# Magnetic-dipole band spectrum of a ferromagnetic superlattice

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The dependence of the magnetic-dipole oscillation frequencies on the quasiwave vector is evaluated for magnetic superlattices—magnet layers which are separated by nonmagnetic layers.

Studies dealing with the properties of magnetic superlattices—layered structures—which are comprised of a magnetic material and a nonmagnetic material have recently received considerable attention.[1,2] An interest in these superlattices stems from the promise they hold in present-day microelectronics and their use as tools in the observation of various physical phenomena.

We will consider here the linear *rf* properties of a one-dimensional superlattice consisting of a ferromagnetic dielectric and 2 nonmagnetic dielectric (Fig. 1). The spectrum of bulk and surface excitations of an infinite plate has been known for a long time.[3,4] Two layers of a ferromagnet which are separated by a nonmagnetic gap were analyzed in Refs. 5–7. We will show here that a band pattern of excitation spectra, instead of a discrete set of solutions typical of single plates, arises in a superlattice, i.e., a system consisting of very many layers.

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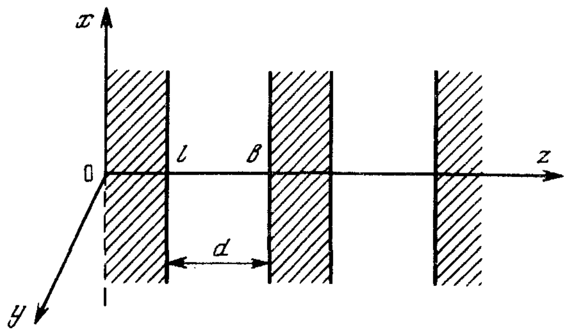


FIG. 1. Superlattice structure. $b = l + d$ is the superlattice period, $l$ is the layer thickness of the magnet, and $d$ is the distance between the layers.

A complete system of equations describing the natural oscillations of the magnetic field consists of magnetostatics equations, layer-boundary conditions, and the Bloch condition:

$$\varphi(z+b) = e^{ikb}\varphi(z), \tag{1}$$

where $\varphi$ is a scalar magnetic potential, and $k$ is a quasiwave vector $[-(\pi/b) \leqslant k \leqslant \pi/b]$. The magnetic susceptibility is given by the Landau-Lifshitz equation in which the dissipation and spatial dispersion are disregarded. The following conditions must, however, be satisfied in this case:

$$\lambda_D, \quad \frac{c}{\omega} \gg \lambda \gg \sqrt{\frac{J}{\mu M_0}}\, a\ . \tag{2}$$

Here $\lambda$ is the wavelength, $\lambda_D$ is the damping length of the wave, $c$ is the velocity of light, $\mu_0$ is the magnetization saturation, and $\sqrt{(J/\mu M_0)a}$ is the exchange length ($a$ is the lattice constant, $J$ is the exchange integral, and $\mu = g\hbar$ is the Bohr magneton).

The solution inside the superlattice cell (inside the magnet) is

$$\varphi_m^{(i)} = e^{i\vec{\kappa}\vec{\rho}}(A_m e^{iqz} + B_m e^{-iqz}), \tag{3}$$

and the solution outside the magnet is

$$\varphi_m^{(e)} = e^{i\vec{\kappa}\vec{\rho}}(C_m e^{\kappa z} + D_m e^{-\kappa z}), \tag{4}$$

where $m$ is the cell number, and $\vec{\rho}$ and $\vec{\kappa}$ are the two-dimensional vectors with the components $x$, $y$ and $\kappa_x \kappa_y$, respectively. The vector $\vec{\kappa}$ is assumed to be the specified parameter of the problem: the lattice parameter.

The excitation spectrum of the system is different for different orientations of the layers with respect to the static magnetic field $\mathbf{H}_0$ inside the magnet. If the field is parallel to the $z$ axis of the superlattice, the magnet layers have only the bulk waves,[4] i.e., the wave vector $q$ cannot be an imaginary vector. If the field is perpendicular to the $z$ axis, in addition to the bulk waves, the Damon-Esbach[3] surface waves can arise at the layer boundaries.

Let us first consider the case in which the field and the anisotropy axis are parallel to the $z$ axis. The dispersion relation which relates $q$ to $\kappa$ and $k$ is

$$\cos kb = \cosh \kappa d \cos ql - \frac{q^2 - \kappa^2}{2\kappa q} \sinh \kappa d \sin ql .$$

In other words, $q = q_{n\varkappa}(k)$, where $n = 1,2...$ is the index of the solution of Eq. (4). The parity of Eq. (4) with respect to $q$ gives rise to a double degeneracy of the solution. Because of the isotropy in the plane of the plate, the solution does not depend on the direction of the vector $\vec{\kappa}$. The frequency of the natural magnetostatic oscillations of the system—the functions $q$ and $k$ for a given $\vec{\kappa}$ and a given solution index—is

$$\omega^2_{n\kappa}(k) = \Omega_0 \left[ \Omega_0 + 4\pi g M_0 \frac{\kappa^2}{\kappa^2 + q^2_{n\kappa}(k)} \right] , \tag{5}$$

where $\Omega_0 = gH_0$. Because of the dependence of $q_{n\varkappa}$ on $k$, the discrete frequencies (for a fixed value of $\varkappa$), which are characteristic of an insulated plate, change to *nonoverlapping zones.* Equation (5) implies that all of the frequencies lie inside the interval from $\Omega_0$ to $\sqrt{\Omega_0(\Omega_0 + 4\pi g M_0)}$, which corresponds to the interval for the bulk spin waves inside an unbounded homogeneous magnet with a zero wave vector.

Analysis of Eq. (4) shows that if the plates are spaced a considerable distance apart ($\varkappa d \gg 1$), the discrete values of $q^0_{n\varkappa}$ and $\omega_{n\varkappa}$ obtained in Ref. 4 yield exponentially narrow bands (the analog of the "strong coupling" in the band theory):

$$q_{n\kappa}(k) = q^0_{n\kappa}\left(1 - 2e^{-\kappa d} \frac{\cos kb - e^{-\kappa d}\cos q^0_{n\kappa} l}{q^0_{n\kappa} l \pm 1} \sin q^0_{n\kappa} l\right) . \tag{6}$$

If $\pi n \leqslant |q^0_{n\kappa} l/2| \leqslant \pi(n + \frac{1}{2})$, the sign is plus and if $\pi(n + \frac{1}{2} \leqslant |q^0_{n\kappa} l/2| \leqslant \pi(n+1)$ the sign is minus.

Of greater interest is the case in which the values of $d$ are small. The value $d = 0$ means that there are no nonmagnetic layers and that the sample becomes homogeneous. The quasiwave vector $k$ in this case becomes an ordinary wave vector, and $\kappa^2/(\kappa^2 + q^2)$ is $\sin^2\theta$, where $\theta$ is the angle between $\vec{\kappa}$ and $\mathbf{M}_0$. If the $\kappa d \neq 0$ but is much less than unity, from (4) we have

$$\cos kb = \cos ql - \frac{1}{2}\left(qd - \frac{\kappa}{q}\kappa d\right)\sin ql . \tag{7}$$

We see (Fig. 2) that a "weak coupling" occurs only for moderately large values of $q$. With an increase in the solution index, there is a transition to "strong coupling." The narrow allowed bands are separated by wide band gaps.

Let us now consider the case in which the external magnetic field and the easy magnetization axis lie in the plane of the plate and are directed along the $x$ axis (Fig. 1). The wave vector $q$ now can be both real and imaginary. A real $q$ corresponds to bulk excitations, while an imaginary $q$ corresponds to surface excitations at the layer

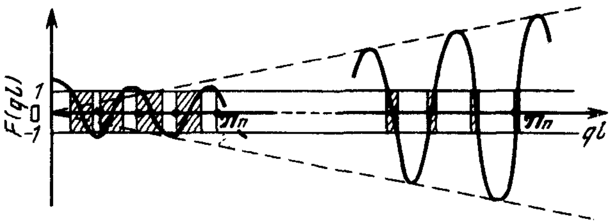


FIG. 2. The right side of Eq. (7) plotted as a function of $ql$. The domain of definition of $\cos kb$ "cuts out" the allowed bands of $ql$ (the hatched parts).

boundaries. For a real $q$ the analog of Eq. (4) is

$$\cos kb = \cosh \kappa d \cos ql - \frac{1}{2\kappa q}\left\{q^2 - \kappa^2 - \frac{4\pi g M_0}{\Omega_0}(\kappa^2 + q^2)\frac{\kappa_y^2}{\kappa_x^2}\right\}\sinh \kappa d \sin ql\,, \quad (8)$$

and for an imaginary[8] $q(q \equiv i\gamma)$ it is

$$\cos kb = \cosh \kappa d \cosh \gamma l + \frac{1}{2\kappa\gamma}\left\{\gamma^2 + \kappa^2 + \frac{4\pi g M_0}{\Omega_0}(\kappa^2 - \gamma^2)\frac{\kappa_y^2}{\kappa_x^2}\right\}\sinh \kappa d \sinh \gamma l. \quad (9)$$

Equation (8) has an infinite set of solutions of $q_{n\vec{\kappa}}(k)$ for the given components $\kappa_x$ and $\kappa_y$. The frequencies of the intrinsic bulk magnetostatic oscillations are

$$\omega^2_{n\vec{\kappa}}(k) = \Omega_0\left[\Omega_0 + 4\pi g M_0 \frac{\kappa_y^2 + q^2_{n\vec{\kappa}}(k)}{\kappa^2 + q^2_{n\vec{\kappa}}(k)}\right]. \quad (10)$$

For specified $\vec{\kappa}$ and $k$ Eq. (9) has a unique solution $\gamma = \gamma_{\vec{\kappa}}(k)$. The wave frequency corresponding to it is

$$\omega^2_{\vec{\kappa}}(k) = \Omega_0\left[\Omega_0 + 4\pi g M_0 \frac{\kappa_y^2 - \gamma^2_{\vec{\kappa}}(k)}{\kappa^2 - \gamma^2_{\vec{\kappa}}(k)}\right]. \quad (11)$$

For large values of $d$, $\gamma^0_{\vec{\kappa}}$ for a single plate (and, hence, $\omega^0_{\vec{\kappa}}$) extends, by analogy with (6), to an exponentially narrow band. For arbitrary value of $d$ the dependence of $\gamma$ on $k$ and $\kappa$ can be found from an exact solution of Eq. (9). The wave which propagates perpendicular to $\mathbf{H}_0$ ($\kappa_x = 0$) is clearly identifiable. In this case we have $\gamma = \pm k_y$. The band with respect to the frequency still forms:

$$\omega^2 = \Omega_0[\Omega_0 + 4\pi g M_0] + \frac{(4\pi g M_0)^2}{2}\,\frac{\sinh|\kappa_y|d \sinh|\kappa_y|l}{\cosh|\kappa_y|b - \cos kb}. \quad (12)$$

The properties of the bulk excitations which arise with $\mathbf{H}_0 \perp z$ are similar to the case in which $\mathbf{H}_0 \| z$.

In analyzing the expressions which we have obtained here it should be kept in mind that the restricting inequalities (2) do not allow too large values of $\kappa$ and $q$, i.e., large $n$, to be analyzed.

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