A moving domain wall and the orientation of the order parameter in ^{3}He-B

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The motion of a domain wall in a previously analyzed two-domain structure in a superfluid B phase of ^{3}He is shown to lift the double degeneracy of this structure. This circumstance can be exploited to unambiguously orient the order parameter in ^{3}He-B.

A stable configuration of a spin, which precesses in a slightly nonuniform magnetic field $\mathbf{H}_0$, in a superfluid B phase of ^{3}He constitutes a two-domain structure.[1,2] In one of the domains the spin $\mathbf{S}$ deviates from an equilibrium orientation by an angle $\theta_0 = \arccos(-1/4)$ and in the other domain it has an equilibrium direction. The orientation of the order parameter in ^{3}He-B is determined by a unit vector $\mathbf{n}$ (in cm^3). The direction of this vector is different in different domains: $\mathbf{n}\perp\mathbf{H}_0$ in a precessing domain but in an equilibrium domain we could have either $\mathbf{n}\|\mathbf{H}_0$ or $\mathbf{n}\|(-\mathbf{H}_0)$. If the relaxation process is ignored, the two-domain structure indicated above is a stationary structure in a coordinate system which rotates around $\mathbf{H}_0$ with a precession frequency ω_p of the entire structure. Here the two possible orientations of $\mathbf{n}$ correspond to the same energy. We will show here that this degeneracy is lifted by the motion of the domain wall, whose direction of motion determines which of the two orientations is preferred from the energy standpoint.

The shape of the domain wall moving with a velocity v is described by the solution of the spin-dynamics equations in which the variables depend on the coordinate z and time t in the combination z-vt. The z axis is antiparallel to $\mathbf{H}_0$. The arguments which reiterate those advanced in Sec. 2 of Ref. 2 show that this solution must be the minimum of the functional

$$\mathcal{F}^{(\pm)} = \frac{c_\parallel^2}{\lambda}\int\Big\{\frac{1}{2}[\rho_{11}(\psi')^2 + 2\rho_{12}^{(\pm)}\psi' u' + \rho_{22}(u')^2] + (1-u)\zeta + w\left[\frac{d\phi}{du}u' - (1-u)\psi'\right]\Big\}d\zeta, \qquad (1)$$

which performs the function of energy. The functions $\psi(\zeta)$ and $\cos\beta\equiv u(\zeta)$, which must be determined, specify the orientation of the spin $\mathbf{S}$ in the coordinate system rotating with a frequency ω_p, ψ is the longitude, and β is the latitude reckoned from $-\mathbf{H}_0$. We are using here a dimensionless coordinate $\zeta = (z$-$vt)/\lambda$ and a dimensionless velocity $w = v\omega_p \lambda / c_{\parallel}^2$, the prime denotes differentiation with respect to ζ, $\lambda = (c_{\parallel}^2/\omega_p \vee \omega_L)^{1/3}$ is the length scale, and $c_{\parallel}$ and $c_{\perp}$ are the velocities of the spin waves (see Ref. 2). We will use below the following relation between these velocities: $\mu = c_{\perp}/c_{\parallel}$. The coefficients ρ_{11},... depend on u:

$$\rho_{11} = 2(1-u)[u + \mu^2(1-u)], \quad \rho_{12}^{(\pm)} = -(2\mu^2 - 1)(1-u)(d\phi/du)^{(\pm)},$$

$$\rho_{22} = (2\mu^2 - 1)(d\phi/du)^2 + 1/(1-u^2).$$

The derivative $(d\phi/du)^{(\pm)} = \pm [3/((1+u)^2(1+4u))]^{1/2}$ arises as a result of elimination of the third angle ϕ from the equations of motion, which must be used in order to determine unambiguously the orientation of the order parameter. The constraint used to eliminate ϕ, $u + (1+u)\cos\phi = 1/2$, correlates with each $u > -1/4$ two values of ϕ of opposite signs. A passage through the domain wall causes ϕ to change from 0 to $\pm\theta_0$, which corresponds to a change in the orientation of **n** from $\mathbf{n} \perp \mathbf{H}_0$ to $\mp \mathbf{n} \parallel \mathbf{H}_0$.

Varying $\mathscr{F}$ with respect to ψ and imposing the condition under which there is no spin current in the equilibrium domain, we find

$$\rho_{11}\psi' = w(1-u) - \rho_{12}^{(\pm)} u'. \tag{2}$$

Varying $\mathscr{F}$ with respect to u and using expression (2) for ψ', we find the equation for the variable u

$$u'' + \frac{\rho_{11}}{\Delta}\{\zeta + \frac{w^2}{2}\frac{d}{du}\left[\frac{(1-u)^2}{\rho_{11}}\right] + \frac{1}{2}(u')^2 \frac{d}{du}\left(\frac{\Delta}{\rho_{11}}\right)\} = 0, \tag{3}$$

where $\Delta \equiv \rho_{11}\rho_{22} - \rho_{12}^2$. Since Eq. (3) and its boundary conditions do not depend on the sign of ρ_{12}, the functional dependence $u(\zeta)$ is the same for both domain walls (Fig. 1). For a given $u(\zeta)$ the functional dependence $\psi(\zeta)$ can be determined by integrating Eq. (2), which depends on the relationship between the signs of w and ρ_{12} (Fig. 2). In pulsed NMR experiments the motion of the wall occurs as a result of spin relaxation, where the velocity w is a negative velocity on the order of $D\omega_p/c_{\parallel}^2$, where D is the spin diffusion coefficient. In the experiments of Borovik-Romanov *et al.*[1] this ratio is ~ 0.1. Substitution of Eq. (2) into Eq. (1) shows that with $w < 0$ a solution in which $(d\phi/du) > 0$ is preferred from the energy standpoint, which is consistent with the antiparallel orientation of **n** and $\mathbf{H}_0$ in an equilibrium domain. The difference in energies of the two solutions is proportional to the velocity $\mathscr{F}^{(-)} - \mathscr{F}^{(+)} = -kwc_{\parallel}^2/\lambda$, where

$$k = 2\sqrt{\frac{3}{(5\mu^2 - 1)(1-\mu^2)}} \arctan\sqrt{\frac{5(1-\mu^2)}{5\mu^2 - 1}}.$$

With $\mu^2 = 3/4$ we have $k \approx 2.48$. The energy barrier which divides the states with different domain walls is equal to $\sim c_{\parallel}^2/\lambda$. Accordingly, an undesirable configuration may take the form of a metastable configuration at low velocities w.

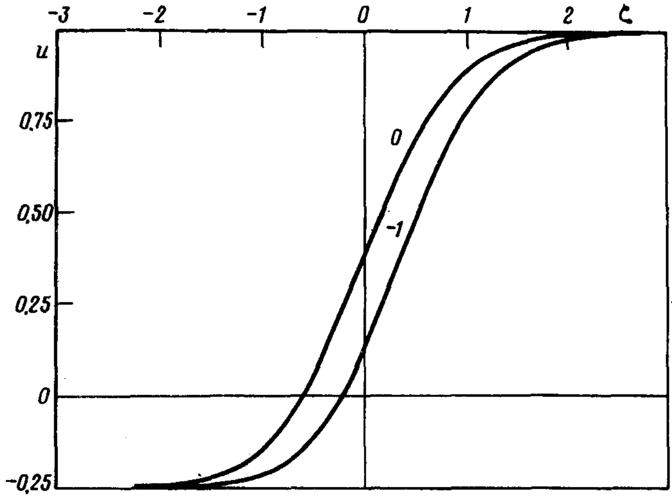

FIG. 1. The dependence $u(\zeta)$ for the two domain walls obtained by numerically integrating Eq. (3) for two velocities $w=0$ and $w=-1$. In the calculations it was assumed that $\mu^2=3/4$.

To orient **n** in a certain direction, we accordingly suggest the following procedure. The initial momentum deflects the spin by an angle $\beta>\theta_0$ everywhere in the test chamber; here $\phi=0$ everywhere. With a relaxation of the spin at the wall of the chamber which is situated in a stronger magnetic field, an equilibrium domain begins

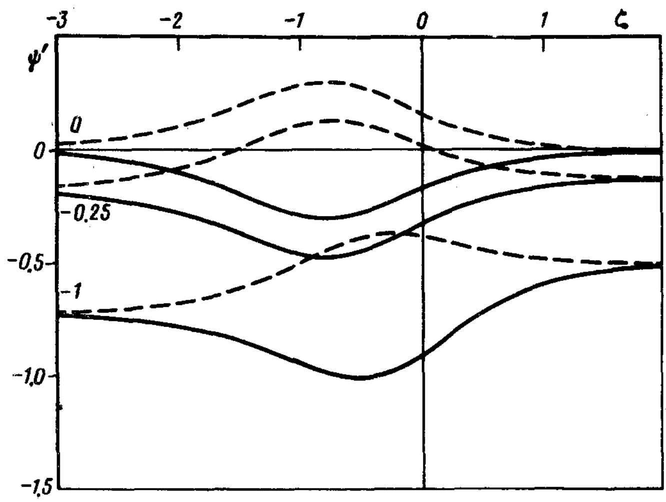

FIG. 2. The dependences $\psi'^{(+)}(\zeta)$ and $\psi'^{(-)}(\zeta)$ for the three velocities $w=0$, $w=-0.25$, and $w=-1$. The dashed curves represent a desirable domain wall and the solid curves denote an undesirable domain wall ($\mu^2=3/4$).

to form. It is reasonable to expect that at the chamber wall ϕ in this case will deviate in the direction $\phi > 0$, which corresponds to a more desirable configuration from the energy standpoint. With further relaxation, the domain wall will move in the direction of weaker fields, leaving behind an equilibrium domain in which **n** is antiparallel to $\mathbf{H}_0$. The formation of an undesirable configuration should not be ruled out entirely in this case. The probability for its formation can be estimated by analyzing the particular process by which a domain wall is formed. Since such an analysis has not been performed, the procedure which we have suggested here should be viewed only as a possible approach. It is important to note, however, that there is a preferred orientation of **n**, which can be explained on the basis of symmetry considerations. The motion-related change in the energy of the domain wall is determined by scalar combinations comprised of the velocity **v**, the normal **s** to the wall, the field $\mathbf{H}_0$, and the orientation of **n** in the case of an equilibrium domain, i.e., in the case where various orientations of **n** are possible. In the one-dimensional case which we are considering here, all these combinations, specifically, $(\mathbf{Hn})(\mathbf{vs})$, $(\mathbf{Hv})(\mathbf{ns})$, $(\mathbf{Hs})(\mathbf{nv})$, and $(\mathbf{Hn})(\mathbf{vn})(\mathbf{sn})$, degenerate to a single combination $H_z s_z v_z n_z$. The symmetry thus allows in the energy of a moving domain wall a term which changes sign when the domain wall changes its direction of motion and when **n** changes its direction in an equilibrium domain.

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