Approximate incorporation of gravitation in the theory of galaxy formation

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(Submitted 21 March 1988)
Pis'ma Zh. Eksp. Teor. Fiz. **47**, No. 8, 365–367 (25 April 1988)

Ya. B. Zel'dovich's well-known theory of the formation of galaxies from the primordial cloud of a Big Bang does not incorporate the effect of gravitation. An approximate method for incorporating gravitation is proposed in the present letter. Some very simple pancake solutions are pointed out.

1. In 1970, Ya. B. Zel'dovich analyzed[1] a very simple model for the coalescence of galaxies from the dust of a primordial Big Bang. In that model it is assumed that the dust grains move freely by inertia, so their coordinates are $x_i = x_i^0 + v_i^0 t$, where x_i^0 is the initial position, and v_i^0 the initial velocity, of the dust grains. In Lagrangian variables, the dust density then varies in accordance with

$$\rho = \rho_0 / \det | \partial x_i / \partial x_j^0 |, \quad \partial x_i / \partial x_j^0 = \delta_{ij} + \nu_{ij} t, \qquad \nu_{ij} = \partial v_i^0 / \partial x_j^0 , \tag{1}$$

and in the simplest case the matrix ν_{ij} can be assumed to be symmetric. Converting the determinant to principal axes, we can then find the expression

$$\rho = \rho_0 / (a_1 + b_1 t)(a_2 + b_2 t)(a_3 + b_3 t). \tag{2}$$

One of the numbers a_i/b_i may turn out to be negative; in such a case we would have $\rho \to \infty$ in the limit $t \to -a/b$, so we would have a flat or pancake-shaped galaxy.

2. This model totally ignores gravitation, which could apparently play an important role during the self-contraction of various dust formations. To correct this situation, we start from the equations of gas dynamics with gravitation:

$$\partial \rho / \partial t + \operatorname{div} \rho \mathbf{v} = 0, \quad \partial \mathbf{v} / \partial t + (\mathbf{v} \nabla) \mathbf{v} = - \nabla \varphi ; \qquad \Delta \varphi = 4\pi G \rho , \tag{3}$$

where G is Newton's gravitational constant. As in Ref. 1, we are ignoring the pressure. Equations (3) are nevertheless quite complicated and—the main point—indefinite because of Seelinger's paradox, which leads to an infinite value for the potential φ in the case of a uniform density ρ in an infinite space. One way out of this difficulty might be to introduce a finite region, which is filled with a density ρ, and to analyze a time-varying solution with an expanding sphere (the universe). This approach would correspond to Miln's model,[2] which is a nonrelativistic analog of the well-known Friedmann solution of the equations of the general theory of relativity.

However, since we are interested not in the entire universe but only the problem of the self-contraction (collapse) of nucleating galaxies, which may be thought of as small local fluctuations, we take the following approach in order to incorporate gravitation approximately: We assume that in the equation $\Delta\varphi = 4\pi G\rho$ we can replace the

Laplacian by the expression $\Delta \to k^2$, where $k = 2\pi/\lambda_0$ and λ_0 is an average distance between galaxies at the time of their nucleation. We then have $\varphi = -\rho G\lambda_0^2/\pi$ and Eqs. (3) become

$$\partial \rho/\partial t + \operatorname{div} \rho \bar{v} = 0, \qquad \partial \bar{v}/\partial t + (\bar{v}\nabla)\bar{v} = (1/\pi)G\lambda_0^2 \nabla \rho . \tag{4}$$

These equations have, in particular, the homogeneous solution $\rho = \text{const}$ with $\bar{v} = 0$. This approximate method for incorporating gravitation thus makes it possible to avoid Seelinger's paradox.

3. Equations (4) could have many different solutions, which depend on the form of the initial perturbations. For the discussion below, however, we will examine the simplest self-similar solution of these equations, in the form of a triaxial ellipsoid,

$$\rho(t, x, y, z) = \rho_0(t)\left(1 - \frac{x^2}{X^2} - \frac{y^2}{Y^2} - \frac{z^2}{Z^2}\right), \tag{5}$$

with semiaxes $X(t)$, $Y(t)$, and $Z(t)$ whose lengths depend on the time. Substituting (5) into Eqs. (4), we find the equations

$$\left.\begin{aligned} &v_x = x\dot{X}(t)/X, \quad v_y = y\dot{Y}/Y, \quad v_z = z\dot{Z}/Z, \quad \rho_0(t) = M_0/XYZ \\ &\ddot{X} = -\partial U/\partial X, \quad \ddot{Y} = -\partial U/\partial Y, \quad \ddot{Z} = -\partial U/\partial Z, \quad U = -C/XYZ, \end{aligned}\right\} \tag{6}$$

where M_0 and $C = (2/\pi)M_0 G\lambda_0^2$ are positive constants. We thus see that the quantities X, Y, and Z can be thought of as the three Cartesian coordinates of a "point" with a unit mass which is moving in a force field with a potential "energy" $U = -C/XYZ$. Since this field is not spherically symmetric, the equations of the motion of the point, (6), would have to be integrated numerically. There is, however, the possibility of the special case $X = Y = Z = R(t)$, in which ellipsoid (5) is a sphere with a radius which varies in accordance with

$$\ddot{R} = -3C/R^4, \quad \dot{R} = -\sqrt{2C}\, R^{-3/2}, \quad R(t) = (25C/2)^{1/5}(-t - |t_*|)^{2/5}, \tag{7}$$

where $-\infty < t < -|t_*|$. At the "initial time" $t = -\infty$, the velocity at which the sphere is contracting is zero, in contrast with Zel'dovich's model,[1] and at the critical time $t = -|t_*|$ the sphere contracts to a point.

In general, we should assume that the "particle" X, Y, Z is not moving exactly along the radius and that at some instant it reaches one of the three planes $X = 0$, $Y = 0$, $Z = 0$ (Fig. 1). In such an event, ellipsoid (5) degenerates into a "pancake," which is the model of a plane galaxy. Zel'dovich's version,[1] which ignores gravitation, can be found from the equations of the present letter by setting $G = 0$ and $U = 0$. In this case we have $\ddot{X} = \ddot{Y} = \ddot{Z} = 0$, so that the point moves exclusively by inertia, at a constant velocity caused by the initial jolt (fluctuation). As a result, we find Zel'dovich's expression for the density, (2). Our model with an increasing collapse velocity apparently gives a better description of the process by which the formation of galaxies begins.

4. In conclusion we think it is useful to point out that equations of a more general

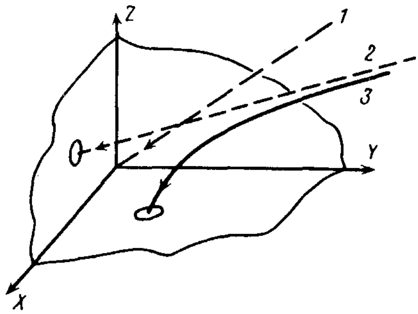

FIG. 1. Motion of a "point" XYZ (the axes of an ellipsoid). 1—Along the radius, (7), with $X=Y=Z$ (a sphere); 2—along a straight line in the case $U=0$ (Zel'dovich's solution, which ignores gravitation); 3—along a curve in the case $U=-C/XYZ$, with gravitation.

type [cf. (4)] were analyzed in Refs. 3 and 4:

$$\partial \rho_* / \partial t + \operatorname{div} \rho_* \bar{v} = 0, \qquad \partial \bar{v} / \partial t + (\bar{v} \nabla) \bar{v} = m c_0^2 \nabla \rho_*^{1/m} , \tag{8}$$

where $\rho_* = \rho/\rho_0$ is the dimensionless density divided by its initial unperturbed value. Equations (8) describe about 50 unstable media with various values of the "azimuthal number" m, which is equal to 1 in our problem of gravitational collapse, (4).

For arbitrary m, Eqs. (8) could also have a partial solution corresponding to a triaxial ellipsoid with a density

$$\rho_* = \rho_0(t) \, [1 - (x/X)^2 - (y/Y)^2 - (z/Z)^2]^m , \tag{9}$$

which again reduces to Eqs. (6). The equation of motion of the point, $\ddot{\mathbf{R}} = \hat{\nabla} C/XYZ$, arises in problems of the evolution of all of these 50 unstable media, so it is of fundamental importance.

The possibility of a solution of the type in (9) for these 50 media was pointed out to us by S. I. Anisimov and also by S. Yu. Shasharina, to whom the author is extremely grateful.

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Translated by Dave Parsons