Kinetic description of edge magnetoplasmons in 2D systems

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The existing classical theory of edge magnetoplasmons in 2D charged systems runs into an applicability limit in strong magnetic fields. It is shown in the present paper that in the region $\omega_c \tau \gg 1$ the length scale near the boundary of the 2D system at which the charge of the edge magnetoplasmons is localized is on the order of the Larmor radius. The effect of a roughness of the boundary of a 2D system on the properties of edge magnetoplasmons is discussed.

A question which remains unsettled in the theory of edge magnetoplasmons is that of the structure of these magnetoplasmons in a strong magnetic field, i.e., in the region $\omega_c \tau \gg 1$, where ω_c is the cyclotron frequency, and τ is the momentum relaxation time. The quantum-mechanical version of the theory, which has been outlined by Volkov and Mikhaĭlov,[1,2] is still in a primitive state. The classical treatment of the problem by the same investigators is based, in particular, on the use of a local Ohm's law. As the magnetic field increases, however, the local nature of Ohm's law may be violated near the boundary of a 2D system. There are accordingly the questions of the applicability limits of the local theory of edge magnetoplasmons and of a modification of the theory to eliminate these limitations. We discuss these problems in the present letter.

A. We assume for definiteness that a 2D electron system occupies the half-plane $x \geqslant 0$ and has an equilibrium density $n(x) = n_s \theta(x)$ with a sharp boundary running along the Y axis. The magnetic field H is directed along the normal to the $x = 0$ plane. A dielectric substrate with a constant $\varkappa$ fills the half-space $z \leqslant 0$.

The approach for calculating the dispersion law $\omega(q)$ for edge magnetoplasmons in the local approximation can be outlined as follows. The continuity equation

$$e\delta\dot{n} + \sigma_{xx}\Delta\varphi = 0, \qquad \sigma_{xx} = \sigma_{yy}\,, \tag{1}$$

where $\delta n = \delta n(x) e^{i(qt+\omega t)}$ and $\varphi = \varphi(x) e^{i(qy+\omega t)}$ are the local density and electric potential in the edge magnetoplasmon, ω and q are the frequency and wave number of the oscillations, σ_{ik} is the conductivity tensor of the electron system in the magnetic field, and Δ is the 2D Laplacian, reduces to the following expression after an integration over x in the range $0 \leqslant x < \infty$:

$$\omega Q = -\,\sigma_{xx}\varphi'(0), \qquad Q = e\int_0^\infty \delta n(s)ds\,, \qquad \varphi'(x) \equiv \partial\varphi/\partial x\,. \tag{2}$$

In writing (2) we used the inequality $\varphi''_x \gg \varphi''_y$, which holds in the long-wavelength approximation in q. Also noting that the boundary condition that no current j_x flows across the $x = 0$ line relates the values of $\varphi'(0)$ and $\varphi(0)$, i.e.,

$$\sigma_{xx}\ \varphi'(0) + iq\sigma_{xy}\ \varphi(0) = 0, \tag{3}$$

we can put (2) in the form

$$\omega = q\,\sigma_{xy}\,\varphi(0)\,/\,Q. \tag{4}$$

The problem of the spectrum of edge magnetoplasmons thus reduces to a determination of the expression for $\varphi(0)$ in terms of Q. Following Refs. 1 and 2, we write

$$\varphi(0) \approx \frac{2Q}{\kappa}\ln\frac{1}{ql_\sigma}\ , \qquad l_\sigma = -2\pi i\,\sigma_{xx}\,/\,(\kappa\,\omega\,). \tag{5}$$

According to (5), most of the charge in an edge magnetoplasmon is concentrated over a distance l_σ in a local description. In the Drude model, where $\sigma_{xy} = e^2 n_s m_*^{-1}\omega_c^{-1}$, $\sigma_{xx} = i\omega\sigma_{xy}\omega_c^{-1}$, $\omega \ll \omega_\xi$, and $\omega_c = eHm_*^{-1}c^{-1}$, where m_* is the effective mass of an electron, the quantity l_σ falls off with increasing H as $l_\sigma \propto H^{-2}$.

B. A necessary condition for the existence of a local Ohm's law is, in particular, that the electric field vary smoothly over the mean free path of the conduction electrons. In a situation with $\omega_c\tau \gg 1$ the role of the typical mean free path is played by the Larmor radius $r_c = v_F\omega_c^{-1}$, where v_F is the Fermi velocity, given by $v_F \sim \hbar n_s^{1/2} m_*^{-1}$. The range of applicability of the local theory of edge magnetoplasmons is thus limited by the requirement

$$l_\sigma \gg r_c \quad \text{or} \quad H \ll H^*, \quad H^* = \frac{m_* c}{e}\,V_c\,, \quad V_c = \frac{e^2}{\kappa}\,n_s^{1/2}\ . \tag{6}$$

In the case of a 2D electron system with the parameters of GaAs, and when we use the Drude formulas for the tensor σ_{ik} we find that the typical magnetic field H^*, found from the estimate $l_\sigma \approx r_c$, is $H^* \gtrsim 1\ T$ in order of magnitude. It is pertinent to note here that actual measurements of σ_{xx} at a finite frequency $\omega \ll \omega_c$ under the condition $\hbar\omega_c \gg T$, in one of the minima of the Shubnikov-de Haas oscillations,[3] yield values of $\sigma_{xx}^{\min}$ which are considerably smaller than the estimates which follow from the simple Drude model. In other words, the Drude formulas give us a crude upper estimate of H^*.

If requirement (6) is not satisfied, a theory of edge magnetoplasmons must be constructed through the use of a kinetic equation for the electron distribution function, to replace Eq. (1). This formalism makes it possible to also incorporate in the theory, in a natural way, the appearance of edge magnetic levels, which significantly change the conducting properties of magnetized systems near a free boundary of the sample (we are thinking of the static skin effect in a magnetic field, which is well known in the physics of metals[4]). We will discuss the theory of edge magnetoplasmons in the kinetic regime in detail in a separate paper. Here we will simply note that in the region $r_c > l_\sigma$ the role of the distance over which most of the charge of the edge magnetoplasmons is concentrated now becomes the role of the Larmor radius r_c.

C. It is worthwhile to point out that roughness of the boundary of a 2D system, $\xi(y)$, affects the properties of edge magnetoplasmons: $\xi'(y) \ll 1$, $\int \xi dy = 0$. The requirement that no current flow across the free boundary takes the form $j_x + \xi'(y) j_y |_{x-0} = 0$ in this case. We then find an expression for the local charge $Q(y)$ which generalizes the definition of Q in (2) and (3):

$$i\omega Q = \sigma_{xy} \frac{\partial \varphi}{\partial y} \Big/ \left[1 + \xi'(y) \frac{\sigma_{xy}}{\sigma_{xx}} \right], \qquad \sigma_{xy} \gg \sigma_{xx} . \tag{7}$$

According to (7), in regions with $\xi' \sigma_{xy}/\sigma_{xx} \gtrsim 1$, which are easily arranged in strong fields H, the local charge density of edge magnetoplasmons is substantially lower than at extrema with $\xi(y) = 0$, where this density has a scale value characteristic of the problem $\xi(y) \equiv 0$. Consequently, the average linear charge of the edge magnetoplasmons at a free boundary with $\xi(y) \neq 0$ is lower than in the case $\xi(y) \equiv 0$.

The charge distribution of the edge magnetoplasmons which arises makes it possible to use an integral form of the continuity equation in seeking the dispersion law for the edge magnetoplasmons. This integral form is $i\omega \overline{Q} \approx j_x |_{x \approx \lambda}$, where the current j_x which forms the charge $\overline{Q}$, is calculated in the interior of the 2D system at distances $x \approx \lambda$ from the free edge, and λ is a typical roughness period, at which the inequality $\xi'(y) \sigma_{xy} | \sigma_{xx} > 1$ starts to hold. In the region $x \gtrsim \lambda$ the fast oscillations of the potential with wave numbers $\lambda^{-1} \gg q$ are smoothed over (a self-averaging effect), and the integral form of the continuity equation leads to the following dispersion law for edge magnetoplasmons:

$$\omega \approx \frac{2q\sigma_{xy}}{\kappa} \ln \frac{1}{q\lambda} ,$$

where λ is the length of the characteristic harmonic $\xi(y)$, for which the condition $\xi'(y) \sigma_{xy} | \sigma_{xx} \gtrsim 1$ holds.

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[3]J. I. Lee, B. B. Goldberg, and M. Heiblum, Solid State Commun. **64**, 447 (1987).

[4]I. M. Lifshits, M. Ya. Azbel', and M. I. Kaganov, Electronic Theory of Metals, Nauka, Moscow, 1971.

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