Renormalization theorems for heterotic strings

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Four zero modes of Grassmann θ fields associated with a subalgebra of the global space-time supersymmetry are retained in multiloop calculations in the Green-Schwarz formalism. It thus becomes possible to prove theorems stating the vanishing of zero-, one-, two-, and three-point functions.

1. A problem in superstring theory today is that of developing a Green-Schwarz formalism[1] which has an explicit supersymmetry in space-time. In the present letter we describe what appears to us to be a suitable approach to multiloop calculations on the basis of this formalism, and we explain why it seems obvious to us that zero-, one-, two-, and three-point functions vanish when this approach is taken. Our arguments are based on the ideas developed in Ref. 2.

2. The Green-Schwarz action[1] is written in the following way:

$$\int d^2z\left[\frac{1}{2}\sqrt{g}\,g^{ab}\Pi_a^\mu\Pi_{b\mu} - i\epsilon^{ab}\,\partial_a X^\mu\,(\bar\theta\gamma_\mu\partial_b\theta)+\mathcal{L}'\right]=\frac{1}{2}\int d^2z$$

$$\times\sqrt{g}\,g^{ab}\,\Pi_a^\mu\Pi_{b\mu} + i\int d^3z\,\epsilon^{abc}\,\Pi_a^\mu(\partial_b\bar\theta\gamma_\mu\partial_c\theta)+\int d^2z\,\mathcal{L}'\,;$$

$$\Pi_a^\mu \equiv \partial_a X^\mu - i\bar\theta\gamma^\mu\partial_a\theta\,.$$

It contains only tensors with an integer spin on the world sheet. We showed in Ref. 2 that whenever the "light-cone gauge"

$$\theta^+ \equiv \frac{1}{2}\gamma^-\gamma^+\theta = 0 \qquad (2)$$

can be imposed on the space-time spinors θ (which are scalars on the world sheet), the theory (after a quantization in the conformal gauge for the metric) is described by the quadratic action

$$\int d^2z\,[\partial X^+\bar\partial X^- + \partial X^i\bar\partial X^i + \eta_{\bar z}^-\,\partial_z\vartheta^- + b\bar\partial c+\bar b\partial c + \mathcal{L}'\,]\,, \qquad (3)$$

where the fields $\eta_{\bar z}^-$ are 1-differentials on the world sheet (and spinors in space-time) with the unnatural norm

$$\|\,\eta_{\bar z}\|^2 = \int |\eta_{\bar z}|^2\,\frac{\sqrt{g}}{|\bar\partial X^+|^2}\,.$$

A critical complication in the case of multiloop calculations is the prohibition against the imposition of gauge condition (2). Specifically, gauge transformations act on θ^+ in accordance with

$$\delta\theta^+ = \Pi^+\gamma^- k\,, \qquad \gamma^+ k = 0, \tag{4}$$

where k is a spinor in space-time and a 1-differential (vector) on the world sheet. Being a 1-differential, $\bar\partial X^+$—the nilpotent part of $\Pi_{\bar z}^+$—must have zeros on any Riemann surface of type $p \geqslant 2$. According to Euler's theorem, the total number of such zeros cannot be less than $2p-2$. We denote the positions of the zeros of $\bar\partial X^+$ by Q_m, where $m = 1,..., M \geqslant 2p-2$. Because of these zeros, the values of θ^+ at the points Q_m do not change under gauge transformations (4). As a result, a gauge condition in the form in (2) cannot be imposed on $\theta(z)$. Instead, one could require

$$\theta^+(z) = \sum_{m=1}^{M} \xi_m \delta^2(z - Q_m). \tag{5}$$

We will carry out the integration initially over the θ fields, assuming that the X fields are fixed. The positions of the points $Q_m\{X\}$ are accordingly assumed to be given at this stage. The integration over the θ fields leads to a result which depends on $Q_m\{X\}$. It should subsequently be integrated over the X fields.

3. In the conformal gauge, an action with the necessary ghost admixtures is

$$S_{quant} = S_{cl}(X,\theta) + S_{gh}(X,\theta,\ldots) + \pi(\gamma^+\theta - \sum_{m=1}^{M} \xi_m \delta^2(z - Q_m)). \tag{6}$$

The quantity S_{gh} in (6) can be found from Eqs. (35) and (36) of Ref. 3. After the integration over π, the action contains no more than a quadratic dependence on ξ_m:

$$S_{quant} = \int O_{(0)} + \sum_{m=1}^{M} \xi_m \int \delta^2(z - Q_m) O_{(1)}$$
$$+ \sum_{m,n=1}^{M} \xi_m \xi_n \int\int \delta^2(z - Q_m)\delta^2(z' - Q_n) O_{(2)}. \tag{7}$$

Here $\int O_{(0)}$ is action (3); i.e., $\int O_{(0)} = S_{\text{quant}}|_{\theta^+ = \pi = 0}$. The operators $O_{(1)}$ and $O_{(2)}$ are found from (6) as variational derivatives of S_{quant} with respect to θ^+ under the conditions $\pi = 0$ and $\theta^+ = 0$.

The operators $O_{(1)}$ and $O_{(2)}$ are therefore certain combinations of the fields X and θ^- and ghosts. After an integration over the Grassmann variables ξ_m, we find the correlation function of these fields, $\langle\hat O\rangle$, which is an obvious combination of $O_{(1)}$ and $O_{(2)}$. The expectation value is found as a functional integral with the action $S_0 = \int O_{(0)}$, which is given in (3).

4. We will now show that, in contrast with the nontrivial expressions for the amplitudes for the scattering of four and more particles, the renormalization theorems for zero-, one-, two-, and three-point functions are insensitive to the detailed form of the correlation function $\langle\hat O\rangle$.

The vanishing of the zero-, one-, two-, and three-point functions in the Green-Schwarz formalism stems from the existence of four zero modes of the field ϑ^- in action (3). The insertion of the operator $\hat{O}$, however, might in principle absorb the zero modes and lead to a violation of the renormalization theorem. Actually, this is not what happens, and the reason is the global space-time supersymmetry.

Let us decompose our original field θ in two $SO(8)$ spinors: $\theta = \theta^+ + \theta^-$, where $\theta^\pm = \frac{1}{2}\gamma^\mp \gamma^\pm \theta$. We then decompose in the four four-component complex SU(4) spinors $\theta^+ = \eta^+ + \vartheta^+$, $\theta^- = \eta^- + \vartheta^-$. In gauge (2), η^+ and ϑ^+ are absent, and η^- is related to $\eta_{\bar{z}}^-$ in action (3): $\eta_{\bar{z}}^- = \bar{\partial} X^+ \eta^- \gamma^-$. (see Ref. 2 for the details). The four constant zero modes of action (3) in which we are interested here are the modes of the four-component field ϑ^- (for type $p = 1$, $\eta_{\bar{z}}^-$ also has four constant zero modes, but this is not true for $p \geqslant 2$). Renormalization theorems are equivalent to the assertion that action (6) has the same constant zero modes ϑ^- as (3) has.

Action (6) is equal to the sum of action (1), where θ^+ has been replaced by (5), and a contribution which depends on the ghost fields and which arises in the course of the quantization process. Our purpose here is to show that with a suitable choice of the fields X the action given by (6) does not change when we shift the fields ϑ^- ($\delta\vartheta^- = \epsilon^- = \text{const}$) without affecting the other fields.

Before gauges are fixed, the theory has a global space-time supersymmetry $\delta\vartheta^- = \epsilon^- = \text{const}$, $\delta X^\mu = i\bar{\epsilon}^- \gamma^\mu \theta$, i.e., $\delta X^+ = 0$, $\delta X^- = i\bar{\epsilon}^- \gamma^- \eta^-$,

$$\delta X^i = i\bar{\epsilon}^- \gamma^i \eta^+ \quad (\text{and } \delta\vartheta^+ = \delta\eta^+ = \delta\eta^- = 0) .$$

Within the path integral we can always introduce a shift:

$$X^\mu \to \tilde{X}^\mu , \; \tilde{X}^+ \equiv X^+ , \; \tilde{X}^- \equiv X^- - i\bar{\vartheta}^- \gamma^- \eta^- ; \; \tilde{X}^i = - i\bar{\vartheta}^- \gamma^i \eta^+ .$$

The Jacobian of this transformation is unity. The action is now invariant under a shift $\delta\vartheta^- = \epsilon^- = \text{const}$, and we have $\delta\tilde{X}^\mu = 0$, $\delta\vartheta^+ = \delta\eta^+ = \delta\eta^- = 0$. In the space of fields there are accordingly some well-defined constant harmonics,

$$\vartheta_0^- = \text{const}, \; \tilde{X}_0^\mu = 0, \; \vartheta_0^+ = 0, \; \eta_0^+ = 0, \; \eta_0^- = 0, \; \text{ghost fields} = 0, \quad (10)$$

which are obviously orthogonal with respect to all the other harmonics and do not contribute to the action. They are accordingly genuine zero modes, and their number is exactly four, since ϑ has four components. (The action remains invariant under the transformations

$$\delta\eta^- = \zeta^- = \text{const}, \; \delta X^\mu = i\bar{\zeta}^- \gamma^\mu \theta , \quad \delta\vartheta^+ = \delta\eta^+ = \delta\vartheta^- = 0,$$

but they act in a nontrivial way on X and thus do not necessarily lead to the existence of zero modes: The mode $\eta^- = \text{const}$, $\tilde{X}^\mu = \text{const}\gamma^\mu\theta$ is not necessarily orthogonal with respect to the other harmonics of the Laplace equation.)

5. We believe that a splitting of θ^- into SU(4) components, $\theta^- = \eta^- + \vartheta^-$, is a necessary procedure since for an arbitrary type p there are only four constant zero modes θ^- (not eight, as in the case $p = 1$). The presence of four zero modes tells us immediately that the zero- and the one-point functions do not have quantum corrections, since they contain fewer than two vertices (since each pair of vertices generally contributes four powers of θ). A more detailed study of this question with allowance for U(1) symmetry, i.e., with the replacement of ϑ^- by η^-, leads to the conclusion that the first nonzero matrix element does indeed contain at least four vertices. It has thus been established that the leading loops in the case $p \geqslant 1$ do not contain quantum corrections to the zero-, one-, two-, and three-point functions.

We regard this discussion as a completely clear and reliable derivation of renormalization theorems for a heterotic string in any number of loops. We believe that this paper confirms the applicability of the Green-Schwarz formalism to a real analysis of multiloop expressions.

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