Josephson medium in high-temperature superconductivity: vortices and critical magnetic fields (theory)

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(Submitted 21 March 1988)
Pis'ma Zh. Eksp. Teor. Fiz. **47**, No. 8, 415–418 (25 April 1988)

If the volume of a high-temperature superconductor is divided into grains whose boundaries are Josephson junctions, the Meissner state will be disrupted in a very weak field H_{c1}—weaker than the field of the earth. The experimental values of the lower critical field, which are significantly higher than this value, should be associated with another critical field, H_i, at which vortices begin to penetrate into the volume of the grains.

The experimental data presently available on high-temperature superconductors indicate that these superconductors apparently have very nonuniform structures and must be described by a model which was used previously to describe granular superconductors (Refs. 1–3, for example). In this model, the superconductor is broken up into superconducting grains, between which there are weak links (Josephson junctions). According to Deutscher and Müller,[4] this breakup of the volume of a high-temperature superconductor into grains occurs not only in ceramics but also in single crystals, in which the grain boundaries coincide with twin boundaries. In this letter we examine the implications of this model for the structure of vortices and for the critical magnetic fields.

If the link between grains is sufficiently weak, it can be assumed that the phase, like the modulus, of the order parameter remains uniform within a grain, and the entire change in the phase occurs at boundaries separating grains. Such a system can then be described by the XY model, whose nodes correspond to grains, while the phase difference between grains corresponds to the angles between the spins of the nodes in the XY model.[1–4] We use the equations of a self-consistent field for this model in the continuum limit, in which the sequence of jumps at the boundaries between grains is replaced by a continuously and smoothly varying phase (φ) of the order parameter, averaged over the grains. In the London region, the order parameter is characterized exclusively by the phase φ, and the average value of the average energy for a Josephson medium can be written

$$\mathcal{F} = \int \left[\frac{g}{2} \left(\vec{\nabla} \varphi - \frac{2\pi \mathbf{A}}{\phi_0} \right)^2 + \frac{1}{8\pi\mu} (\mathrm{curl}\, \mathbf{A})^2 \right] dV . \tag{1}$$

Here $\mathbf{A}$ is the vector potential which determines the induction $\mathbf{B} = \mathrm{curl}\, \mathbf{A}$, which is equal to the magnetic field averaged over a grain, μ is the magnetic permeability of the Josephson medium (we will be discussing it below), $\phi_0 = hc/2e$ is the fluxoid, and g is

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the stiffness, equal in order of magnitude to $E_J d$ for a weak link: $E_J d \ll (\hbar^2/m) n_s$, where E_J is the energy of the Josephson junction per unit area, d is the grain size, and n_s is the superfluid density within a grain. Using (1), and working in the standard way, we find expressions for the penetration depth λ, the vortex energy ϵ_v, and the lower critical field $H_{c1} = 4\pi\epsilon_v/\phi_0$:

$$\lambda^2 = \frac{\phi_0^2}{16\pi^3 g\mu}, \quad \epsilon_v = \frac{\phi_0^2}{(4\pi\lambda)^2\mu} \ln\frac{\lambda}{d} = \pi g \ln\frac{\lambda}{d}, \quad H_{c1} = \frac{4\pi^2 g}{\phi_0} \ln\frac{\lambda}{d} \sim \frac{E_J d}{\phi_0} \ln\frac{\lambda}{d}. \tag{2}$$

Here we have taken account of the circumstance that the size of the vortex core, i.e., the coherence length in this model, is on the order of the grain size d. If d is much smaller than the London penetration depth $\lambda_L = \phi_0\sqrt{m}/2h\sqrt{\pi n_s}$, the magnetic permeability μ will be close to unity: $1 - \mu \sim d^2/\lambda_L^2$. Furthermore, the penetration depth λ will be significantly greater than not only λ_L but also $\lambda_J \sim \phi_0/\sqrt{E_J \lambda_L}$, which is the depth to which the magnetic field penetrates into an isolated Josephson junction (this case was studied in Ref. 5). In the other limit, $d \gg \lambda_L$, the permeability $\mu \sim \lambda_L/d$ is small because the field penetrates only slightly into the volume of the grains, but the field penetrates a long way along the surfaces of Josephson junctions, to a depth $\lambda \sim \lambda_J$.

The average theory of a Josephson medium which follows from (2) can be used as long as the penetration depth λ is greater than d. If the condition $\lambda_J \gg \lambda_L$ holds, on the other hand, and if we also have $d \gg \lambda \sim \lambda_J$, then the sizes of the vortex and the field H_{c1} will be the same as for an isolated Josephson junction (Fig. 1a). In the region $d \gg \lambda_J^2/\lambda_L$ and $\lambda_J \ll \lambda_L$, on the other hand ($E_J d$ and $E_J \lambda_L \gg (\hbar^2/m) n_s$), multiple Josephson junctions will have no effect of any sort on the properties of the superconductor, in which the penetration depth is the London depth, and the vortices are Abrikosov vortices. Where the average theory of a Josephson medium is valid, however, i.e., at $\lambda \gg d$, the size of the vortices (the penetration depth) is very large, and the structure of the vortices differs at different spatial scales. At scales exceeding the grain size d, where the average theory works, the magnetic field and the phase vary smoothly with distance. At scales on the order of d and below, however, the phase (and the magnetic field, if $d \gg \lambda_L$) is highly nonuniform: The entire change in this phase occurs at the boundaries between grains. This circumstance may be responsible for a pronounced pinning of such vortices. We will call them "hypervortices" to distinguish them from Abrikosov and Josephson vortices. They are shown in Figs. 1b and 1c for the cases $d \gg \lambda_L$ and $d \ll \lambda_L$. According to (2), the expressions for H_{c1} in these two cases differ only in the value of λ in a logarithm, so the estimate given below for H_{c1} applies to both cases.

In a grainy system, the resistance begins to decrease at the superconducting transition temperature T_c in the interior of the grains, and the resistance disappears at a temperature $T_J \sim E_J d^2$, where the links between grains establish an infinite long-range order.[2,3] The latter relation can be used to estimate E_J in the expression for H_{c1} [see (2)]. If we have $E_J \sim (T_c - T)^\kappa$ near the transition, then we have $H_{c1} \sim (T_J/\phi_0 d)[(T_c - T)/(T_c - T_J)]^\kappa$, where the index κ is equal to 1 for a Josephson junction

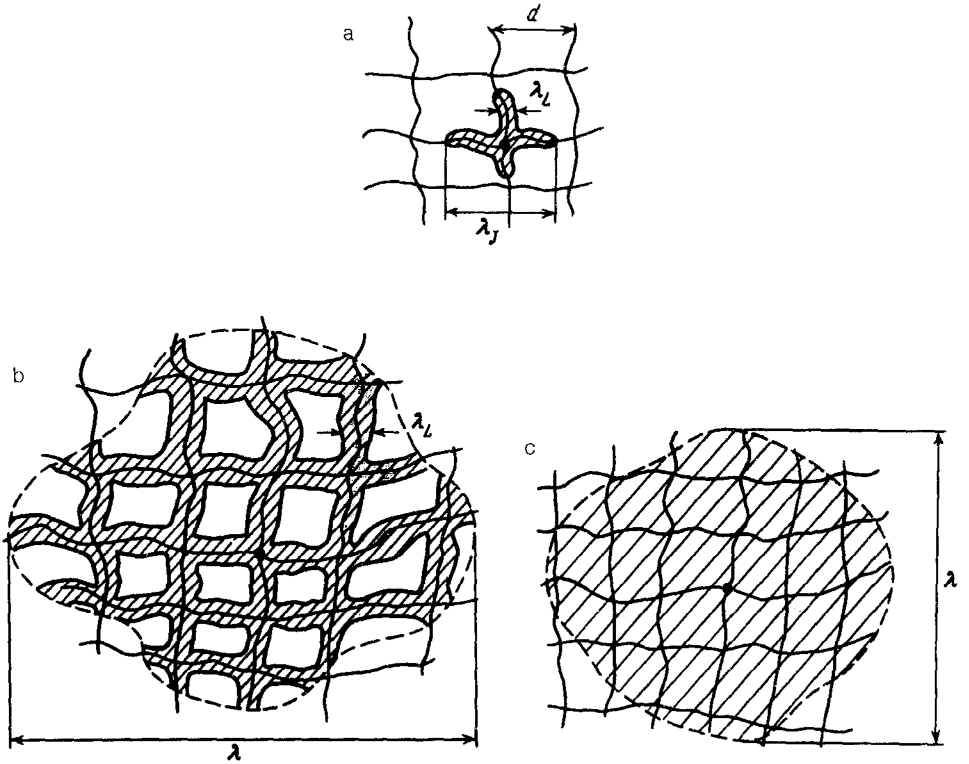

FIG. 1. Vortices in a Josephson medium. The hatching shows the region in which the magnetic field penetrates. The large point is the center (core) of a vortex, where the phase is undefined. a—Josephson vortex, $\lambda_J = \phi_0/\sqrt{E_J\lambda_L} \gg \lambda_L$, $d \gg \lambda_J$, $H_{c1} \sim \phi_0/\lambda_J\lambda_L$; b—cellular hypervortex, $\lambda = \lambda_J \gg d$, $d \gg \lambda_L$, $H_{c1} \sim \phi_0/\lambda^2\mu \sim E_J d/\phi_0$; c—continuous hypervortex, $\lambda = \lambda_J\sqrt{\lambda_L/d}\,\lambda_L \gg d$, $\lambda_J \gg \sqrt{d\lambda_L}$, $H_{c1} \sim E_J d/\phi_0$.

and 2 for a weak link through a normal metal (a proximity effect).[1)] Setting $T \sim 100$ K and $d \sim 10^{-4}$ (Ref. 4), and ignoring the temperature factor $[(T_c - T)/(T_c - T_J)]^\kappa$, we find H_{c1} to be on the order of 10^{-3} Oe. Incorporating the temperature factor, even in the case of a very sharp transition, with $T_c - T \sim 1$ K and $\kappa = 2$, increases this estimate by no more than two orders of magnitude for liquid-nitrogen temperatures. In other words, H_{c1} does not exceed the magnetic field of the earth. On the other hand, the lower critical fields found experimentally are usually considerably higher. Accordingly, the model of a Josephson medium can be associated with the experimental results only if we assume that the high-temperature superconductor is in a mixed state even in a "zero" field (a measure of which would be the magnetic field of the earth), but this mixed state would be of a special kind, different from the mixed state of homogeneous type-II superconductors. These differences stem from the specific features of the hypervortices which apparently are very immobile: The phase contour lines which pass within the boundaries of grains must "jump across" grains as a hypervortex moves. The core of the hypervortex moves only along curved two-dimensional boundaries of the grains. The effect is to facilitate the pinning of the hypervortex and to increase the resistance to its motion. The low mobility of hypervortices should lead to a low resistivity, which would simulate a true superconducting state with significant critical currents. In this case, however, we might observe an incomplete Meissner effect if the condition $d \lesssim \lambda_L$ held, and the magnetic susceptibility might depend on the history of the sample, specifically, on the magnetic field in which the sample was cooled. These aspects of behavior have been seen experimentally in several places. Just what are we to conclude from the magnetic field on the order of 10 or 100 Oe which has been reported as the field H_{c1} in some experimental papers? In our opinion, this is an intermediate field H_i at which the vortices begin to penetrate into the interior of grains. If the condition $d \ll \lambda_L$ holds, then the magnitude of this field is $H_i \sim \phi_0/d^2$; i.e., it is an upper critical field H_{c2} in the theory of an average Josephson medium in which the grain size d is the coherence length. However, the field H_i is not the field at which superconductivity is disrupted but the field above which we cannot use the theory of an average Josephson medium, since the distance between vortices, $b \sim \sqrt{\phi_0/H}$, becomes smaller than d. Deutscher and Müller[4] identified the field H_i as the field H_{c1} above which a superconducting glass phase appears. In the case $d \gg \lambda_L$, the intermediate field H_i, at which vortices begin to penetrate into the interior of grains, is the same as the field $H_{c1} = (\phi_0/4\pi\lambda_L^2)\ln(\lambda_L/\xi)$ for a homogeneous superconducting medium within the grains with a coherence length ξ. The theory of an average Josephson medium ceases to apply in weaker fields $H_i^* \sim \phi_0/d\lambda_L$ (inductions $B_i^* \sim \phi_0/d^2$), in which the distance between vortices, $b \sim \sqrt{(\phi_0/H)(d/\lambda_L)}$, reaches a value on the order of d. In the field interval $H_i > H > H_i^*$ the vortices occupy the surfaces of the boundaries between grains, and the distance between them is $b \sim \phi_0/H\lambda_L < d$.

The most valid test of the idea proposed above would be an experiment in a very weak magnetic field with shielding against the field of the earth. Such an experiment would make it possible to identify the actual lower critical field for the Josephson medium on the basis of the shape of the current-voltage characteristic and the diamagnetic response.

I am indebted to K. B. Efetov, A. I. Larkin, and Yu. N. Ovchinnikov for useful discussions.

[1)] The estimate of H_{c1} was originally cruder, ignoring the temperature dependence of E_J, so it was valid only far from T_c and for a fairly diffuse transition. A. I. Larkin suggested that I incorporate the temperature factor in the estimate of H_{c1}.

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Translated by Dave Parsons