Static resonant voltage in metals

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The existence of a paramagnetic cyclotron instability in metals as a result of excitation in them of a finite-amplitude cyclotron standing wave is predicted. The instability gives rise to a strong nonuniform electron heating, which produces a static electric field in the metal. The lineshape of the static resonant voltage is determined.

Because of the high carrier density, the electron subsystem in metals is known to be unable to reach a level of nonequilibrium where the deviation of the average electron energy from the Fermi energy would be considerably higher than the phonon temperature. This situation occurs because the electric field strength in metals is always low: The static electric field is restricted by the Joule heating of carriers and in the high-frequency case the electric field is low due to the skin effect. Near the cyclotron frequencies, however, the metal becomes transparent to electromagnetic radiation because of the existence of weakly decaying oscillations of the electron-hole plasma of the metal.[1,2] Excitation of long-wavelength cyclotron waves[1,2] in a metal may give rise to the conditions for the appearance of a cyclotron parametric resonance[3,4] at which there is an appreciable superheating of the electron gas. At cyclotron parametric resonance in the field of a standing cyclotron wave the electron distribution function, a nonequilibrium function, is a function of the coordinates. This circumstance gives rise to average energy gradients and a carrier concentration which in turn produce a static electric field and a static voltage of a resonant nature.

The components of a long-wavelength cyclotron standing wave in a metal plate in a magnetic field $\mathbf{H}_0$ can be determined from the expressions (an extraordinary wave)[1,2]

$$\begin{aligned} H_z &= H_1 e^{-x/\delta} \cos(kx+\alpha)\cos\omega t; && \tan\alpha = (k\delta)^{-1}; \\ E_y &= H_1(\omega/\mathrm{cosk})[1+(k\delta)^{-2}]^{-1} e^{-x/\delta}\sin kx\cdot\sin\omega t; && k=\pi n/L; \\ E_x &= -H_1(\Omega_0/\mathrm{cosk})[1+(k\delta)^{-2}]^{-1} e^{-x/\delta}\sin kx\cdot\cos\omega t; && n = 1,2,3\,.. \end{aligned} \qquad (1)$$

The x axis runs parallel to $\mathbf{n}$, $\mathbf{n}$ is the normal to the surface of the sample, the z axis runs parallel to $\mathbf{H}_0$, H_1 is the amplitude of the field of the cyclotron wave, ω and δ are the frequency and degree of damping of the wave, L is the thickness of the sample, $\Omega_0 = eH_0/mc$, and e and m are the charge and mass of the carriers. The components of the field of the cyclotron wave are described by expressions (1) in the long-wave limit, where

$$kR \ll 1, \qquad \Omega_0\,\omega \gg \nu_i\,. \qquad 2)$$

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Here $R = v_F/\Omega_0$, where v_F is the Fermi velocity. The spectrum of long-wavelength cyclotron waves and their damping have been studied extensively.[1,2] For these waves to exist, the following condition in (2) must be satisfied $k\delta \sim \omega/\nu_i \gg 1$, where ν_i is the relaxation rate of the electron momentum. Cyclotron waves have been detected in several metals and semimetals.[1,2,5–8] Let us consider the case in which the long-wavelength cyclotron waves are excited in a metal, where the quadratic dispersion law applies:

$$\epsilon = \frac{p_x^2 + p_y^2}{2m_1} + \frac{p_z^2}{2m_2}\ ; \tag{3}$$

(in the case of holes in bismuth, for example, $m_1 = 0.064 m_0$ and $m_2 = 0.703 m_0$, where m_0 is the mass of a free electron[7]).

As a result of frequency modulation of the collective cyclotron rotation in the field of a cyclotron wave [Eqs. (1)], the electron system develops resonance features like those of a parametric resonance which occurs in the mechanical vibrations.[9] These resonance features appear if the following condition holds:

$$\Omega_0 \approx s\omega/2 \qquad (s = 1,\ 2,\ \ldots). \tag{4}$$

Let us consider the case in which the kinetic instability has not yet developed and the energy pumped into the electron system is sufficiently small:

$$T/\epsilon_F \ll 2\kappa_{1,2}^2(x)/5\nu_e\nu_i(1+\Delta_{1,2}^2) \ll 1; \tag{5}$$

$$\kappa_1(x) = (3/4)\Omega_1 e^{-x/\delta}\cos kx \quad \text{for } \Omega_0 \approx \omega/2; \quad \Delta_1 = (2\Omega_0 - \omega)/\nu_i;$$

$$\kappa_2(x) = \Omega_1 e^{-x/\delta}\sin kx \quad \text{for } \Omega_0 \approx \omega\ ; \quad \Delta_2 = (\Omega_0 - \omega)/\nu_i\ ;$$

$\Omega_1 = eH_1/mc$; ν_e is the energy relaxation; T is the phonon temperature; ϵ_F is the Fermi energy; and $\Delta_{1,2}$ is a deviation from resonance.

The inequality on the left side of expression (5) is a threshold condition imposed on the cyclotron wave amplitude, at which the parametric instability develops. The inequality on the right side of expression (5) shows that the superheating of the electron system is small in comparison with the Fermi energy. We see that the main cyclotron parametric resonance ($\Omega_0 \approx \omega/2$) is accompanied by a strong superheating of electrons in the antinodes of the magnetic field of a standing wave and that inequality (5) breaks down near the nodes and the electrons are at equilibrium. At the next resonance ($\Omega_0 \approx \omega$) the situation is reverse. A calculation shows that at resonance the electron gas in the sample is heated nonuniformly. The effective temperature of this gas is given by

$$T_{eff} = \begin{cases} T + (2/5)\mu\kappa_1^2(x)/\nu_e\nu_i(1+\Delta_1^2); & s = 1; \\ T + (2/3)m\Omega_0^2\kappa_2^2(x)/k^2\nu_e\nu_i(1+\Delta_2^2); & s = 2. \end{cases} \tag{6}$$

Here μ is the chemical potential of the nonuniformly heated quasiparticles. The presence of a nonuniformly heated carrier gas gives rise to the appearance of a static electric field $E_i(x)$, which can be found from the Poisson equation. If condition (5) holds, the isotropic part of the electron distribution function can be written as a Fermi function $F=\{\exp[(\epsilon-\mu)/T_{\rm eff}]+1\}^{-1}$. The chemical potential μ, which can be found on condition that the total current vanishes, is

$$\mu(x) = \mu_0 - e\varphi(x) - (\pi^2/4)T_{eff}^2(x)/\epsilon_F. \tag{7}$$

The value of μ_0 is determined from the normalization condition: $n_0=\langle n(x)\rangle$, $n(x)$ is the carrier density in the case of nonuniform heating, n_0 is the total number of quasiparticles, $\varphi(x) = -\int_0^x dx' E_i(x')$, and $\langle\cdots\rangle = L^{-1}\int_0^L dx(\cdots)$. The induced electric field $E_i(x)$ or the potential $\varphi(x)$ is the solution of the Poisson equation,

$$\varphi'' = \lambda^{-2}\{\varphi - \langle\varphi\rangle + (\pi^2/4e\epsilon_F)[T_{eff}^2 - \langle T_{eff}^2\rangle]\}, \tag{8}$$

with the boundary conditions $E_i(0)=E_i(L)$ [$\lambda=(\epsilon\epsilon_F/4\pi n_0 e^2)^{1/2}$ is the Debye screening length, and ϵ is the static dielectric constant].

Let us now consider the static resonant voltage which was measured experimentally. For the main resonance we have

$$\mathscr{E}_1 = \frac{\epsilon_F}{e}\left[\frac{9\pi}{80}\frac{\Omega_1^2}{\nu_e\nu_i}\frac{1}{1+\Delta_1^2}\right]^2(1-e^{-4L/\delta}), \tag{9}$$

if $|\Omega_0-\omega/2|\ll\Omega_0$. We see that the static voltage is of a resonant nature in the function of the static magnetic field H_0 and it increases rapidly with increasing amplitude of the field of the pump wave, $\mathscr{E}_1 \approx H_1^4$.

For the next resonance $s=2(\Omega_0\approx\omega)$ the voltage is much lower than that in (9): $\mathscr{E}_2 \sim (\lambda/R)^4\mathscr{E}_1$. This circumstance stems from the fact that effective temperature (6) in this case is zero at the sample's boundaries.

At low temperatures a static resonant voltage can be measured in a bismuth sample ($L\approx1$ mm) with an electron mean free path $l\sim0.1$–1 mm in magnetic fields $H_0\approx1$ kOe. The pumping amplitude would have to be $H_1 \gtrsim 0.01$ Oe at frequencies 10^{10} s^{-1}. The static resonant voltage in this case would be $\mathscr{E}_1\sim0.1$ mV.

[1]E. A. Kaner and V. G. Skobov, Adv. in Phys. **17**, 605 (1968).
[2]P. M. Platzman and P. A. Wolff, Waves and Interactions in Solid State Plasma, Suppl. 13 to Solid State Phys., Academic Press, New York (1973).
[3]I. E. Aronov, E. A. Koner, and A. A. Slutskin, Solid State Commun. **38**, 245 (1981).
[4]I. E. Aronov and É. A. Kaner, Pis'ma Zh. Eksp. Teor. Fiz. **40**, 223 (1984) [JETP Lett. **40**, 992 (1984)].
[5]V. S. Édel'man, Zh. Eksp. Teor. Fiz. **63**, 169 (1972) [Sov. Phys. JETP **36**, 89 (1973)].
[6]V. P. Naberezhnykh, D. É. Zherebchevskiĭ, and V. L. Mel'nik, Zh. Eksp. Teor. Fiz. **63**, 169 (1972) [Sov. Phys. JETP **36**, 89 (1973)].
[7]M. R. Trunin, Zh. Eksp. Teor. Fiz. **88**, 1834 (1985) [Sov. Phys. JETP **61**, 1087 (1985)].
[8]M. R. Trunin and V. S. Édel'man, Zh. Eksp. Teor. Fiz. **92**, 988 (1987) [Sov. Phys. JETP **65**, 560 (1987)].
[9]L. D. Landau and E. M. Lifshitz, Mechanics, 2nd ed., Pergamon Press, Oxford (1969).

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