Coherent amplification of pulses by a nonresonant two-level medium

É. M. Belenov, P. G. Kryukov, A. V. Nazarkin, A. N. Oraevskiĭ, and A. V. Uskov
P. N. Lebedev Physics Institute, Academy of Sciences of the USSR

(Submitted 23 February 1988; resubmitted 25 March 1988)
Pis'ma Zh. Eksp. Teor. Fiz. **47**, No. 9, 442–444 (10 May 1988)

A solution of the Maxwell-Bloch equations, which describes the coherent propagation of a light pulse amplified by a two-level medium, is presented. All the energy stored in the medium is removed by the pulse, and the frequency of the pulse increases as it is amplified.

1. The dynamics of a coherent propagation of an intense light pulse of strength $\mathcal{E}(z,t) = E_0(z,t)\cos(\omega_0 t - k_0 z + \varphi_0(z,t))$, which is at resonance with the frequency ω_0 for the transition of two-level particles, is described by the system of equations

$$\mathcal{E}_{zz} - \mathcal{E}_{tt}/c^2 = (4\pi N/c)\mathcal{P}_{tt}, \tag{1}$$

$$_{tt} + \omega_0^2 \mathcal{P} = -(2\mu^2\omega_0/\hbar)\mathcal{E} n, \qquad n_t = (2/\hbar\omega_0)\mathcal{E}\mathcal{P}_t . \tag{2}$$

Here N is the particle density, μ is the dipole moment of the transition, and n is the difference in the poulations of the levels (n varies between ± 1). The relaxation terms were dropped in the constitutive equations (2) (specifically, we assumed $\mathcal{P}_t/T_2 = n/T_1 = 0$, where T_2 and T_1 are the polarization and particle-number relaxation times). The absence of relaxation terms allows us to describe, in terms of Eqs. (1) and (2), effects such as the induced self-transparency (in the case of absorbing medium)[1–3] or the total removal (in an amplifying medium) of the particle energy by the field during the propagation of the pulse.[2,4,5] Here the increase in the pulse energy with increasing z is linked with the increase ($\sim z$) in the number of field photons and the increase in the power level is related to the pulse compression. The envelope $E_0(z,t)$ of the field is an oscillating function, whose oscillating frequency increases as $\sim z$. For large values of z the oscillating frequency is higher than the transition frequency and the envelope approximation [for $E_0(z,t)$ and $\mathcal{P}(z,t)$], which assumes that the condition $|(\varphi_0)_t| \ll \omega_0$ is satisfied, can no longer be used to describe the dynamics of the propagation of a pulse.

We describe below the evolution of an intense pulse in an amplifying medium in a field in which the slowly varying amplitude and phase cannot be singled out and when the following condition for the field-induced polarization is satisfied:

$$|\mathcal{P}_{tt}| \gg \omega_0 |\mathcal{P}| . \tag{3}$$

The frequency $\omega(z)$ of the photons which comprise the pulse field changes with increasing z and increases without bound in the limit $z \to \infty$. The pulse loses the energy it accumulated in the medium and the field energy increases without an increase in the number of pulse photons.

2. By discarding the term $\omega_0^2 \mathscr{P}$ in Eq. (2) we can integrate the constitutive equations for any field shape $\mathscr{E}(z,t)$. Specifically, $n = \cos \psi$, where the function

$$\psi = \frac{2\mu}{\hbar} \int_{-\infty}^{t} \mathscr{E}(z, t)\, dt \tag{4}$$

satisfies the sine-Gordon equation

$$\psi_{\xi\eta} = \Omega^2 \sin \psi, \qquad \Omega^2 = \frac{2\pi N}{\hbar c^2} \omega_0 , \tag{5}$$

$$\xi = t + z/c, \qquad \eta = t - z/c . \tag{6}$$

Equation (5) has a solution with a self-similar variable $u = \Omega^2 \xi \eta$ (see Refs. 2 and 4), ψ varies in accordance with the equation

$$u\, \psi_{uu} + \psi_u - \sin \psi = 0, \tag{7}$$

and the field strength $\mathscr{E}(z,t)$ is related to the function ψ by

$$\mathscr{E}(z, t) = \frac{\hbar \Omega^2}{\mu} t\, \psi_u . \tag{8}$$

Equation (7) has regular solutions[2] at $u = 0$. These solutions are such that ψ_u is an oscillating, sign-changing function of the wave-packet type, which does not vanish near the point $u = 0$. Since the field strength $\mathscr{E}(z, t)$ is nonvanishing only at $\eta \approx 0$, we can write expression (12) in the form

$$\mathscr{E}(z, t) = \frac{\hbar \Omega^2}{c\mu} z\, \psi_u \left[2 \frac{\Omega^2}{c} z \left(t - \frac{z}{c} \right) \right] . \tag{9}$$

Mathematically, the evolution of the field strength $\mathscr{E}(z, t)$ of the pulse (9) reproduces the evolution, which was described in Refs. 2, 4, and 5, of the envelope of the pulse E_0 (z, t) which propagates in an amplifying resonant medium with the relaxation constants $T_1 = T_2 = \infty$.

Field (9) consists of subpulses of area $\sim \pm 2\pi$, the total area of the subpulses $\psi(z, \infty)$, equal to π, remains the same, and the field removes all the energy stored in the matter. The physics of the field evolution changes dramatically, however, as a result of switching to propagation conditions under which the effect can be described only in terms of the total field strength and total field polarization. In the first case (envelope model) the pulse contracts as it propagates, but its frequency remains the same. In the second case the motion of the strong pulse cannot be described by the envelope model, the pulse contracts and its frequency changes. According to (9), its frequency is $\omega(z) = (2\Omega^2/c)z$.

In the first case the field energy increases as a result of adding photons to the pulse of the same frequency as the pulse. In the second case the number of pulse photons remains constant. The pulse energy increases due to the addition of the energy $\hbar\omega_0$ emitted by the particle as a result of an induced transition between the levels, to the instantaneous energy $\hbar\omega(z)$ of the field photons. We thus conclude that the evolution of a short, strong light pulse which propagates in a two-level medium is similar to the evolution of a light pulse which propagates in a gravitational field. A quantum description reduces the effects to an increase in the "weight" of the field photons and a classical description reduces it to a compression (by $\sim z$) of the pulse and an increase ($\sim z$) in the amplitude of its oscillations.

3. Condition (3) under which the model considered above can be used reduces to the satisfaction of the inequality $(\mu\mathscr{E}/\hbar)^2 \gg \omega_0^2$. Introducing the light flux density $q = (c/4\pi)\,\mathscr{E}^2$, we find

$$q \gg \frac{\hbar\omega_0^2}{4\pi\mu^2}c. \tag{10}$$

Assuming $\omega_0 = 10^{14}$ rad/s and $\mu = 5\times10^{-18}$ abs. units, we find $q \gg 10^{11}$ W/cm^2.

Let us estimate the characteristic distance at which the frequency $\omega(z)$ of the photons of the pulse is much higher than the frequency ω_0 of the transition. According to (5) and (9), we have

$$z \gg c\hbar/4\pi N\mu^2. \tag{11}$$

With $N = 10^{17}$ cm^3 and $\mu = 5\times10^{-18}$ abs. units, condition (11) is equivalent to the requirement that $z \gg 1$ cm. In other words, a passage of a distance of 1 cm causes the frequency to be shifted by the amount $\sim\omega_0$.

[1]S. L. McCall and E. L. Hahn, Phys. Rev. **21**, 1151 (1968).
[2]G. L. Lamb, Rev. Mod. Phys. **43**, 99 (1971).
[3]I. A. Poluéktov, Yu. M. Popov, and V. S. Roĭtberg, Usp. Fiz. Nauk **114**, 97 (1974) [Sov. Phys. Usp. **17**, 673 (1974)].
[4]V. E. Zakharov, Pis'ma Zh. Eksp. Teor. Fiz. **32**, 603 (1980) [JETP Lett. **32**, 589 (1980)].
[5]S. V. Manakov, Zh. Eksp. Teor. Fiz. **50**, 495 (1983) (sic.).

Translated by S. J. Amoretty