojum and an antiboojum in a thin layer of a nematic liquid crystal ($h \approx 10$ μm) with a hybrid orientation of **n**; a—texture of the liquid crystal in polarized light (the ends of the same string, 11 mm long, are shown); b—decorated sample (the chains run along lines of the director in the T plane); c—pair of boojums in a thick sample ($h \approx 60$ μm) (there is no string). The length of the horizontal bar in parts a–c is 200 μm.

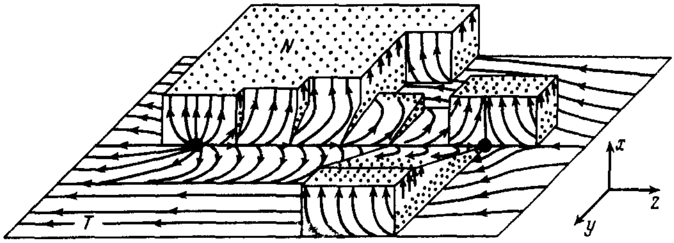

FIG. 2. Overall distribution of the director field inside and outside the string.

turned out that the boojums have topological charges of opposite sign[7] and that the nonuniform distribution of the director between boojums is stretched out into a string. Figure 2 shows the **n** configuration in a liquid-crystal layer with a string connecting a boojum and an antiboojum. The distortions of the **n** field propagate a distance $D = 100$–$300\ \mu$m across the structure. The length of the strings can reach 1 cm (Fig. 1, a and b). Since the **n** field is uniform away from the string in the T layer (in the yz plane), and since this field undergoes a rotation through an angle of 2π within the string, each string is seen as four parallel extinction branches in polarized light. As h is reduced, the string width D also decreases. The formation of the strings is unrelated to any macroscopic flows of the nematic liquid crystal.[1)] As time elapses, these defects eventually disappear because the boojums close on each other along the string axis and annihilate. The boojum closing velocity v does not depend on the distance between boojums; it changes only during a slowing by mechanical impurities (Fig. 3). Regions AB of curves a and b in Fig. 3 correspond to situations in which only one of the boojums is moving; the second is pinned.

The configuration of the **n** field, shown in Fig. 2, is too complicated for a descrip-

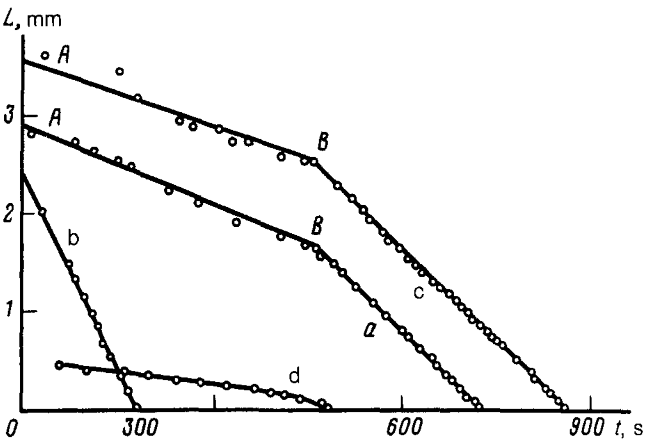

FIG. 3. Time evolution of the distance between the boojums for MBBA on a glycerine substrate (a,b) and for ZhK-440 on an APGT substrate (c,d). a,b,c—The boojums are connected by strings 320, 160, and 260 μm wide, respectively; d—there is no string (thick cell).

tion of its dynamics, so we will instead examine a corresponding system: part of a nonsingular vortex in a volume at whose ends a hedgehog and an antihedgehog are concentrated. This $\mathbf{n}$ configuration is achieved by rotating the pattern in Fig. 2 in the yz plane, around the axis of the string. In this case, it follows from the properties of the Frank energy $F = (K/2)\int dr(\nabla \mathbf{n})^2$ (K is the Frank modulus) under scale transformations that the energy of the interaction of two point singularities depends linearly on the distance between them, L (Ref. 9, for example):

$$U(L) = AL, \tag{1}$$

where $A = 4\pi q K$ is the energy per unit length of a vortex with a topological charge q (Ref. 10; in our case, $q = 1$). In the plane perpendicular to the vortex axis, F is the scale-invariant, and the vortex propagates over the entire infinite volume. The finite thickness of the liquid-crystal layer disrupts this scale invariance, apparently leading to the result that the thickness of the observable strings becomes finite. For the part of the vortex in the volume corresponding to Fig. 2 we can write

$$A = \frac{2\pi K h}{(R^2 + h^2)^{1/2}} \approx 2\pi K \frac{h}{R} . \tag{2}$$

Here it is assumed that Eq. (1) remains valid in a finite sample. This assumption is confirmed by an experiment with $R \approx D/4 \gg h$ [the approximate equality in (2)], where R is the distance from the vortex axis at which $\mathbf{n}$ rotates through an angle of $\pi/2$.

We treat the dynamics of a string as a dissipative motion of boojums which are interacting in accordance with law (1) with force (2) and which experience a frictional force proportional to the boojum closing velocity v. Since we have $v = \text{const}$, for one boojum in the "center-of-mass" system we have $\mathbf{n} = \mathbf{n}[x, y, z - (v/2)\,t\,]$. In the coordinate system moving with the singularity, we have $\mathbf{n} = \mathbf{r}/r$ in the region $0 \leqslant x \leqslant h$, $|y| \leqslant D/2$, $|z| \leqslant a$ and we have $\mathbf{n} = \text{const}$ outside this region. The dissipative function ψ of the two boojums in the center-of-mass system is then given by (we are assuming that the friction acts identically on the boojum and the antiboojum)

$$\Psi = 2\frac{\gamma}{2}\int d\mathbf{r}\left(\frac{\partial \mathbf{n}}{\partial t}\right)^2 = \frac{\gamma}{2}\frac{v^2}{2}\int d\mathbf{r}\left(\frac{\partial \mathbf{n}}{\partial z}\right)^2 \approx 1.3\,\pi h \gamma \frac{v^2}{2} , \tag{3}$$

where γ is an effective orientational viscosity. In deriving the approximate equality in (3) we assumed $R \gg a, h$ and $a \approx h$. From the equation of motion

$$\frac{\partial U}{\partial L} = -\frac{\partial \Psi}{\partial (dL/dt)} \tag{4}$$

we then find the boojum closing velocity ($dL/dt = -v$):

$$v = 3K/2R\gamma . \tag{5}$$

Since the actual distribution of $\mathbf{n}$ in a string and boojums differs from that used in deriving (2) and (3), we introduce the following expression for the boojum closing velocity V in place of (5), for a comparison with experiment

$$V = \frac{3K}{2\alpha R\gamma} = \frac{6K}{\alpha D\gamma}, \tag{6}$$

where the dimensionless constant α is found experimentally.

In summary, we have described the dynamics of the contraction of a string between a pair of boojums, for which the force of the interaction does not depend on the distance between them. It is this circumstance that explains the linear nature of $L(t)$ (Fig. 3). Further evidence that this theory agrees with experimental data comes from the circumstance that the values found experimentally for VD for strings of various thicknesses are approximately the same. For MBBA, for example, the value is $VD \approx (2.12 \pm 0.05) \times 10^{-3}$ mm^2/s. Substituting these values into (6), along with the known values $K = 7 \times 10^{-12}$ N and $\gamma = 77$ cP (Ref. 11) for MBBA (at room temperature), we find $\alpha = 0.256 \pm 0.005$.

We wish to stress the importance of the three-dimensional nature of the deformations of the director for the appearance of strings. In thick samples ($h \gtrsim 50\ \mu$m) with hybrid boundary conditions, strings were not observed (Fig. 1c); furthermore, they were not observed in samples of any thickness if the boundary conditions were the same. The explanation apparently lies in the two-dimensional nature of the **n** distribution in these cases. In the two-dimensional case, the scale time for the annihilation of singularities is proportional to the square of the distance between them, as a result of the logarithmic interaction of the singularities ($U \propto \ln L$), as described by the XY model (see also Refs. 5 and 6). The $L(t)$ dependence (curve d in Fig. 3) for the singularities shown in Fig. 1c corresponds qualitatively to the two-dimensional case.

[1)]Configurations resembling strings were observed by Rapini *et al.*[8] These configurations were seen, however, only in a nematic liquid crystal subjected to an electric field and shear deformations simultaneously.

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