

# Resonant emission of a chain of Josephson vortices on an array of inhomogeneities

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A theory is derived for the collective effect of resonant emission as a chain of vortices moves in a long Josephson junction along which there are structural inhomogeneities in a periodic arrangement. Experiments reveal structural features on the current-voltage characteristic of a junction with artificial inhomogeneities in an external magnetic field. These structural features are associated with the occurrence of resonances of this sort.

Fine-structure steps were recently observed experimentally<sup>1</sup> on the current-voltage characteristic of long Josephson junctions with an array of inhomogeneities which were deliberately fabricated. These steps correspond to resonances which arise as a single Josephson vortex (fluxon) moves in the field of its emission at inhomogeneities.<sup>2</sup> In the present paper we derive a theory for, and report the experimental observation of, a collective resonance effect in the emission of plasma waves by a chain of fluxons as this chain moves in a Josephson junction with an array of inhomogeneities.

The effect discussed in the present letter differs from the effect which stems from a one-fluxon resonance with inhomogeneities<sup>1</sup> in that in a chain of fluxons the emission may be amplified because of the coherent summation of waves emitted by each fluxon. A chain of fluxons in a Josephson junction can be produced by an external magnetic field  $H$ ; the distance between neighboring fluxons is  $b \sim H^{-1}$ .

A long Josephson junction with an embedded lattice is described by the well-known dimensionless equation for the magnetic flux  $\varphi$ :

$$\varphi_{tt} - \varphi_{xx} + \sin \varphi = -\alpha \varphi_t - \gamma - \mu \sum_{n=-\infty}^{+\infty} \delta(x - an) \sin \varphi, \quad (1)$$

where  $\alpha$  is a dissipation coefficient,  $\gamma$  is the current density of a current flowing through the junction, and  $a$  is the period of the lattice.

The resonant condition is

$$m(2\pi v/b) = n \omega_{pl}, \quad (2)$$

where  $m$  and  $n$  are integers,  $v$  is the fluxon velocity, and  $\omega_{pl}$  is the frequency of the emitted waves.

The voltage across a Josephson junction is known to satisfy  $V \sim b^{-1}v$ , so the current-voltage characteristic is determined by the functional dependence  $v(\gamma)$  which follows from the energy balance equation:

$$2\pi\gamma v = 8\alpha v^2 + W(v), \quad (3)$$

where  $W(v)$  is the radiative-loss power per fluxon, and  $8\alpha v^2$  is the power of the dissipative loss. At velocities satisfying Eq. (2), the work performed by the current on the chain is expended on the excitation of plasma waves, not on the acceleration of the chain. At the corresponding voltages, steps of nearly constant voltage should accordingly arise on the current-voltage characteristic. For model (1), according to Ref. 2, we have

$$v_{m,n} = \{ [1 - (4\pi n b / m a)]^2 + (2\hbar b / m)^2 \}^{1/2}. \quad (4)$$

Using the "resonant" perturbation theory for the sine-Gordon equation,<sup>3</sup> and treating  $\mu$  in (1) as a small parameter, we can also estimate the height of the steps in (4):  $\gamma \sim \mu^2 / \alpha b$ .

In the present experiments we studied Nb-NbO<sub>x</sub>-Pb Josephson junctions with an array of inhomogeneities fabricated deliberately by photolithography. The inhomogeneities were fabricated by modulating the thickness of the insulating interlayer of the junctions.<sup>1</sup> Figure 1 shows current-voltage characteristics measured at various values of the external magnetic field, which was directed in the plane of the interlayer of the junctions. Curves 1 and 2 were found for a uniform control junction in fields  $H = 3.63$  Oe and 2.70 Oe, respectively. These curves have a characteristic peak which corresponds to the case in which the chain of vortices in the Josephson junction has a velocity close to the limiting velocity  $v_0 = 1$ . As the external field is increased, this peak shifts up the voltage scale. Curves 3 and 4 were found for Josephson junctions

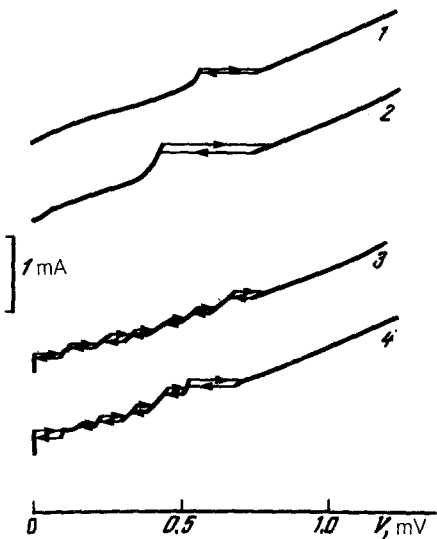


FIG. 1.

with five inhomogeneities, separated by a distance  $a \sim b$ , in fields  $H = 3.21$  Oe and 2.75 Oe, respectively. We see that there is an entire set of structural features on the current-voltage characteristic at the various voltages. Estimates based on expression (4) show that the observed structural features agree within a satisfactory error, on the order of 5–8%, with the positions of resonant peaks with  $m/n = 2/3, 3/4, 1/2$ , etc. The uncertainty in the estimates of the parameter values required for the calculations stems from the error in the determination of the Josephson penetration depth from experimental data.

In conclusion we would like to point out that the effect described here might be interpreted as a synchronization of a chain of short Josephson junctions which are connected in parallel. A long Josephson junction with an array of inhomogeneities might be thought of as a multicontact, low-inductance interferometer. The structural features observable on the current-voltage characteristic could then be thought of as an effect of a synchronization of Fiske steps<sup>4</sup> in short junctions on the basis of this model. An important point in this analysis is that for a phase shift of  $2\pi$  between the oscillations in different junctions the power radiated by  $N$  short Josephson junctions will be proportional to  $N^2$ . This situation is reminiscent of superradiance.

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