Superconductivity of an electron liquid near crystallization

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An effective attraction arises between electrons in an electron liquid near crystallization, and a superconducting transition may occur.

As the density of an electron liquid, n, is lowered to a certain n_{cr}, the ground state of the liquid becomes inhomogeneous and a charge density wave or a spin density wave arises. As the density is reduced further, a Wigner crystal forms. A special model of an electron liquid, for which a charge density wave arises in a region of pronounced compression, was analyzed in Ref. 1. It was shown that near the instability ($n \gtrsim n_{cr}$) an attraction arises between electrons, and a superconducting transition may occur. In the present letter we show that proximity to the transition to an inhomogeneous state (a charge density wave or a spin density wave) always results in the appearance of an effective attraction between electrons, regardless of the particular model used for the electron liquid. For definiteness, we consider an isotropic electron liquid against a positive static background near the transition to a state with a charge density wave. At $n \leqslant n_{cr}$ the electron liquid is homogeneous, but the density fluctuations with wave

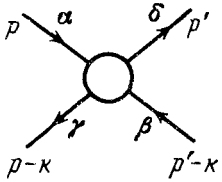

FIG. 1.

vectors $|\mathbf{k}| \sim k_0$ are large, where k_0 determines the period of the inhomgeneous phase. We will follow the approach taken in Refs. 2 and 3. The large value of the density fluctuations means that a static two-particle vertex with a momentum transfer $|\mathbf{k}| \sim k_0$ has a resonance singularity in a particle-hole channel. This vertex does not depend on the spins, since we are assuming that both the phases (the homogeneous and inhomogeneous phases) are paramagnetic. We write the total static vertex (Fig. 1) as the sum of resonant and smooth parts:

$$\Gamma_{\alpha\beta\gamma\delta}(\mathbf{p}, \mathbf{p}', \mathbf{k}) = D(\mathbf{k})\delta_{\alpha\gamma}\delta_{\beta\delta} - D(\mathbf{p}-\mathbf{p}')\delta_{\alpha\delta}\delta_{\beta\gamma} + \Gamma^{(0)}_{\alpha\beta\gamma\delta}. \quad (1)$$

Here α, β, γ, and δ are spin indices; the momenta $\mathbf{p}$ and $\mathbf{p}'$ lie on the Fermi surface; and D and $\Gamma^{(0)}$ are respectively the resonant and smooth parts of the vertex. The resonant part of the static vertex can be written as follows at momentum transfers $|\mathbf{k}| \sim k_0$ ($|\mathbf{k}^2 - k_0^2| \ll k_0^2$):

$$D(\mathbf{k}) = A[(\mathbf{k}^2 - k_0^2)^2 + r^2]^{-1}. \quad (2)$$

In this expression, r vanishes in the limit $n \to n_{cr}$ for $k_0 \sim p_F$, where p_F is the Fermi momentum. A vertex like (2) was found in Ref. 1 for the electron-liquid model analyzed there.

We can show that a necessary condition for stability of the system is that A be negative. For this purpose we find the dynamic correction to the resonant part of the static vertex at frequencies $|\omega| < k_0 v_F$. This correction is determined by the main sequence of diagrams of the random phase approximation (*RPA*), whose sum is

$$D^R(\mathbf{k}, \omega) = A[(\mathbf{k}^2 - k_0^2)^2 + r^2 - A\delta\Pi^R(\mathbf{k}, \omega)]^{-1}. \quad (3)$$

Here $\delta\Pi^R(\mathbf{k}\omega) = -i\pi\nu(\omega/kv_F)\theta(2p_F - k)$ is the frequency correction to the polarization operator, $\nu = mp_F a^2/2\pi^2$ is the state density, and v_F is the Fermi velocity. The mass m and the jump a in the Fermi distribution are determined by the nonresonant vertex $\Gamma^{(0)}$. It can be seen from expression (3) for the retarded vertex part, $D^R(\mathbf{k}, \omega)$, that this part has a pole in the limits $k \to k_0$, $r \to 0$, $|\omega| \ll k_0 v_F$. This pole lies in the lower ω half-plane only if $A < 0$. A positive sign of A would correspond to an unstable state. Resonant interaction (2) in a stable electron liquid near crystallization must therefore be attractive. We write the vertex (3), averaged over the Fermi surface, as follows:

$$-\overline{\nu D(\mathbf{k}, \omega_n)} = \left(\frac{\Omega}{\omega_0 + |\omega_n|}\right)^{1/2} = \lambda\left(\frac{\omega_0}{\omega_0 + |\omega_n|}\right)^{1/2}, \quad (4)$$

where $\omega_0 = r^2 k_0 v_F/\pi\nu|A|$, $\Omega = \pi\nu|A|k_0 v_F/(2p_F)^4$ and ω_n is the Matsubara frequency. If $\lambda = -\nu D(\mathbf{k},0) > \nu\Gamma^{(0)}$ the system will be superconducting. The repulsion is insignificant when we are sufficiently close to the crystallization point, and T_c is found from the Éliashberg equation with interaction (4):

$$\Delta(\epsilon_n) = \pi T_c \sum_{\epsilon_m} \left(\frac{\Omega}{\omega_0 + |\epsilon_n - \epsilon_m|} \right)^{1/2} \frac{\Delta(\epsilon_m)}{|\epsilon_m + ia\Sigma(\epsilon_m)|}, \tag{5}$$

where

$$ia\Sigma(\epsilon_n) = \pi T_c \sum_{\epsilon_m} \left(\frac{\Omega}{\omega_0 + |\epsilon_n - \epsilon_m|} \right)^{1/2} \operatorname{sgn} \epsilon_m . \tag{6}$$

Equation (5) is analogous to the Éliashberg equation for the model of an electron-phonon interaction with an optical phonon mode ω_0, but the interaction function in the phonon model is $\Omega^2/(\omega_0^2 + \omega_n^2)$, instead of (4), where Ω is determined by the electron-phonon interaction. The phonon model was analyzed in Refs. 5 and 6. Following Ref. 5, and solving Eq. (5) numerically, we find, in the limit $\omega_0 \to 0$ (strong coupling)

$$T_c = 0.26\,\Omega = 0.26\,\omega_0\lambda^2 . \tag{7}$$

The values of T_c at finite ω_0 are essentially the same as those given by limiting expression (7), which remains valid at $\lambda \leqslant 1$. This behavior (a wide region of strong coupling) is explained on the basis of the weak (square-root) dependence of interaction (4) on ω_0.

The quantity $\lambda = (\Omega/\omega_0)^{1/2}$ is analogous to the electron-phonon interaction constant. It determines the renormalization of the mass, $m^* = (1+\lambda)m$ and the jump in the Fermi distribution, $a^* = a/(1+\lambda)$. The first-order diagram for the eigenenergy part [Fig. 2a, where the wavy line corresponds to the resonant interaction $D(\mathbf{k}, \omega)$] is smooth if $\Omega \ll \varepsilon_F$ and $\lambda T \ll \varepsilon_F$. This condition also justified the *RPA*, which we used in (3). At high temperatures, $T \gg \omega_0$ the damping of the electron excitations depends linearly on the temperature, $1/\tau = 2\pi\lambda T$, as can be seen from (6). Accordingly, the resistance caused by scattering by critical density fluctuations (3) is also a linear function of the temperature. In the limit $n \to n_{cr}$, λ increases without bound, and lp_F becomes ~ 1. It thus appears that a transition to an inhomogeneous state should be preceded by a localization regime.

Let us examine the renormalization of the vertex $\Gamma^{(0)}$. The diagrams of the type

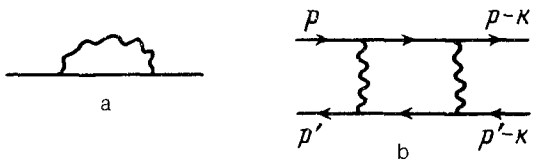

FIG. 2.

in Fig. 2b, with a small momentum transfer $\mathbf{k}$, are important here. In these cases, the poles of the resonant vertices, $D(\mathbf{k}\omega)$, close on each other. These diagrams correspond to a long-range attraction, and in an uncharged Fermi liquid they may lead to a first-order phase transition at finite value of r (Ref. 4). In this case, the region $\lambda > \nu\overline{\Gamma}^{(0)}$ may be inaccessible in the liquid phase. In an electron liquid, this danger is eliminated by the long-range Coulomb repulsion.

If the electron liquid is near the transition to a state of a spin density wave,[7] the fluctuations in the spin density will be large, and resonant vertex (2) will appear in the spin scattering channel[3]:

$$\Gamma_{\alpha\beta\gamma\delta}(\mathbf{p}, \mathbf{p}', \mathbf{k}) = D(\mathbf{k})\sigma_{\alpha\gamma}\sigma_{\beta\delta} - D(\mathbf{p}-\mathbf{p}')\sigma_{\alpha\delta}\sigma_{\beta\gamma} + \Gamma^{(0)}. \quad (8)$$

In this case, a pairing with an orbital angular momentum $l=0$ cannot occur. A pairing with $l=1$, in contrast, is possible, under the condition $k_0^2/2p_F^2 < 1$, as is a pairing with $l=2$, under the condition $1 - 1/\sqrt{3} < k_0^2/2p_F^2 < 1 + 1\sqrt{3}$. In either of these cases, the strong-coupling limit is not realized, and T_c vanishes in the limit $\omega \to 0$.

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