States of charmonium with a nonzero orbital angular momentum

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A phenomenological theory of charmonium states with a nonzero orbital angular momentum is derived in QCD.

A physical picture of the structure of mesons consisting of c and $\bar{c}$ quarks can be outlined as follows. Since valence quarks in mesons with an orbital angular momentum L which is not too large move slowly, the average gluon fields have time to adjust to this slow motion. As a result, the adiabatic approximation is applicable in describing the forces which confine quarks.[1] In cases in which the quarks are separated by a distance small in comparison with the characteristic distance of QCD, $R_c \approx (1\ \text{GeV})^{-1}$, this average field describes a Coulomb attraction with a small, slowly growing coupling constant $\alpha_s(r)$. At $r \approx R_c$, perturbation theory breaks down: There is a transition to the regime of strong coupling. In other words, the gluodynamic string which is stretched out between the quarks begins to dominate the interaction of quarks at $r > R_c$. For levels with $L \neq 0$, the quarks are for the most part separated from each other by a large distance $\langle r \rangle \geqslant 2R_c$, where the interaction is dominated by low-energy excitations of the string (transverse vibrations of the string), and the potential energy is

$$U(r) = kr - \alpha_* / r. \tag{1}$$

The energy $U(r)$ has a property which is very important for our purposes: With increasing L, both the radius of the orbit and the width of the region occupied by the quantum-mechanical paths of a quark increase. As a result, distances $r < R_c$ play a lesser role, and the region in which expression (1) holds plays a greater role.

It is a trivial matter to generalize (1) to the case of nonrelativistic quarks, with paths which do not intersect (there are the dominant paths in charmonia with $L \neq 0$). The amplitude for the transition of a quark along a given path is proportional to the Wilson expectation value:

$$W[C] = \langle \operatorname{tr} P_C \exp \{ i \int_C d\tau \dot{x}_\mu(\tau) A_\mu(x(\tau)) \} \rangle .$$

For a rectangular contour, this expectation value is $\exp[iTU(r)]$, where T is the time over which quarks went from the initial state to the final state. The simplest generalization of expression (1) to the case of interest here is

$$W[C] = \exp\left\{ ikA_{min}[C] + \frac{1}{2\pi} \oint\oint dx_\mu dy_\nu D_{\mu\nu}(x-y) \right\}, \tag{2}$$

where $A_{\min}[C]$ is the area of the minimal surface which can be attached to contour C, and $D_{\mu\nu}(x) = \alpha_* \delta_{\mu\nu}/(x^2 - i0)$. The expectation value $W[C]$ given by (2) incorporates the basic properties of loop expectation values in gauge theories,[2] and it gives us expression (1) as the interaction energy of static quarks.

We turn now to a description of a phenomenological theory of charmonium states with $L \neq 0$. We begin with the amplitude for a transition of this system from the "in" state to the "out" state over a time T. We take an average over the fields of quarks and gluons with wavelengths $< 1/m$. The renormalizability of QCD guarantees that the form of the action will not change in the logarithmic approximation, while the parameters which appear in it are replaced by renormalized parameters. As in QED, we have an additional term $\bar{\psi}\sigma^{\mu\nu}F_{\mu\nu}\psi/m$, where $\sigma_{\mu\nu} = i/2[\gamma^\mu,\gamma^\nu]$ is responsible for the anomalous chromomagnetic moment of the quark. We are ignoring small contributions of higher order in $1/m$ and also $(\bar{\psi}\psi)^2$. The effective action of the quarks in a long-wavelength field of gluons is bilinear in the quark field:

$$\bar{\psi}\,(G^{-1} = -m + i\hat{\nabla} + \frac{g}{4m}\sigma^{\mu\nu}F_{\mu\nu}(A))\,\psi\ , \tag{3}$$

where m is the structure mass of the c quark, $\hat{\nabla} = \gamma^\mu(\partial_\mu - iA_\mu)$, and g is the anomalous "magnetic" moment of the c quark. Carrying out a Foldy-Wautheisen transformation in (3) for $\bar{\psi}$ and ψ we find an expansion of the quark part of the action in p^2/m^2:

$$G^{-1} = i\gamma_0\partial_t + H_0 + H_{int}\,; \qquad H_0 = \gamma_0\left[m + \frac{\mathbf{P}^2}{2m}\right] + A_0\ , \tag{4}$$

$$H_{int} = -\gamma_0\frac{\mathbf{P}^4}{8m^3} - \frac{(1+g)}{2m}\gamma_0\vec{\sigma}\mathbf{B} - \frac{(1+2g)}{8m^2}[i\vec{\sigma}\,\vec{\nabla}\times\mathbf{E} + 2\vec{\sigma}\mathbf{E}\times\mathbf{P} + \vec{\nabla}\mathbf{E}], \tag{5}$$

where $\mathbf{P} = \vec{\nabla}/i$, $\vec{\nabla} = \vec{\partial} - i\mathbf{A}$, and σ are the Pauli matrices. In the leading approximation, it is a straightforward matter to derive a Feynman representation for the transition amplitude:

$$K(in \to out) = \int\limits_{in \to out} Dx_+(t)Dx_-(t)\exp\left\{ i2mT + i\int_0^T dt\, m(\dot{x}_+^2 + \dot{x}_-^2)/2 \right\} W[C], \tag{6}$$

where $x(t)$ and $\mathrm{x}(t)$ are the coordinates of the quark and antiquark, respectively, and $W[C]$ in (6) incorporates an average over long-wavelength fluctuations of the gluon fields. For the dominant quark paths, it is given by (2). The contributions from the term with $D_{\mu\nu}(x)$ are of the same form as the well-known electromagnetic interaction of charged particles, and in the leading approximation in v^2/c^2 they reduce to a "Coulomb" potential. In the first approximation in v^2/c^2, the solution of the equations of motion of a string, $\sigma A_{\min}[C] = 0$, is

$$x_0(t,\sigma) = t\,; \qquad x(t,\sigma) = R + \sigma r/2 \tag{7}$$

where $0 \leqslant t \leqslant T$, $-1 \leqslant \sigma \leqslant 1$, $r = [x_+(t) - x_-(t)]$, and $R = [x_+(t) - x_-(t)]/2$. Substituting (7) into the expression for the area of the minimal surface, we find the following result in the c.m. frame ($\dot{R}=0$):

$$A_{min} = r - r^3\dot{n}^2/24 , \tag{8}$$

where $r=|\mathbf{r}|$ and $n=\mathbf{r}/r$. The quantity R plays the role of a collective coordinate of the center of mass in integral (6). We now seek states of mesons which have a zero momentum. In this case, the integration over R can be carried out explicitly; it reduces to the conservation of the resultant momentum of the meson. In the leading approximation, the Hamiltonian of the system is therefore

$$H_0 = \frac{p_r^2}{m} + \frac{\mathbf{L}^2}{2I} + kr - \frac{\alpha_*}{r} , \tag{9}$$

where $I=mr^2(1+kr/6m)/2$ is the moment of inertia of the system (the quark plus the string). Since the moment of inertia of a string is small in comparison with that of quarks, we can expand $1/I$ in a series in (9). The correction of first order in k/m^2 has the shape of a Coulomb potential; it changes α_* in (9) to

$$\alpha(L) = \alpha_* + k\mathbf{L}^2/6m^2 . \tag{10}$$

As a result, the effective Coulomb coupling constant depends on the orbital angular momentum. Restricting the discussion to the zeroth-approximation Hamiltonian H_0, we find the potential model of Ref. 3, but with an L-dependent α.

The point of primary interest in charmonium theory is calculating the spin-orbit and spin-spin splittings. In mesons, with $L\neq 0$ the spin splitting is greatly suppressed, since the quarks are separated by large distances, where α is independent of r, and the string does not give rise to a spin-spin interaction. It is a straightforward matter to refine expression (6), through the incorporation of a correction of first order in $H_{\rm int}$. The first term in (5) leads to a trivial change in the kinetic energy of the quark, while the other terms modify the Wilson expectation value in expression (6). In the first approximation in $1/m^2$, they reduce to various combinations constructed from

$$\langle\, \mathrm{tr}F_{\mu\nu}(x)P_c \exp\{ i\int_c d\tau \dot{x}_\mu(\tau)A_\mu(x(\tau))\}\,\rangle = \frac{\delta W[C_x]}{\delta S_{\mu\nu}(x)} . \tag{11}$$

Equation (11) is the well-known Mandel'shtam formula, and $\delta/\delta S_{\mu\nu}(x)$ is the variational derivative of $W[C]$ with respect to the oriented area. The derivative of $A_{\rm min}$ $[C]$ with respect to $\delta S_{\mu\nu}(x)$ is $t_\mu\nu(x)$ (Ref. 2), the orientation tensor of the tangent plane. For the case of slow motion of the quarks, we can write the following expressions in the coordinate system which we have adopted:

$$t_{0i} = \frac{n_i}{(1-\dot{n}^2r^2/4)^{1/2}} ; \qquad t_{ij} = \frac{r(\dot{n}_i n_j - \dot{n}_j n_i)}{2(1-\dot{n}^2r^2/4)^{1/2}} . \tag{12}$$

Using (6) and (12), we find the contribution of the string to the spin-orbit interaction:

TABLE I. Choice of the parameters of the model.

k, GeV^2	m_c GeV	α_*	g
0.16	1.458	0.54	0.33
0.17	1.446	0.55	0.28
0.176	1.438	0.556	0.25
0.18	1.434	0.56	0.23

$$H_{LS}\,(\text{string}) = -\frac{k}{2m^2 r}\left(1+\frac{\mathbf{L}^2}{2m^2r^2}\right)\mathbf{LS}\,, \tag{13}$$

where $\mathbf{S} = (\vec{\sigma}_1 + \vec{\sigma}_2)/2$ is the total spin of the meson. It is easy to see that the contributions from the differentiation of the D function in Wilson expectation value (2) are the same as the corresponding contributions which are found in QED for positronium.[4] The correction to Hamiltonian H_0 is therefore

$$H_1 = H_{LS}(\text{string}) + \frac{\alpha_*}{m^2}\left(-\frac{\mathbf{p}^2}{r}+\frac{\mathbf{L}^2}{2r^3}\right) - \frac{\mathbf{p}^4}{4m^2}$$

$$+\frac{(3+4g)\alpha_*}{2m^2r^3}\,\mathbf{LS} + \frac{3\alpha_*(1+g)^2}{2m^2r^3}\,S_iS_j(n_in_j-\delta_{ij}/3). \tag{14}$$

TABLE II. Comparison of the charmonium mass spectrum for $k = 0.176$ GeV^2 with the experimental $N^{(2S+1)}L_J$ level, where N is the principal quantum number, S is the total spin, L is the orbital angular momentum, and J is the total angular mometum (the levels selected for fixing the parameters of the model are underscored).

Level	Theory	Experiment	Level	Theory	Experiment
1^1P_1	3.525	3.525	1^1D_1	3.816	–
1^3P_J, $J=0$	<u>3.415</u>	3.415	1^3D_J, $J=1$	3.805	3.770
$J=1$	<u>3.511</u>	3.511	$J=2$	3.820	–
$J=2$	<u>3.556</u>	3.556	$J=3$	3.817	–
2^1P_1	3.931	–	2^1D_2	4.661	–
2^3P_J, $J=0$	3.826	–	2^3D_J, $J=1$	4.152	4.159
$J=1$	3.916	–	$J=2$	4.169	–
$J=2$	3.962	–	$J=3$	4.171	

The terms describing the contact interaction, i.e., the terms which are proportional to $\delta(r)$, have been discarded here, since we have $\Psi_L(0) = 0$ for states with $L \neq 0$.

Hamiltonian (9), (10), (13), (14) contains the four parameters k, m, α_*, and g. The string tension k is known from the slope of the Regge paths, and the three other parameters were determined from data on the $1P$ level of charmonium. Calculations were carried out for several values of k. The results are shown in Tables I and II (the experimental data were taken from Ref. 5).

From the comparison with experiment we see that the position of the 2^2D_1 level is essentially the same as the experimental position, while the position of the 1^3D_1 level is only 30 meV above the experimental level. At the accuracy of our model, we could not hope to find a better agreement. The data presently available on heavy mesons are not a sufficient basis for a reliable test of the hypotheses which have been adopted. On the other hand, these data are sufficient for determining the parameters of the c quark and α_* (Table I) and for thus predicting the positions and splittings of the as yet unknown P, D, etc., levels of charmonium.

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