Mesoscopic fluctuations of the superfluid current density in disordered superconductors

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The mesoscopic fluctuations of the superfluid current density in disordered superconductors, which are produced as a result of random distribution of impurities, are studied. These fluctuations can exceed the average value of the superfluid current density. Methods for observing these fluctuations are discussed.

Considerable attention has recently been given to the study of mesoscopic fluctuations of the current density in disordered normal metals.[1,2] In this letter we show that the density of the superfluid current in superconductors experiences similar mesoscopic spatial fluctuations resulting from a disordered arrangement of impurities.

We begin with the expression for the superfluid current density $\mathbf{J}_s(\mathbf{r})$ in a disordered superconductor

$$J_{si}(\mathbf{r}) = \int S_{ik}(\mathbf{r},\mathbf{r}')\, v_{sk}(\mathbf{r}')d\mathbf{r}' . \qquad (1)$$

Here $\mathbf{v}_s = 1/2m[\hbar\vec{\nabla}\varphi - (2e/c)\mathbf{A}]$ is the superfluid velocity, $\mathbf{A}(\mathbf{r})$ is the vector potential, and φ and Δ are the phase and modulus of the order parameter. To find $S_{ik}(\mathbf{r},\mathbf{r}')$, we must sum perturbation-theory diagrams which are illustrated in Fig. 1a (Ref. 3). In Fig. 1a the heavy lines correspond to the normal and anomalous electronic Green's functions in a superconductor in the field of a scattering potential $u(r)$. We wish to emphasize that expression (1) can be used for an arbitrary $u(r)$ even before taking an average over random occurrences of the scattering potential.

Averaging (1) over occurrences of the random potential, we find the known

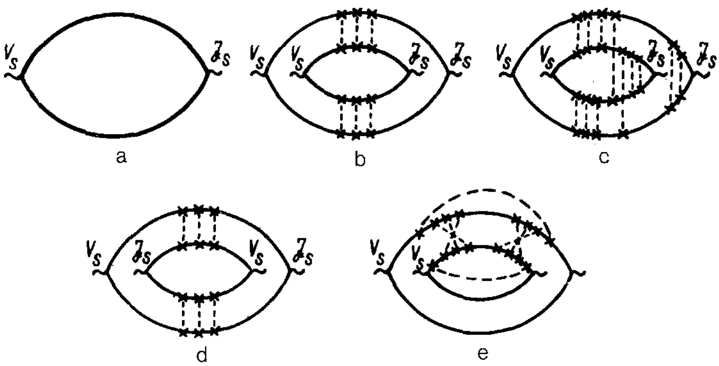

FIG. 1.

results.[3] For $\zeta(0) \gg l \gg \hbar p_F^{-1}$ we find

$$\langle \mathbf{J}_s \rangle = eN_s \langle \mathbf{v}_s \rangle ; \qquad N_s = \frac{N}{3}\left(\frac{l}{\zeta(T)}\right)^2 \tanh \frac{\Delta(T)}{2T} \quad . \tag{2}$$

Here $\zeta(T) = \sqrt{D\hbar/\Delta(T)}$, l and D are the mean free path and the diffusion coefficient of an electron in a normal metal, N is the electron density, p_F is the Fermi momentum, and the angle brackets denote averaging over random occurrences of the scattering potential.

In the disordered superconductors the quantity $\mathbf{J}_s(\mathbf{r})$ experiences spatial fluctuations, so that $\delta \mathbf{J}_s = \mathbf{J}_s - \langle \mathbf{J}_s \rangle$ might be even larger than $\langle \mathbf{J}_s \rangle$. An estimate for δJ_s can be found by calculating $\langle (\delta \mathbf{J}_s)^2 \rangle$, which can be expressed, according to (1), in terms of the correlation function $K = \langle \delta S_{ik}(\mathbf{r},\mathbf{r}_1) \delta S_{em}(\mathbf{r},\mathbf{r}_1') \rangle$, where $\delta S_{ik} = S_{ik} - \langle S_{ik} \rangle$. To evaluate $K(r,r_1,k_1')$, we must sum, by analogy with the procedure used in Ref. 1, the perturbation-theory diagrams in Fig. 1b, where the solid lines denote the normal and anomalous Green's functions in the superconductor, which are averaged over the occurrences of the random potential, and the dashed lines denote scattering by the impurities. As a result, for $\zeta(0) \gg l$ we find

$$\langle (\delta \mathbf{J}_s)^2 \rangle \approx \langle \mathbf{J}_s \rangle^2 \frac{\zeta(0)\hbar^2}{l(p_F^2 l^2)} \quad . \tag{3}$$

If $\zeta(0) \gg l(p_F l/\hbar)^2$, we have $\langle (\delta \mathbf{J}_s)^2 \rangle \gg \langle \mathbf{J}_s \rangle^2$. The strong fluctuations of the superfluid current (3) are a consequence of the long-range nature of the decay of $\delta S_{ik}(\mathbf{r},\mathbf{r}')$. With $l < |\mathbf{r} - \mathbf{r}'| < \zeta(0)$, we find

$$\langle (\delta S_{ik}(\mathbf{r},\mathbf{r}'))^2 \rangle \sim |\mathbf{r} - \mathbf{r}'|^{-2} \ . \tag{4}$$

We should point out that (3) and (4) are analogous to the corresponding expressions for the current density fluctuations in a normal metal.[4–6]

To evaluate the correlation function $\langle \delta J_{si}(\mathbf{r}) \delta J_{sj}(\mathbf{r}') \rangle$ for $l \ll |\mathbf{r} - \mathbf{r}'| \ll \zeta(0)$, we

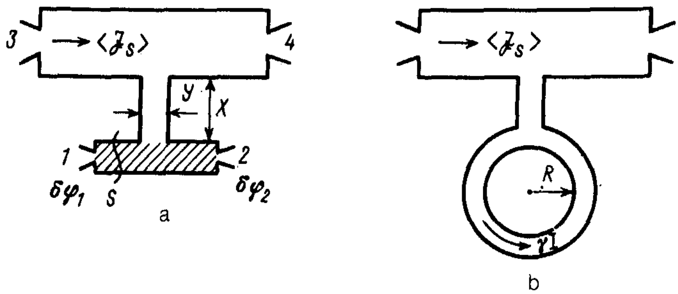

FIG. 2.

must take the sum of the diagrams in Fig. 1c. Another approach, which is equivalent to the first one, involves, according to Ref. 6, the solution of the system of equations

$$\delta \mathbf{J}_s = eN_s \delta \mathbf{v}_s + \mathbf{J}^S_{\rm sc}; \qquad \mathbf{v}_s = \langle \mathbf{v}_s \rangle + \delta \mathbf{v}_s \tag{5}$$

$$\mathrm{div}\ \delta \mathbf{J}_s = 0.$$

Here the quantity $\langle J^s_{\rm sc}(r) J^s_{\rm sc}(r') \rangle \approx \delta_{ik} \delta(\mathbf{r} - \mathbf{r}') \zeta(0) \hbar^4 / (l p_F^2)^2 \langle \mathbf{J}_s \rangle^2$ is determined solely by the diagrams in Fig. 1b.

The mesoscopic fluctuations of the superfluid current are particularly striking in the experimental geometry suggested in Ref. 7, where the sample has the shape illustrated in Fig. 2a. The current in the sample passes through contacts 3 and 4. Contacts 1 and 2 are used to measure the quantity $\Delta\varphi = \delta\varphi_1 - \delta\varphi_2$, where $\delta\varphi_{1,2}$ are fluctuations of the order-parameter phase, which are averaged over the contact dimensions. If $x, y > l$ and $x \gg y$, (see Fig. 2a), we will have $\langle \Delta\varphi \rangle \sim \exp\{-x/y\}$, a quantity which can be ignored. Integrating (5) over the volume v of the sample between contacts 1 and 2 (the hatched area in Fig. 2a), we find

$$\langle (\Delta\varphi)^2 \rangle \approx \left(\frac{m}{eN_s S} \right)^2 \int_v d\mathbf{r}\, d\mathbf{r}' \,\langle J_{{\rm sc},i} (\mathbf{r})\, J_{{\rm sc},i} (\mathbf{r}') \rangle . \tag{6}$$

Here A and $V = SL$ are the cross-sectional area and the volume of the hatched part of the sample (Fig. 2a). The correlation function $\langle J^s_{\rm sc}(\mathbf{r}) J^s_{\rm sc}(\mathbf{r}') \rangle$ is given by the diagrams in Fig. 1b. For $\zeta(0) > x$ we find

$$\langle (\Delta\varphi)^2 \rangle \approx \left(\frac{\hbar^2}{p_F^2 l} \right)^2 \frac{\zeta(0) V}{S^2} \left(\frac{\langle \mathbf{J}_s \rangle m}{eN_s} \right)^2 \approx \left(\frac{\hbar^2}{p_F^2 l} \right)^2 \frac{\zeta(0) V}{S^2} \langle \nabla \varphi \rangle^2 . \tag{7}$$

At $x > \zeta(0)$, an additional factor $\exp(-2x/\zeta(0))$ appears in (7).

If the sample has a shape illustrated in Fig. 2b, then a current δI with a random sign and random magnitude, depending on the $u(r)$, will flow in the ring. Here

$$\langle (\delta I)^2 \rangle \approx \frac{S\zeta(0)}{L^2}\left(\frac{\hbar^2}{lp_F^2}\right)^2 \min[L, \zeta(0)]\langle \mathbf{J}_s\rangle^2 ; \qquad L = 2\pi R. \tag{8}$$

Assuming $\zeta(0) \sim L \sim 10^{-5}$ cm, $\lambda \approx 10^{-7}$ cm, $S \approx 10^{-12}$ cm^2, $l \approx 10^{-6}$ cm, and $\langle J_s \rangle \approx 10^5$ A/cm^2, we find $\delta I \sim 10^{-10}$ A and $\Delta\varphi \sim 10^{-3}$.

If $|\mathbf{r} - \mathbf{r}'| \gg \zeta(0)$, we can find the correlation function $\langle \delta J_{si}(\mathbf{r}) \delta J_{sk}(\mathbf{r}') \rangle$ by solving Eq. (5). To evaluate $\langle J^s_{sc}(\mathbf{r}) J^s_{sc}(\mathbf{r}') \rangle$, however, we must now sum the diagrams in Fig. 1 (b, c, d, and e). The mesoscopic fluctuations of δJ_s at the scale of order $\zeta(0)$ in this case can be described by the density fluctuations of the superfluid electrons. These fluctuations can be determined from the expression $\delta J_{si} = eN_s \delta v_{si} + e\delta N_{s,ik} \langle v_{sk} \rangle$

$$\left\langle \left(\frac{\delta N_s}{N_s}\right)^2 \right\rangle \approx \left\langle \left(\frac{\delta G}{\langle G \rangle}\right)^2 \right\rangle \approx \left(\zeta(0)\frac{lp_F^2}{\hbar^2}\right)^{-2} \tag{9}$$

Here G is the conductance of a normal metallic cube $\zeta(0)$ in size. At distances greater than $\zeta(0)$ the fluctuations of δN_s are independent.

The quantities δJ_s and $\Delta\varphi$ can be measured by point contacts.

By analogy with Refs. 8–10, the quantities $\delta \mathbf{J}_s$, $\Delta\varphi$, and δI are anomalously sensitive to changes in the random potential due to the diffusion of the impurities or relaxation of the two-level systems in glasses. The other way one can observe the fluctuations of $\delta \mathbf{J}_s$ and $\Delta\varphi$ is by placing the sample into a capacitor with a voltage U.[11] The electric field penetrates into the metal to a depth corresponding to the Debye length, causing the conditions governing the scattering at the surface to change. The probability amplitude of the diffusion of an electron over a distance $\zeta(0)$ in this case will acquire an additional phase and the quantities $\delta \mathbf{J}_s$, $\Delta\varphi$, and δI will oscillate with the amplitudes (3), (7), and (8) as the voltage U is changed.

Finally, the fluctuations which we have studied can determine the pinning of vortices in type-II superconductors and also the critical-current fluctuations in quasi-one-dimensional superconducting channels.

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