Low-energy Lagrangian for pseudoscalar mesons and the linear σ model

I. V. Andreev and M. M. Tsypin
Institute of Physics, Academy of Sciences of the USSR

(Submitted 8 April 1988)
Pis'ma Zh. Eksp. Teor. Fiz. **47**, N. 11, 548–550 (10 June 1988)

The linear $SU(3)\times SU(3)$ σ model is shown to describe satisfactorily the low-energy interactions of pseudoscalar mesons if the corrections taking into account the vector-meson exchange are incorporated. Scalar mesons with a mass of about 1500 MeV arise in this theory.

Linear σ model based on chiral symmetry and a partial axial current conservation was proposed many years ago.[1,2] It turned out, however, that it does not describe accurately enough the low-energy interactions of pseudoscalar mesons.[3] We will show here that in the low-energy limit this model describes satisfactorily the experimental data even at the tree level if a correction term describing the vector-meson exchange is added to its Lagrangian. We will therefore find the coefficients in the low-energy expansion of the Lagrangian of the model and we will compare them with the values obtained from the analysis of the experimental data.[4]

The most general renormalizable Lagrangian of the linear $SU(3)\times SU(3)$ σ model has the form[2]

$$L = \frac{1}{4}\mathrm{tr}(\partial_\mu M^+ \partial^\mu M) - \frac{1}{4}\mu^2 \mathrm{tr}(M^+ M) - \kappa \mathrm{Re}\det M$$

$$-\frac{1}{8}(\lambda_1 - \frac{1}{3}\lambda_2)(\mathrm{tr} M^+ M)^2 - \frac{1}{8}\lambda_2 \mathrm{tr}((M^+ M)^2) - \frac{f}{4}\mathrm{tr}(\chi M + M^+ \chi). \qquad (1)$$

Here $M = \lambda_0(s_0 + ip_0) + \lambda_a(s_a + ip_a)$ is a 3×3 matrix which describes nine scalar s_0 and a_a fields and nine psuedoscalar p_0 and p_a fields, λ_a, $a = 1...8$ are Gell-Mann matrices, $\lambda_0 = \sqrt{2/3}\ I$, and $f = 93$ MeV is the vacuum expectation value of s_0. The term with $\det M$ breaks the axial $U(1)$ symmetry explicitly and gives the mass of the η' meson, $m_\eta^2 \approx 6\kappa f + 2/3 m_K^2$. The last term in (1) breaks that symmetry explicitly and accounts for the nonzero mass of the octet of the pseudoscalar mesons.

Lagrangian (1) gives the masses of the scalar mesons

$$m_{s0}^2 = 6\lambda_1 f^2 - 2\kappa f + O(m_K^2), \quad m_{sa}^2 = 2\lambda_2 f^2 + 4\kappa f + O(m_K^2). \qquad (2)$$

If $\lambda_1, \lambda_2, \to \infty$, then $m_{s0}, m_{sa} \to \infty$ and model (1) will reduce to a nonlinear σ model. Here $M = -fU$, where U is a unitary matrix, and the Lagrangian can be written in the form

$$L = (f^2/4)\mathrm{tr}(\partial_\mu M^+ \partial^\mu M) + \kappa f^3 \mathrm{Re}\det U + (f^2/4)\mathrm{tr}(\chi M + M^+\chi) \cdot \qquad (3)$$

We will consider the case in which the scalars have large masses (much larger than those of the pseudoscalars and their momenta) but not infinite masses. The scalar-exchange diagrams in this case would account for a contribution on the order of $1/m_{sa}^2$ to the amplitudes of the scattering of the pseudoscalars by each other, consistent with the correction terms on the order of $1/m_{sa}^2$ in (3). To determine these terms, we will write M in the form $M = -fU(I + \Delta)$, where Δ is a small Hermitian correction to the unit matrix. Substituting it into the equation of motion for $M + M^+$ which is derived from (1), we find

$$m_{s0}^2 \mathrm{tr}\,\Delta \approx \mathrm{tr}[\partial_\mu U^+ \partial^\mu U + \frac{1}{2}(U^+\chi + \chi U - 2\chi)] - 6\kappa f(1 - \mathrm{Re}\det U),$$

$$m_{sa}^2 \langle \Delta \rangle \approx \langle \partial_\mu U^+ \partial^\mu U + \frac{1}{2}(U^+\chi + \chi U)\rangle, \qquad (4)$$

where $\langle ... \rangle$ denotes the traceless part. We will follow the same procedure with the μ' meson.[4] We write $U = V\exp(i\lambda_0\varphi_0)$, $\det V = 1$. Substituting this notation into the equation of motion for $M - M^+$, we find

$$\varphi_0 \approx \frac{i}{2\sqrt{6}m_\eta^2}, \mathrm{tr}(\chi V - V^+\chi). \qquad (5)$$

Substituting (4) and (5) into (1), we obtain a low-energy Lagrangian for the octet of pseudoscalar mesons with the corrections on the order of $1/m_{sa}^2$ and $1/m_\eta^2$. We add to this Lagrangian a correction on the order of $1/m_V^2$, which describes the contribution from the exchange of vector mesons with a mass m_V. This correction is of the Skyrme type[5]

$$\Delta L = \frac{1}{2}\gamma\,\mathrm{tr}([V^+\partial_\mu V,\ V^+\partial_\nu V][V^+\partial^\mu V,\ V^+\partial^\nu V]) . \tag{6}$$

Here $\gamma = 1/16g^2 \approx 1.7\times 10^{-3}$, where $g \approx 6$ is a coupling constant of the vector and pseudoscalar mesons.

As a result, we obtain a low-energy Lagrangian of the form

$$\begin{aligned} L =& (f^2/4)(1-2\delta)\,\mathrm{tr}(\partial_\mu V^+\partial^\mu V) + (f^2/4)(1-\delta)\mathrm{tr}(\chi V + V^+\chi) \\ &+ (\alpha + \gamma/2)\mathrm{tr}(\partial_\mu V^+\partial^\mu V)\,\mathrm{tr}(\partial_\nu V^+\partial^\nu V) + \gamma\,\mathrm{tr}(\partial_\mu V^+\partial^\nu V)\mathrm{tr}(\partial^\mu V^+\partial_\nu V) \\ &+ (\beta - 3\gamma)\,\mathrm{tr}(\partial_\mu V^+\partial^\mu V\partial_\nu V^+\partial^\nu V) + \alpha\,\mathrm{tr}(\partial_\mu V^+\partial^\mu V)\mathrm{tr}(\chi V + V^+\chi) \\ &+ \beta\,\mathrm{tr}(\partial_\mu V^+\partial^\mu V(\chi V + V^+\chi)) + (\alpha/4)\mathrm{tr}(\chi V + V^+\chi)\mathrm{tr}(\chi V + V^+\chi) \\ &+ \epsilon\,\mathrm{tr}(\chi V - V^+\chi)\mathrm{tr}(\chi V - V^+\chi) + (\beta/4)\mathrm{tr}(\chi V\chi V + \chi V^+\chi V^+) , \end{aligned} \tag{7}$$

which takes into account terms up to fourth power in the momenta and masses of the pseudoscalars. The first two terms correspond to the standard nonlinear σ model [Lagrangian (3)] and the remaining eight terms are the corrections. Here we have introduced the constants

$$\begin{aligned} &\alpha = (f^2/6)(1/m_{so}^2 - 1/m_{sa}^2) . \qquad \beta = f^2/2m_{sa}^2 , \\ &\delta = \frac{2}{3m_{so}^2}\,\mathrm{tr}\,\chi = \frac{2}{3m_{so}^2}(2m_K^2 + m_\pi^2), \quad \epsilon = -f^2/48m_\eta^2 . \end{aligned} \tag{8}$$

The numerical values of the coefficients of the eight correction terms,[4] obtained from the analysis of the experimental data, in the order of their appearance in (7) are: $L_1 = 0.9 \pm 0.3$, $L_2 = 1.7 \pm 0.7$, $L_3 = 4.4 \pm 2.5$, $L_4 = 0 \pm 0.5$, $L_5 = 2.2 \pm 0.5$, $L_6 = 0 \pm 0.3$, $L_7 = -0.4 \pm 0.15$, and $L_8 = 1.1 \pm 0.3$ (all values are given in units of 10^{-3}). The values $L_1 - L_6$ are consistent with the following choice of parameters

$$\alpha = (0 \pm 0.5)\cdot 10^{-3}, \quad \beta = (2.2 \pm 0.5)\times 10^{-3}. \tag{9}$$

For the coefficients L_7 and L_8 of the terms quadratic in the pseudoscalar square mass, Lagrangian (7) gives values which are underestimated by a factor of 2. Lagrangian (7) with parameters (9) thus satisfactorily describes low-energy interactions of π, K, and η mesons. The masses of scalar mesons in this case should be $m_{s0} \approx m_{sa} = 1450 \pm 150$ MeV.

The width of the Γ decay of such scalar mesons into two pseudoscalars, on the basis of theory (1) in the tree approximation, is estimated to be $\Gamma > m_s$, so that these mesons most likely would not be seen as resonances. We note that the coefficients L_1, L_2, and L_3 can be calculated directly from QCD,[6,7] yielding $\alpha = 0$ and $\beta = \gamma = N_c/192\pi^2 \approx 1.6 \times 10^{-3}$, where $N_c = 3$ denotes the number of colors. In the framework of our model, this result corresponds to the scalar meson mass $m_{sa} = m_{s0} = 4\sqrt{2}\pi f = 1650$ MeV.

[1]M. Gell-Mann and M. Levy, Nuovo Cim. **16**, 705 (1960).
[2]B. W. Lee, Chiral Dynamics, New York, 1972.
[3]J. Gasser and H. Leutwyler, Ann. Phys. **158**, 142 (1984).
[4]J. Gasser and H. Leutwyler, Nucl. Phys. **B250**, 465 (1985).
[5]I. J. R. Aitchison, C. M. Fraser, and P. J. Miron, Phys. Rev. **D33**, 1994 (1986).
[6]A. A. Andrianov and Yu. V. Novozhilov, Phys. Lett. **B153**, 422 (1985).
[7]P. Simic, Phys. Rev. Lett. **55**, 40 (1985).

Translated by S. J. Amoretty